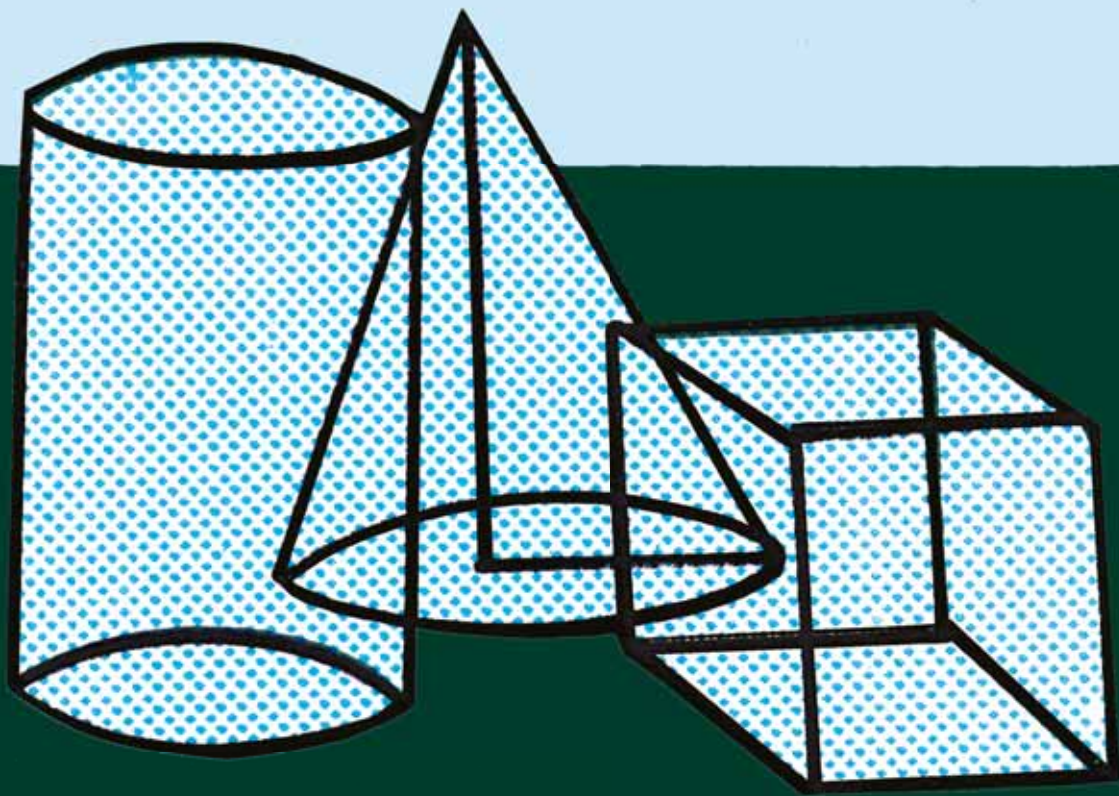


GANIT PRAKASH

Class X



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ভারতের সংবিধান

প্রস্তাবনা

আমরা, ভারতের জনগণ, ভারতকে একটি সার্বভৌম সমাজতান্ত্রিক ধর্মনিরপেক্ষ গণতান্ত্রিক সাধারণতন্ত্র রূপে গড়ে তুলতে সত্যনিষ্ঠার সঙ্গে শপথ গ্রহণ করছি এবং তার সকল নাগরিক যাতে: সামাজিক, অর্থনৈতিক ও রাজনৈতিক ন্যায়বিচার; চিন্তা, মতপ্রকাশ, বিশ্বাস, ধর্ম এবং উপাসনার স্বাধীনতা; সামাজিক প্রতিষ্ঠা অর্জন ও সুযোগের সমতা প্রতিষ্ঠা করতে পারে এবং তাদের সকলের মধ্যে ব্যক্তি-সম্মত ও জাতীয় ঐক্য এবং সংহতি সুনিশ্চিত করে সৌভ্রাতৃত্ব গড়ে তুলতে; আমাদের গণপরিষদে, আজ, 1949 সালের 26 নভেম্বর, এতদ্বারা এই সংবিধান গ্রহণ করছি, বিধিবদ্ধ করছি এবং নিজেদের অর্পণ করছি।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens : JUSTICE, social, economic and political; LIBERTY of thought, expression, belief, faith and worship; EQUALITY of status and of opportunity and to promote among them all – FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation; IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

Fundamental Rights and Fundamental Duties of Citizens of India

Fundamental Rights (Article 14-35 of Indian Constitution)

1. Right to Equality

- The law of the country considers all citizens equally and protects everybody equally.
- The State shall not discriminate against any citizen on grounds only of religion, race, caste, sex, place of birth or any of them.
- There shall be equality of opportunity for all citizens in matters relating to employment or appointment to any office under the State.
- Abolition of untouchability is proclaimed and prohibited.
- Prohibition of acceptance and use of all titles except military and academic.

2. Right to Freedom

- Freedom of speech and expression.
- Freedom to assemble peacefully without arms.
- Freedom to form associations/unions/cooperative societies.
- Freedom to move freely throughout the territory of India.
- Freedom to reside in any part of the country.
- Right to carry on any trade or profession/occupation.
- The convicted person will be punished in the existing law.
- A person cannot be convicted for the same offence more than once.
- No person accused of an offence shall be compelled by the State to bear witness against himself.
- Right to life and personal liberty.
- No person can be arrested without any valid reason and the arrested person should be given the scope to defend oneself.

3. Right against Exploitation

- Prohibition of labour without payment or buying and selling of any person.
- Prohibition of employment of children below the age of 14 in any hazardous industry or factories or mines.

4. Right to Freedom of Religion

- Freedom of conscience, the freedom to profess, practice, and propagate religion to all citizens.
- Right to form and maintain institutions for religious and charitable intents and acquire the immovable and movable property.
- There can be no taxes, the proceeds of which are directly used for the promotion and/or maintenance of any particular religion/religious denomination.

- No religious instruction shall be provided in State-run educational institutions and no religious worship cannot be imparted against the consent of the student in such institutions.

5. Cultural and Educational Rights

- All citizens of our country have the right to protect, preserve, and propagate their language, script and culture.
- The State shall not deny admission into educational institutes maintained by it or those that receive aids from it to any person based on race, religion, caste or language.
- All religious and linguistic minorities have the right to establish and administer educational institutions of their choice.

6. Right to Constitutional Remedies

- A person can move to Supreme Court and High Court if he/she wants to get his/her fundamental rights protected.

Fundamental Duties in India **(Article 51A of Indian Constitution)**

1. Abide by the Indian Constitution and respect its ideals and institutions, the National Flag and the National Anthem.
2. Cherish and follow the noble ideals that inspired the national struggle for freedom.
3. Uphold and protect the sovereignty, unity and integrity of India.
4. Defend the country and render national service when called upon to do so.
5. Promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities and to renounce practices derogatory to the dignity of women.
6. Value and preserve the rich heritage of the country's composite culture.
7. Protect and improve the natural environment including forests, lakes, rivers and wildlife and to have compassion for living creatures.
8. Develop scientific temper, humanism and the spirit of inquiry and reform.
9. Safeguard public property and to abjure violence.
10. Strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement.
11. Provide opportunities for education to his child or ward between the age of six and fourteen years.

PREFACE

In the year 2011, the Government of West Bengal had constituted an 'Expert Committee' in the field of school Education to renew and reconsider the school curriculum, syllabii and textbooks following the guidelines of National Curriculum Framework 2005 and the Right to Education Act 2009. This book is a result of honest endeavour and tireless effort by the experts of the concerned subject in the committee.

This book has been designed according to the syllabus of class X and has been named 'Ganitprakash'. This book follows the principles of studying Mathematics as a language so that students when exposed to problems on the language of Mathematics, can easily understand which Mathematical process, formula or method needed to be applied in solving the problem.

Arithmetic, Algebra and Geometry sections have been explained in such a simple and lucid language so that all pupils can grasp the subject of Mathematics easily. The scope of Mathematics as an effective instrument in successfully solving a pupil's personal, family and different kinds of social problems has been emphasized.

On behalf of the board, I sincerely express my gratitude and thank all eminent teachers, educationists, subject experts and well known illustrators who have put in sincere effort to prepare this important book.

The West Bengal Board of Secondary Education remains obliged and pay thanks to Hon'ble Education Minister Dr. Partha Chatterjee, Government of West Bengal, Department of Education, W.B; West Bengal Right to Education Commission and Paschimbango Sarbosiksha Mission in making this project a success.

Hope this book 'Ganitprakash' published by Government of West Bengal will play an important role in making science related topics attractive and interesting. This will also help in raising the standard of the study of Mathematics at Madhyamik level and provide great impetus to the students. In this way, the Board intends to fulfil its responsibility towards society.

I humbly request all educationists, teachers and others interested in education to bring to our notice, with an open mind, any mistake they find in the book so that the same can be rectified in the next edition. This will improve the book and help the student community at large. There is a saying in English, "even the best can be bettered." We will gladly accept all concrete criticism and feedback for the betterment of the quality of the book from all interested in and attached to the field of education.

December – 2015
77/2Park Street
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Kalyanmoy Ganguly

President

West Bengal Board of Secondary Education

FOREWORD

The honourable Chief Minister of West Bengal, Smt Mamata Banerjee constituted an 'Expert Committee' in the field of school education in 2011. This committee was given the responsibility to review, reconsider and revise the syllabii and textbooks. Based on the recommendations of the committee, curriculum, syllabii and textbooks have been prepared. From the very onset, we have followed the guidelines and vision of the National Curriculum Framework 2005 and the Right to Education Act 2009 (RTE Act, 2009) from this year according to the new Syllabii and curriculum of class X, the textbook has been published.

The Mathematics book at the upper Primary level is called 'Ganitprakash'. The students are guided how to solve the Mathematical problems step by step. For the benefits of the students in every sphere, the basic concepts have been presented in a lucid language and supplemented with 'Hands on Trial' activities. All care and effort had been taken to make the book interesting and engaging. We have kept in mind the student's ability of application while compiling the book. Hope this book will be whole heartedly accepted by the student community. It is necessary to say that, from class I to class X, according to the new syllabii and curriculum, the different Concept and Exercises have been placed in order in this textbook, as the pupils will be gradually higher to the upwards easily by following this book and be competent.

Selected educationists, teachers and subject experts have prepared the book within a very short time. The sole controller of Madhyamik Examination, West Bengal Board of Secondary Education has obliged us by approving the book. The West Bengal Board of Secondary Education, Education Department of West Bengal Government, 'Paschimbango Sarsbosiksha Misson', West Bengal Right to Education Commission have provided us with great help on several instances. We sincerely thank them.

Respected Minister of Education, Dr. Partha Chatterjee has obliged us with his valuable suggestions and comments. We express our gratitude to him.

For the improvement of the quality of the book, the advice and suggestions of all who love education, are welcome.

December – 2015

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Syllabus

1. Quadratic equation in one variable

- i) Concept of quadratic equation in one variable
- ii) Concept of quadratic equation in one variable $ax^2+bx+c=0$ (a, b, c are real numbers and $a \neq 0$)
- iii) Solution of quadratic equation with the help factorization. (Roots are rational numbers.)
- iv) Solution of quadratic equation by expressing perfect square.
- v) Concept of Sridhara Acharyya's formula.
- vi) Concept about the nature of roots.
- vii) Concept of construction of a quadratic equation in one variable if roots are known.
- viii) Solution of real problems of quadratic equation in one variable.

2. Simple Interest

- i) Concept of principal, interest, rate of interest in percent per annum, amount, time.
- ii) Concept of the formula ($I = \frac{prt}{100}$)
- iii) Concept of solution of different real problems.

3. Theorems related to circle.

- i) In the same circle or in equal circles, equal chords intercept equal arcs and subtend equal angles at the centre (Proof is not necessary)
- ii) In the same circle or in equal circles, the chords which subtend equal angles at the centre are equal (proof is not necessary).
- iii) One and only one circle can be drawn through three non-collinear points. (Proof is not necessary)
- iv) If a line drawn from the centre of any circle bisects the chord, which is not a diameter, will be a perpendicular on the chord— proof.
- v) A perpendicular drawn from the centre of a circle on a chord, which is not a diameter, bisects the chord - proof.
- vi) Application of above statements.

4. Rectangular Parallelopiped or Cuboid

- i) Concept of the things of the shape of rectangular parallelopiped and cube which are seen in real life.
- ii) Concept of the number of the surfaces, edges, vertices and diagonals.
- iii) Concept of formation of formula of total surface area.
- iv) Concept of formation of formula of volume.
- v) Concept of formation of formula of the length of a diagonal.
- vi) Concept of solution of different real problems.

5. Ratio and proportion

- i) Concept of ratio and proportion in Algebra.
- ii) Concept of different types of ratio and proportion
- iii) Concept of application of different proportional properties in the problems related to proportion

6. Compound Interest (upto 3 years) and uniform rate of increase or decrease

- i) Concept of difference in simple interest and compound interest.
- ii) Concept of formation of formula if the compound interest is given yearly, half-yearly and quarterly.
- iii) Concept of solution of different real problems.

- iv) Concept of formula formation of uniform rate of increase or decrease from the formula of compound interest.
- v) Concept of solution of real problems.

7. Theorems related to angles in a circle

- i) Concept of angle subtended at the centre and in the circle
- ii) The angle subtended at the centre by an arc is twice that of an angle subtended in the circle— proof
- iii) In any circle, angles in the same segment are equal—proof.
- iv) Angle in a semicircle is a right-angle — proof.
- v) If a straight line segment makes equal angles at the two points situated on the same side of it, then the four points are concyclic. (proof is not necessary)
- vi) Application of above statements.

8. Right Circular Cylinder

- i) Concept of right circular cylinders which are seen in real life.
- ii) Concept of curved surface and plane surface of a right circular cylinder.
- iii) Concept of formula formation of curved surface area.
- iv) Concept of formula formation of total surface area.
- v) Concept of formula of volume.
- vi) Concept of solution of real problems of different types.

9. Quadratic Surd

- i) Concept of irrational numbers.
- ii) Concept of quadratic Surds.
- iii) Concept of pure, mixed, like and unlike quadratic Surds
- iv) Concept of rationalising factor
- v) Concept of rationalising factor of denominator.
- vi) Concept of addition, subtraction, multiplication and division of quadratic surds.
- vii) Concept of solution of different real problems of quadratic surds.

10. Theorems related to cyclic quadrilateral

- i) The opposite angles of a cyclic quadrilaterals are supplementary to each other— proof
- ii) If the opposite angles of a quadrilateral are supplementary to each other, then the vertices of quadrilateral are concyclic— (Proof is not necessary).
- iii) Application of above statements.

11. Construction : Construction of circumcircle and incircle of a triangle.

- i) Construction of circumcircle of a given triangle.
- ii) Construction of incircle of a given triangle.
- iii) Construction of a circle about a given triangle (proof is not included in Evaluation)

12. Sphere

- i) Concept of a solid with the shape of sphere and hemisphere which are seen in real life.
- ii) Concept of surfaces of sphere and hemisphere.
- iii) Concept of curved surface area of a sphere
- iv) Concept of curved surface area and total surface area of a hemisphere.
- v) Concept of volumes of sphere and hemisphere.
- vi) Concept of solution of different real problems.

13. Variation

- i) Concept of simple variation, inverse variation and compound variation.
- ii) Concept of different problems related to variation, inverse variation and solution of real problems.

14. Partnership Business

- i) Concept about partnership business
- ii) Concept of simple and mixed partnership business.
- iii) Concept about principal.
- iv) Concept of distribution of dividend
- v) Application of ratio in different real problems related to partnership business.

15. Theorems related to Tangent to a circle.

- i) Concept of tangent and transversal of a circle.
- ii) The tangent and the radius passing through the point of contact are perpendicular to each other — proof
- iii) If two tangents are drawn from an external point, then the two line segments joining external point and point of contact are equal and they make equal angles at the centre— proof.
- iv) Concept of direct common tangent and transverse common tangent.
- v) If two circles touch each other, then two centres of two circles and point of contact are collinear—proof.
- vi) Application of above statements.

16. Right circular cone.

- i) Concept of right circular conical solids which are seen in real life.
- ii) Concept of curved surface and plane surface of a right circular cone.
- iii) Concept of curved surface area of a right circular cone.
- iv) Concept of total surface area of a right circular cone.
- v) Concept of volume of a right circular cone.
- vi) Solution of different real problems.

17. Construction : Construction of tangent to a circle.

- i) Concept of construction of tangent of a circle to a point on the circle.
- ii) Concept of construction of two tangents to a circle from an external point.

18. Similarity

- i) Concept of similar geometric figures.
- ii) A line drawn parallel to any side of a triangle divides other two sides or extended two sides proportionally (proof is not necessary.)
- iii) If any straight line divides two sides or extended two sides of a triangle proportionally, then the straight line will be parallel to third side. (proof is not necessary)
- iv) If two triangles are similar, their corresponding sides are proportional (proof is not necessary)
- v) If the sides of two triangles are proportional then their corresponding angles are equal. (proof is not necessary)
- vi) In two triangles, if an angle of one is equal to an angle of the other and the adjacent sides of the angles are proportional, then the two triangles are similar. (proof is not necessary)
- vii) If in a right angled triangle, a perpendicular is drawn from its angular point to its hypotenuse, then the two triangles obtained are similar with original triangle and they are similar to each other— proof
- viii) Applications of above statements.

19. Problems related to different solid objects.

- i) Solution of real problems related to different solid objects (rectangular parallelepiped, right circular cylinder, sphere, hemisphere, right circular cone)

20. Trigonometry : concept of measurement of angle.

- i) Evolution, growth and explanation of necessity of trigonometry in reality.
- ii) Concept of positive and negative angles.
- iii) Concept of measurement of angle.
- iv) Concept of sexagesimal system and circular system, concept of their relations and application in different problems.

21. Construction : Determination of mean proportional.

- i) Determination of mean proportional of two line segments in geometric method.
- ii) Construction of a square whose area is equal to a rectangle.
- iii) Construction of a square with area equal to a triangle.

22. Pythagoras theorem

- i) Pythagoras theorem – proof.
- ii) Converse of Pythagoras theorem – proof.
- iii) Applications of above theorem.

23. Trigonometric Ratios and Trigonometric Identities.

- i) Concept of different trigonometric ratios with respect to a right angled triangle.
- ii) Concept of relations among different trigonometric ratios.
- iii) Determination of the values of trigonometric ratios of some standard angles (0° , 30° , 45° , 60° , 90°) and concept of applications in different problems.
- iv) Concept of applications of trigonometric ratios in different problems.
- v) Concept of elimination of an angle (viz. θ) from trigonometric ratios.

24. Trigonometric Ratios of complementary angle

- i) Concept of complementary angle.
- ii) Concept of trigonometric ratios of a complementary angle of an angle and concept of solution of different problems.

25. Application of Trigonometric Ratios : Heights and Distances

- i) Concept of angle of elevation and angle of depression.
- ii) Concept of solution of real problems by trigonometric method with the help of right angled triangle, angle of elevation and angle of depression.

26. Statistics : Mean, Median, Ogive, Mode.

- i) Concept of measures of central tendency.
- ii) Concept of average or mean.
- iii) Concept of three methods for determination of mean (a) direct method, (b) short method (c) standard deviation.
- iv) Concept of needs of determination of median.
- v) Concept of the formula required to determine median and concept of solution of different real problems.
- vi) Concept of cumulative frequency curved line or ogive.
- vii) Concept of determination of median from ogive.
- viii) Necessity for determination of mode.
- ix) Concept of determination of formula for mode and concept of solution of different real problems.
- x) Concept of relations among mean, median and mode.

Marks distribution of first summative Evaluation

(Summative-I)

Subject	MCQ	SA	LA**	Total Marks
Arithmetic	2 (1×2)	2 (2×1)	5 (5×1)	9
Algebra	2 (1×2)	2 (2×1)	10 (3+4+3)	14
Geometry	2 (1×2)	4 (2×2)	5 (5×1)	11
Mensuration	-	2 (2×1)	4 (4×1)	6
Total Marks	6	10	24	40
	6 + 10 = 16			

** L.A.

Internal assessment : 10 Marks

Arithmetic		
(i) Simple interest (ii) Compound interest (iii) Uniform rate of increase or decrease	} ——— 1 out of 2 questions : 5×1 marks = 5 marks	
Algebra		
(i) Solution of Quadratic equation in one variable _____ (ii) Application of quadratic equation in real problems [Construction of equation and solution] _____ (iii) Ratio and proportion } (iv) Quadratic Surd }		1 out of 2 questions : 3×1 marks = 3 marks 1 out of 2 questions : 4×1 marks = 4 marks 1 out of 2 questions : 3×1 marks = 3 marks
Geometry		
(i) Theorem related to circle (ii) Theorem related to angle on a circle (iii) Theorems related to cyclic quadrilateral	} Theorem _____ 1 out of 2 questions : 5×1 marks = 5 marks	
Mensuration		
(i) Cuboid (ii) Right circular cylinder		} ——— 1 out of 2 questions : 4×1 marks = 4 marks

Marks distribution of second summative Evaluation

(Summative-II)

Subject	MCQ	SA	LA **	Total Marks
Arithmetic	1 (1×1)	-	5 (5×1)	6
Algebra	2 (1×2)	2 (2×1)	3 (3×1)	7
Geometry	2 (1×2)	2 (2×1)	13 (5+5+3)	17
Mensuration	2 (1×2)	4 (2×2)	4 (4×1)	10
Total Marks	7	8	25	40
	7 + 8 = 15			

** L.A.

Internal assessment : 10 Marks

Arithmetic	
(i) Partnership business	_____1 out of 2 questions : 5×1 marks = 5 marks
Algebra	
(i) Variation	} _____1 out of 2 questions : 3×1 marks = 3 marks
(ii) Quadratic equation in one variable	
Geometry	
(i) Theorems related to tangent to a circle	} Theorem _____1 out of 2 questions : 5×1 marks = 5 marks
(ii) Theorems related to similarity	
(iii) Construction of circumcircle and incircle of a triangle	} Construction _____ 1 out of 2 questions : 5×1 marks = 5 marks
(iv) Application	
	_____1 question : 3×1 marks = 3 marks
Mensuration	
(i) Sphere	} _____1 out of 2 questions : 4×1 marks = 4 marks
(ii) Right circular cone	

Marks distribution of Third Summative Evaluation/Selection Test (Summative-III)

Subject	MCQ (1×6)	V S A		S A 10 out of 12 (2×10)	L A **	
		Fill in the blanks 5 out of 6 (1×5)	True or False 5 out of 6 (1×5)			
Arithmetic	1	1	1	4 (2×2)	5 (5×1)	
Algebra	1	1	1	4 (2×2)	9 (3+3+3)	
Geometry	1	1	1	6 (2×3)	13 (5+3+5)	
Trigonometry	1	1	1	4 (2×2)	11 (3+3+5)	
Mensuration	1	1	1	4 (2×2)	8 (4+4)	
Statistics	1	1	1	2 (2×1)	8 (4+4)	
Total Marks	6	5	5	20	54	90
	6 + 5 + 5 + 20 = 36					

** LA

Internal Formative Evaluation : 10 Marks

Arithmetic		
(i) Simple interest	}	1 out of 2 questions : 5×1 marks = 5 marks
(ii) Compound interest and uniform rate of increase or decrease		
(iii) Partnership business		
Algebra		
(i) Quadratic equation in one variable	_____	1 out of 2 questions : 3×1 marks = 3 marks
(ii) Variation	}	1 out of 2 questions : 3×1 marks = 3 marks
(iii) Quadratic Surd		
(iv) Ratio and proportion	_____	1 out of 2 questions : 3×1 marks = 3 marks
Geometry		
		1 out of 2 theorems : 5×1 marks = 5 marks
Application of theorem for the solution of geometric problems–		1 out of 2 questions : 3×1 marks = 3 marks
Construction :		1 out of 2 questions : 5×1 marks = 5 marks
Trigonometry		
(i) Concept of measurement of angle	}	2 out of 3 questions : 3×2 marks = 6 marks
(ii) Trigonometric Ratio and Trigonometric Identities		
(iii) Trigonometric Ratios of complementary angle		
(iv) Application of Trigonometric Ratios:Heights & Distances–		1 out of 2 questions : 5×1 marks=5marks
Mensuration		
(i) Cuboid	}	2 out of 3 questions : 4×2 marks = 8 marks
(ii) Right circular cylinder		
(iii) Sphere		
(iv) Right circular cone		
(v) Problems related to different solid objects		
Statistics		
Mean, Median, Ogive, Mode		2 out of 3 questions : 4×2 marks = 8 marks

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1

QUADRATIC EQUATIONS WITH ONE VARIABLE

Today is Sunday. We cannot play in Dhrubas' garden. Today their garden will be cleaned and coconuts will be plucked from the coconut trees.



- 1 The number of coconuts plucked from each tree is 1 more than the number of coconut trees in the Dhrubas' garden. The total number of coconuts plucked from Dhrubas' garden is 132. How will we get the total number of coconut trees in their garden?

Let there be x coconut trees in Dhrubas' garden

∴ The no. of coconuts plucked from each tree is $x + 1$

∴ The total no. of coconuts = $x(x + 1)$

From the condition, $x(x+1) = 132$

$$\text{or, } x^2 + x = 132$$

$$\text{or, } x^2 + x - 132 = 0 \text{ _____ (i)}$$



The no. of coconut trees in Dhrubas' garden will satisfy the equation (i)

But what is called the equation (i)?

The equation (i) is called a **quadratic equation in one variable** with real co-efficients

Because, the equation in the form of $ax^2+bx+c=0$, where a, b, c are real numbers and $a \neq 0$ is called a quadratic equation in one variable.

- 2 The length of the Dhrubas' rectangular garden is 5meters more than its breadth and the area of the garden is 204 sq.m. . Let me see, how the breadth of the garden can be found out.

Let the breadth of the garden be x m.

∴ The length of the garden is $(x + 5)$ m.

∴ The area of the garden = $x(x + 5)$ sq. m.

By the condition, $x(x+5) = 204$

$$\text{or, } x^2+5x-204 = 0 \text{ _____ (ii)}$$



The breadth of the garden satisfies the equation (ii) and the equation (ii) is a [Linear / quadratic] equation

[Let me write it myself]

3 Each worker, who was cleaning Dhrubas' garden has been given 30 times more money (in ₹) than the number of workers. If ₹ 1080 is given in total, then let me construct the equation to get the number of persons cleaning the garden.

Let x persons were cleaning the Dhrubas' garden.

∴ Each would get = $x \times ₹ 30 = ₹ 30x$

∴ ₹ $30x \times x = ₹ 30x^2$ is given in total.

∴ According to the condition, $30x^2 = 1080$

$$\text{or, } x^2 = 36$$

$$\text{or, } x^2 - 36 = 0 \quad \text{_____ (iii)}$$

The number of persons, who cleaned the garden, will satisfy the equation (iii) and the equation (iii) is a [linear/quadratic] equation. **[Let me write it myself]**

Application : 1. I see whether the following equations can be written in the form of $ax^2+bx+c=0$, where a, b, c are real numbers and $a \neq 0$.

(i) $(x+1)(x+3) - x(x+2) = 15$ (ii) $x^2 - 3x = 5(2-x)$ (iii) $x - 1 + \frac{1}{x} = 6$ ($x \neq 0$)

(iv) $(x-2)^2 = x^3 - 4x + 4$ (v) $(x-3)^3 = 2x(x^2-1)$

(i) $(x+1)(x+3) - x(x+2) = 15$

or, $x^2 + x + 3x + 3 - x^2 - 2x = 15$

or, $2x + 3 - 15 = 0$

or, $2x - 12 = 0$

or, $x - 6 = 0$ _____ (i)

The form of the equation (i) is not like $ax^2+bx+c=0$ [where a, b, c are real numbers and $a \neq 0$]

∴ the given equation is not a quadratic equation.

∴ though some equations seem to be quadratic equations apparently, it may not always be quadratic equations.

(ii) $x^2 - 3x = 5(2-x)$

or, $x^2 - 3x = 10 - 5x$

or, $x^2 - 3x + 5x - 10 = 0$

or, $x^2 + 2x - 10 = 0$ _____ (ii)

The equation (ii) is like $ax^2+bx+c=0$ [a, b, c are real numbers and $a \neq 0$]

∴ the given equation is a quadratic equation.

(iii) $x - 1 + \frac{1}{x} = 6$

or, $\frac{x^2 - x + 1}{x} = 6$

or, $x^2 - x + 1 = 6x$

or, $x^2 - 7x + 1 = 0$ _____ (iii)

The equation (iii) is a quadratic equation. **[Let me write the reasons by understanding myself].**



$$\begin{aligned} \text{(iv)} \quad (x-2)^2 &= x^3 - 4x + 4 \\ \text{or, } x^2 - 4x + 4 &= x^3 - 4x + 4 \\ \text{or, } -x^3 + x^2 &= 0 \end{aligned}$$



Whether the equation (iv) is a quadratic equation I write it by understanding myself and show the reason. **[Let me do it myself]**

I write by understanding myself if the equation (v) is a quadratic equation. **[Let me write it myself]**

Application : 2. We, the students of class IX and X have deposited ₹ 1824 in the chief Minister's relief fund. As a subscription, each student of class IX and X has given ₹ 1 and paise 50 respectively equal in number of students of their respective classes. If the number of students of class IX exceeds the number of students of class X by 8, then let me find the quadratic equation in one variable.

Let, the number of students of class X = x

∴ The number of students of class IX will be $(x + 8)$

The students of class IX have given ₹ $(x+8) \times 1 \times (x+8) = ₹ (x+8)^2$

The students of class X have given paise $x \cdot 50$. $x = ₹ \frac{1}{2} \cdot x = ₹ \frac{x^2}{2}$

According to the condition, $\frac{x^2}{2} + (x + 8)^2 = 1824$

$$\text{or, } \frac{x^2 + 2(x + 8)^2}{2} = 1824$$

$$\text{or, } x^2 + 2(x^2 + 16x + 64) = 3648$$

$$\text{or, } x^2 + 2x^2 + 32x + 128 = 3648$$

$$\text{or, } 3x^2 + 32x - 3520 = 0 \quad \text{--- (i)}$$

∴ The equation (i) is a quadratic equation.



Application : 3. The length of a rectangular area exceeds its breadth by 36m. Its area is 460 sqm. Let me construct a quadratic equation in one variable from the statement and determine the co-efficients of x^2 , x and x^0 .

Let, the breadth = x m.

∴ The length = $(x+36)$ m. and area = $x(x+36)$ sqm.

According to the condition, $x(x+36) = 460$

$$\text{or, } x^2 + 36x = 460$$

$$\text{or, } x^2 + 36x - 460 = 0 \quad \text{--- (i)}$$

The equation (i) is a required quadratic equation in one variable, here the co-efficient of x^2 is 1, co-efficient of x is 36 and co-efficient of x^0 is -460.

Let me construct the quadratic equation in another way and see what I shall get.

Let, the length of the rectangular area = x m. ∴ the breadth = $(x-36)$ m.

Therefore, area = $x(x-36)$ sqm.

According to the question, $x(x-36) = 460$

$$\text{or, } x^2 - 36x - 460 = 0 \quad \text{--- (i)}$$

∴ The equation (i) is a required quadratic equation in one variable. Here, co-efficient of x^2 is

, co-efficient of x is and co-efficient of x^0 is | **[Let me write it myself]**



Application : 4. The length of a rectangular area is 2 m. more than its breadth and its area is 24 sqm., I construct a quadratic equation in one variable. [Let me do it myself]

Application : 5. I express the equation $x^3 - 4x^2 - x + 1 = (x+2)^3$ in the form of a general quadratic equation and write the co-efficients of x^2 , x & x^0 .

$$x^3 - 4x^2 - x + 1 = (x+2)^3$$

$$\text{or, } x^3 - 4x^2 - x + 1 = x^3 + 6x^2 + 12x + 8$$

$$\text{or, } -10x^2 - 13x - 7 = 0$$

$$\text{or, } 10x^2 + 13x + 7 = 0$$

∴ co-efficient of x^2 is 10, co-efficient of x is and co-efficient of x^0 is [Let me do it myself]



Let us work out 1.1

- Which of the following polynomial is a quadratic polynomial :
(i) $x^2 - 7x + 2$ (ii) $7x^5 - x(x+2)$ (iii) $2x(x+5) + 1$ (iv) $2x - 1$
- Which of the following equations can be written in the form of $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, let us write it.
(i) $x - 1 + \frac{1}{x} = 6, (x \neq 0)$ (ii) $x + \frac{3}{x} = x^2, (x \neq 0)$ (iii) $x^2 - 6\sqrt{x} + 2 = 0$ (iv) $(x-2)^2 = x^2 - 4x + 4$
- Let us determine the power of the variable in which the equation $x^6 - x^3 - 2 = 0$ will become a quadratic equation?
- Let us determine the value of 'a' for which the equation $(a-2)x^2 + 3x + 5 = 0$ will not be a quadratic equation.
 - If $\frac{x}{4-x} = \frac{1}{3x}, (x \neq 0, x \neq 4)$ be expressed in the form of $ax^2 + bx + c = 0 (a \neq 0)$, then let us determine the co-efficient of x .
 - Let us express $3x^2 + 7x + 23 = (x+4)(x+3) + 2$ in the form of the quadratic equation $ax^2 + bx + c = 0 (a \neq 0)$
 - Let us express the equation $(x+2)^3 = x(x^2 - 1)$ in the form of $ax^2 + bx + c = 0 (a \neq 0)$ and write the co-efficients of x^2, x and x^0 .
- Let us construct the quadratic equations in one variable from the following statements.
 - Divide 42 into two parts such that one part is equal to the square of the other part.
 - The product of two consecutive positive odd numbers is 143.
 - The sum of the squares of two consecutive numbers is 313.
- Let us construct the quadratic equations in one variable from the following statements.
 - The length of the diagonal of a rectangular area is 15 m. and the length exceeds its breadth by 3 m.
 - One person bought some sugar at ₹ 80. If he would get 4 kg. more sugar with that money, then the price of 1 kg. of sugar would be less by ₹ 1.
 - The distance between two stations is 300 km. A train went to second from first station with uniform velocity. If the velocity of the train could be 5km/hour more, then the time taken by the train to reach the second station would be lesser by 2 hours.

- (iv) A clock seller sold a clock by purchasing it at ₹ 336. The amount of his profit percentage is as much as the amount with which he bought the clock.
- (v) If the velocity of the stream is 2kms/hr, then the total time taken by Ratanmajhi to cover 21 kms in down stream and the same distance in upstream is 10 hours.
- (vi) The time taken to clean out garden by Majid is 3 hours more than Mahim. Both of them together can complete the work in 2 hours.
- (vii) The unit digit of a two digit number exceeds its tens' digit by 6 and the product of two digits is less than the number by 12.
- (viii) There is a road of equal width around a rectangular playground in the outside of it having the length 45 m. and breadth 40 m. and the area of the road is 450 sqm.

4 The number of coconut trees in Dhrubas' garden satisfies this quadratic equation $x^2+x-132=0$ (i) But how the number of coconut trees can be found out?

The left hand side of the quadratic equation $x^2+x-132=0$ is a quadratic polynomial : $x^2+x-132$

I try to factorise the quadratic polynomial $x^2+x-132$.

$$\begin{aligned} x^2+x-132 &= x^2+12x-11x-132 \\ &= x(x+12)-11(x+12) \\ &= (x+12)(x-11) \end{aligned}$$



∴ The quadratic equation $x^2+x-132=0$ can be re-written as $(x+12)(x-11)=0$

∴ If $(x+12)(x-11)=0$, we get

either, $x+12=0$ ∴ $x=-12$

or, $x-11=0$ ∴ $x=11$

∴ $x=11$, or $x=-12$.

But the number of coconut trees can not be negative ∴ $x \neq -12$

∴ There are 11 coconut trees in Dhrubas' garden.

Since, $pq=0$ ∴ $p=0$ or $q=0$, where p, q are real numbers



5 By putting $x=11$ and $x=-12$ in the quadratic equation (i), let us see what I shall get.

$$x^2+x-132=0 \text{ (i)}$$

By putting $x=11$ and $x=-12$ one by one in the left hand side of the quadratic equation (i), I see that, $(11)^2+11-132=0$ and $(-12)^2+(-12)-132=0$

i.e. $x=11$ and $x=-12$ satisfy the quadratic equation (i).

What do we call the two numbers 11 and -12 of the quadratic equation (i)?

11 and -12 are the two roots of the quadratic equation (i). $x=11$ and $x=-12$ are the solutions of the quadratic equation (i).

A real number α , will be a root of the quadratic equation $ax^2+bx+c=0$ [a, b, c are real numbers and $a \neq 0$] if $a\alpha^2+b\alpha+c=0$. i.e. In that case, $x=\alpha$ will satisfy the quadratic equation $ax^2+bx+c=0$.

∴ The zeroes of the quadratic polynomial ax^2+bx+c [a, b, c are real and $a \neq 0$] will be the roots of the quadratic equation $ax^2+bx+c=0$.

Since, the number of zeroes of the quadratic polynomial is $\square$ $[1/2]$, therefore, the number of the roots of the quadratic equation is $\square$ $[1/2]$.

6 I write the length and breadth of Dhrubas' garden by determining the roots of the quadratic equation $x^2+5x-204=0$ ____ (ii)

$$x^2+5x-204=0 \text{ ____ (ii)}$$

$$\text{or, } x^2+17x-12x-204=0$$

$$\text{or, } x(x+17)-12(x+17)=0$$

$$\text{or, } (x+17)(x-12)=0$$

$$\text{Either } x+17=0 \quad \therefore x=-17$$

$$\text{or, } x-12=0 \quad \therefore x=12$$

Since, $x=-17$ and $x=12$ are the general solutions, of the quadratic equation (ii),

therefore, -17 and 12 are the two roots of the quadratic equation (ii).

Since the breadth of the garden cannot be negative, so, $x \neq -17$

$$\therefore x=12;$$

∴ The breadth of the garden is 12 m. and the length of the garden $= (x+5)$ m. = 17 m.

Application : 6. Which of the adjoining are the roots of the quadratic equation $2x^2-5x-3=0$,
(i) 5 (ii) 3 (iii) $-\frac{1}{2}$. I write them by understanding it.

$$2x^2-5x-3=0 \text{ ____ (I)}$$

(i) Putting $x=5$ in the left side of the quadratic equation (I) we get,

$$2.(5)^2-5.5-3=50-25-3=22 \neq 0$$

∴ $x=5$ does not satisfy the equation (I). So, 5 is not the root of the quadratic equation.

(ii) Putting $x=3$ in the left side of the quadratic equation (I), we get,

$$2 \times 3^2 - 5.3 - 3 = 0$$

∴ $x=3$ satisfy the equation (I). So, 3 is the root of the quadratic equation (I).

(iii) Putting $x=-\frac{1}{2}$ in the quadratic equation (I), we see that $-\frac{1}{2}$ is the root of the quadratic equation (I) [Let me do it myself]

Application : 7. For what value of k , 2 will be the root of the quadratic equation $kx^2+2x-3=0$, let me write by calculating it.

Since, 2 is a root of the quadratic equation $kx^2+2x-3=0$,
therefore, $k \times 2^2 + 2.2 - 3 = 0$

$$\text{or, } 4k + 1 = 0$$

$$\text{or, } k = -\frac{1}{4}$$

∴ 2 will be a root of the quadratic equation if $k = -\frac{1}{4}$.

Application : 8. Let me write by calculating, for what value of k , 1 will be a root of the quadratic equation $x^2+kx+3=0$ [Let me do it myself]



Application : 9. I try to solve the following quadratic equations by factorization.

(i) $6x^2 - x - 2 = 0$ (ii) $25x^2 - 20x + 4 = 0$ (iii) $x^2 + 5x = 0$ (iv) $4x^2 - 9 = 0$

(v) $x^2 + (3 - \sqrt{5})x - 3\sqrt{5} = 0$

(i) $6x^2 - x - 2 = 0$ _____ (I)

or, $6x^2 - 4x + 3x - 2 = 0$

or, $2x(3x - 2) + 1(3x - 2) = 0$

or, $(3x - 2)(2x + 1) = 0$

Either, $3x - 2 = 0$ or, $3x = 2 \therefore x = \frac{2}{3}$ or, $2x + 1 = 0$ or, $2x = -1 \therefore x = -\frac{1}{2}$

i.e. $x = \frac{2}{3}$ and $x = -\frac{1}{2}$ are the solution of the quadratic equation (I). $\therefore \frac{2}{3}, -\frac{1}{2}$ are the two roots of the equation (I)

To determine the roots of the quadratic equation $ax^2 + bx + c = 0$ [where a, b and c are real numbers and $a \neq 0$] the quadratic expression $ax^2 + bx + c$ is to be factorised into two linear factors and after each factor equalising to zero, the two roots are determined.

(ii) $25x^2 - 20x + 4 = 0$ _____ (II)

or, $25x^2 - 10x - 10x + 4 = 0$

or, $5x(5x - 2) - 2(5x - 2) = 0$

or, $(5x - 2)(5x - 2) = 0$

Either, $5x - 2 = 0 \therefore x = \frac{2}{5}$ or, $5x - 2 = 0 \therefore x = \frac{2}{5}$

Since two factors are equal, $\therefore x = \frac{2}{5}$ and $x = \frac{2}{5}$ are the solutions of the equation (II)

$\therefore$ We have got two roots, $\frac{2}{5}$ and $\frac{2}{5}$ here the two roots are equal $\therefore \frac{2}{5}$ and $\frac{2}{5}$ are two roots of the equation (II)

(iii) $x^2 + 5x = 0$ _____ (III)

or, $x(x + 5) = 0$

either, $x = 0$ or, $x + 5 = 0 \therefore x = -5$

i.e. $x = 0$ and $x = -5$ are the solutions of the quadratic equation $x^2 + 5x = 0$.

$\therefore 0$ and -5 are the roots of the quadratic equation $x^2 + 5x = 0$.

If c in the quadratic equation $ax^2 + bx + c = 0$ is 0, then one root must be 0.

(iv) $4x^2 - 9 = 0$ _____ (IV)

or, $(2x + 3)(2x - 3) = 0$

Either, $2x + 3 = 0 \therefore x = -\frac{3}{2}$ or, $2x - 3 = 0 \therefore x = \frac{3}{2}$

$x = -\frac{3}{2}$ or, $x = \frac{3}{2}$ are the solutions of the quadratic equation (IV); $-\frac{3}{2}, \frac{3}{2}$ are the two roots of equation (IV)

I determine the roots of the quadratic equation in another way.

$4x^2 - 9 = 0$

or, $4x^2 = 9$ or, $x^2 = \frac{9}{4} \therefore x = \pm \frac{3}{2}$ [$x = \pm \frac{3}{2}$ i.e. $x = +\frac{3}{2}$ and $x = -\frac{3}{2}$]

$\therefore$ I have got the solutions of the equation (IV) as $x = \frac{3}{2}$ and $x = -\frac{3}{2}$; $\therefore -\frac{3}{2}, \frac{3}{2}$ are the two roots of the equation (IV)



$$(v) x^2 + (3 - \sqrt{5})x - 3\sqrt{5} = 0$$

$$\text{or, } x^2 + 3x - \sqrt{5}x - 3\sqrt{5} = 0$$

$$\text{or, } x(x+3) - \sqrt{5}(x+3) = 0$$

$$\text{or, } (x+3)(x-\sqrt{5}) = 0$$

$$\text{Either, } x+3 = 0 \therefore x = -3 \quad \text{or, } x-\sqrt{5} = 0 \therefore x = \sqrt{5}$$

$\therefore$ The two roots are -3 and $\sqrt{5}$

$\therefore$ Both the roots of the quadratic equation in one variable with real co-efficients are not always rational or irrational simultaneously.

Application : 10. Let us solve : $(x+4)(2x-3) = 6$

$$(x+4)(2x-3) = 6$$

$$\text{or, } 2x^2 + 8x - 3x - 12 - 6 = 0$$

$$\text{or, } 2x^2 + 5x - 18 = 0$$

$$\text{or, } 2x^2 + 9x - 4x - 18 = 0$$

$$\text{or, } x(2x+9) - 2(2x+9) = 0$$

$$\text{or, } (2x+9)(x-2) = 0$$

$$\text{Either, } 2x+9 = 0, \therefore x = -\frac{9}{2}$$

$$\text{or, } x-2 = 0, \therefore x = 2$$

$$\therefore x = -\frac{9}{2} \text{ or, } x = 2.$$

i.e. $x = -\frac{9}{2}$ and $x = 2$ are the solutions of the quadratic equation $(x+4)(2x-3) = 6$.

Application : 11. Let us solve the

quadratic equation $\frac{x}{3} + \frac{3}{x} = 4\frac{1}{4}$, ($x \neq 0$)

$$\frac{x}{3} + \frac{3}{x} = 4\frac{1}{4}$$

$$\text{or, } \frac{x^2 + 9}{3x} = \frac{17}{4}$$

$$\text{or, } 4x^2 + 36 = 51x$$

$$\text{or, } 4x^2 - 51x + 36 = 0$$

$$\text{or, } 4x^2 - 48x - 3x + 36 = 0$$

$$\text{or, } 4x(x-12) - 3(x-12) = 0$$

$$\text{or, } (x-12)(4x-3) = 0$$

$$\text{Either, } x-12 = 0, \therefore x = 12$$

$$\text{or, } 4x-3 = 0, \therefore x = \frac{3}{4}$$

$\therefore$ The solutions of the given equation are

$$x = \frac{3}{4} \text{ \& } x = 12$$



I solve the quadratic equation $\frac{x}{3} + \frac{3}{x} = 4\frac{1}{4}$, ($x \neq 0$) in another way.

$$\text{Let, } \frac{x}{3} = a$$

$$\therefore \text{ The given equation will be, } a + \frac{1}{a} = 4\frac{1}{4}$$

$$\text{or, } a + \frac{1}{a} = 4 + \frac{1}{4}$$

$$\text{or, } (a-4) + \frac{1}{a} - \frac{1}{4} = 0$$

$$\text{or, } (a-4) - \frac{1}{4a}(a-4) = 0$$

$$\text{or, } (a-4) \left(1 - \frac{1}{4a}\right) = 0$$

$$\text{Either, } a-4 = 0 \text{ or, } 1 - \frac{1}{4a} = 0$$

$$\text{if } a-4 = 0, \text{ then } a = 4, \therefore \frac{x}{3} = 4 \therefore x = 12$$

$$\text{Again, } 1 - \frac{1}{4a} = 0, \text{ then } \frac{1}{4a} = 1$$

$$\text{or, } 4a = 1 \quad \text{or, } a = \frac{1}{4} \therefore \frac{x}{3} = \frac{1}{4} \therefore x = \frac{3}{4}$$

$\therefore x = \frac{3}{4}$ and $x = 12$ are the solutions of the given equation.

Application : 12. I solve and write the two roots of the quadratic equation $\frac{a}{x-b} + \frac{b}{x-a} = 2$ ($x \neq b, a$).

$$\begin{aligned}\frac{a}{x-b} + \frac{b}{x-a} &= 2 \\ \text{or, } \frac{a}{x-b} - 1 + \frac{b}{x-a} - 1 &= 0 \\ \text{or, } \frac{a-x+b}{x-b} + \frac{b-x+a}{x-a} &= 0 \\ \text{or, } (a+b-x) \left[\frac{1}{x-b} + \frac{1}{x-a} \right] &= 0 \\ \text{or, } (a+b-x) \left[\frac{x-a+x-b}{(x-a)(x-b)} \right] &= 0 \\ \text{or, } (a+b-x) \left[\frac{(2x-a-b)}{(x-a)(x-b)} \right] &= 0\end{aligned}$$



Either, $a+b-x = 0$ or, $\frac{2x-a-b}{(x-a)(x-b)} = 0$

Either, $a+b-x = 0$, $\therefore x = a+b$

or, $\frac{2x-a-b}{(x-a)(x-b)} = 0$, or, $2x-a-b = 0$, $\therefore x = \frac{a+b}{2}$

$\therefore x = a+b$ and $x = \frac{a+b}{2}$ are the solutions of the given quadratic equation and the two roots are $a+b, \frac{a+b}{2}$

Application : 13. I solve and write the two roots of the quadratic equation $\frac{a}{ax-1} + \frac{b}{bx-1} = a+b$, $[x \neq \frac{1}{a}, \frac{1}{b}]$ [Let me do it myself]

Application : 14. I solve the quadratic equation $\frac{x-3}{x+3} - \frac{x+3}{x-3} + 6\frac{6}{7} = 0$ ($x \neq -3, 3$)

$$\begin{aligned}\frac{x-3}{x+3} - \frac{x+3}{x-3} + 6\frac{6}{7} &= 0 \\ \text{or, } a - \frac{1}{a} + 6\frac{6}{7} &= 0 \text{ ————— (i) [putting, } \frac{x-3}{x+3} = a \text{]} \\ \text{or, } \frac{a^2-1}{a} + \frac{48}{7} &= 0 \\ \text{or, } \frac{a^2-1}{a} &= -\frac{48}{7} \\ \text{or, } 7a^2-7 &= -48a \\ \text{or, } 7a^2+48a-7 &= 0 \\ \text{or, } 7a^2+49a-a-7 &= 0 \\ \text{or, } 7a(a+7)-1(a+7) &= 0 \\ \text{or, } (a+7)(7a-1) &= 0\end{aligned}$$



$\therefore$ One of the factors of $(a+7)$ and $(7a-1)$ must be zero.

Either, $a+7 = 0$, $\therefore a = -7$, or, $7a-1 = 0$, $\therefore a = \frac{1}{7}$

Now from $a = -7$ we get, $\frac{x-3}{x+3} = -7$



$$\text{or, } x-3 = -7x-21$$

$$\text{or, } 8x = -18$$

$$\text{or, } x = -\frac{18}{8} \quad \therefore x = -\frac{9}{4}$$

Again from $a = \frac{1}{7}$, we get $\frac{x-3}{x+3} = \frac{1}{7}$

$$\text{or, } 7x-21 = x+3$$

$$\text{or, } 6x = 24 \quad \therefore x = 4$$

$\therefore x = -\frac{9}{4}$ and $x = 4$ are the solutions of the given quadratic equation.

$\therefore$ The two roots of the equation are $-\frac{9}{4}$ and 4.

Other Method : From (i), we get $a - \frac{1}{a} + \frac{48}{7} = 0$

$$\text{or, } a - \frac{1}{a} + 7 - \frac{1}{7} = 0$$

$$\text{or, } a + 7 - \frac{1}{a} - \frac{1}{7} = 0$$

$$\text{or, } (a+7) - \frac{1}{7a}(a+7) = 0$$

$$\text{or, } (a+7) \left(1 - \frac{1}{7a}\right) = 0$$

$\therefore$ One of $(a+7)$ and $\left(1 - \frac{1}{7a}\right)$ must be zero.

Either, $a+7 = 0$, $\therefore a = -7$, or, $1 - \frac{1}{7a} = 0$, $\therefore a = \frac{1}{7}$

Now, if $a = -7$, then $\frac{x-3}{x+3} = -7$ or, $x-3 = -7x-21$

$$\text{or, } 8x = 3-21$$

$$\text{or, } 8x = -18$$

$$\therefore x = -\frac{9}{4}$$

Again, if $a = \frac{1}{7}$, then $\frac{x-3}{x+3} = \frac{1}{7}$ or, $7x-21 = x+3$

$$\text{or, } 6x = 24, \quad \therefore x = 4$$

That is, $x = -\frac{9}{4}$ and $x = 4$ are the solutions of the given quadratic equation. So, the given equation has two roots $-\frac{9}{4}$ and 4.

Application : 15. I solve the quadratic equation $\frac{x+3}{x-3} + \frac{x-3}{x+3} = 2\frac{1}{2}$ ($x \neq -3, 3$) [Lets do it myself]

Let us work out 1.2

1. In each of the following cases, let us justify & write whether the given values are the roots of the given quadratic equation :

(i) $x^2+x+1 = 0$, 1 and -1 (ii) $8x^2+7x = 0$, 0 and -2 (iii) $x + \frac{1}{x} = \frac{13}{6}, \frac{5}{6}$ and $\frac{4}{3}$

(iv) $x^2 - \sqrt{3}x - 6 = 0$, $-\sqrt{3}$ and $2\sqrt{3}$

2. (i) Let us calculate and write the value of k for which $\frac{2}{3}$ will be a root the quadratic equation $7x^2+kx-3 = 0$.

(ii) Let us calculate and write the value of k for which -a will be a root the quadratic equation $x^2+3ax+k = 0$.

3. If $\frac{2}{3}$ and -3 are the two roots of the quadratic equation $ax^2+7x+b=0$, then let me calculate the values of a and b .

4. Let us solve :

- (i) $3y^2-20=160-2y^2$ (ii) $(2x+1)^2+(x+1)^2=6x+47$ (iii) $(x-7)(x-9)=195$
 (iv) $3x-\frac{24}{x}=\frac{x}{3}, x \neq 0$ (v) $\frac{x}{3}+\frac{3}{x}=\frac{15}{x}, x \neq 0$ (vi) $10x-\frac{1}{x}=3, x \neq 0$
 (vii) $\frac{2}{x^2}-\frac{5}{x}+2=0, x \neq 0$ (viii) $\frac{(x-2)}{(x+2)}+6\left(\frac{x-2}{x-6}\right)=1, x \neq -2, 6$
 (ix) $\frac{1}{x-3}-\frac{1}{x+5}=\frac{1}{6}, x \neq 3, -5$ (x) $\frac{x}{x+1}+\frac{x+1}{x}=2\frac{1}{12}, x \neq 0, -1$
 (xi) $\frac{ax+b}{a+bx}=\frac{cx+d}{c+dx} [a \neq b, c \neq d], x \neq -\frac{a}{b}, -\frac{c}{d}$ (xii) $(2x+1)+\frac{3}{2x+1}=4, x \neq -\frac{1}{2}$
 (xiii) $\frac{x+1}{2}+\frac{2}{x+1}=\frac{x+1}{3}+\frac{3}{x+1}-\frac{5}{6}, x \neq -1$ (xiv) $\frac{12x+17}{3x+1}-\frac{2x+15}{x+7}=3\frac{1}{5}, x \neq -\frac{1}{3}, -7$
 (xv) $\frac{x+3}{x-3}+6\left(\frac{x-3}{x+3}\right)=5, x \neq 3, -3$ (xvi) $\frac{1}{a+b+x}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}, x \neq 0, -(a+b)$
 (xvii) $\left(\frac{x+a}{x-a}\right)^2-5\left(\frac{x+a}{x-a}\right)+6=0, x \neq a$ (xviii) $\frac{1}{x}-\frac{1}{x+b}=\frac{1}{a}-\frac{1}{a+b}, x \neq 0, -b$
 (xix) $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}+\frac{1}{(x-3)(x-4)}=\frac{1}{6}, x \neq 1, 2, 3, 4$
 (xx) $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}, x \neq a, b, c$ (xxi) $x^2-(\sqrt{3}+2)x+2\sqrt{3}=0$

Application : 16. My maternal uncle travelled 84 kms. by cycle and observed that if he would be cycling with the speed of 5kms/hr more, then the time taken to complete the journey is reduced by 5 hrs. let me calculate and write the speed of my maternal uncle's journey in km/hr.

Suppose, my maternal uncle travelled with a speed x km/hr.

According to the condition,

$$\frac{84}{x} - \frac{84}{x+5} = 5$$

or,

$$\frac{84(x+5) - 84x}{x(x+5)} = 5$$

or,

$$\frac{84x + 420 - 84x}{x^2+5x} = 5$$

or,

$$5(x^2+5x) = 420$$

or,

$$x^2+5x = 84$$

or,

$$x^2+5x-84 = 0$$

or,

$$x^2+12x-7x-84 = 0$$

or,

$$x(x+12)-7(x+12) = 0$$

or,

$$(x+12)(x-7) = 0$$

Either, $x+12=0, \therefore x=-12$

or, $x-7=0, \therefore x=7$

But here $x \neq -12$, because the speed can not be negative.

$\therefore x=7$

$\therefore$ My maternal uncle travelled with a speed 7 kms/hr.



Application : 17. My friend Ajay has written a two digit number in his copy of which the tens digit is less by 4 than the unit digit. If the product of the two digits is subtracted from the number, the result is equal to the square of the difference of the two digits of that number. By calculation let us try to write the number which Ajoy can write in his copy.

Let, the unit digit of the two digit number written by Ajay be x ; $\therefore$ the tens digit is $(x-4)$

$\therefore$ The number $= 10(x-4) + x = 11x-40$

According to the given condition, $(11x-40)-x \times (x-4) = \{x-(x-4)\}^2$

$$\text{or, } 11x-40-x^2+4x = 16$$

$$\text{or, } -x^2+15x-56 = 0$$

$$\text{or, } x^2-15x+56 = 0$$

$$\text{or, } x^2-7x-8x+56 = 0$$

$$\text{or, } x(x-7)-8(x-7) = 0$$

$$\text{or, } (x-7)(x-8) = 0$$

Either, $x-7 = 0$, $\therefore x = 7$ or, $x-8 = 0$, $\therefore x = 8$

$\therefore x = 7$ or, $x = 8$

If $x = 7$, then the two digit number will be $= 11 \times 7 - 40 = 37$

If $x = 8$, the two digit number will be $= 11 \times 8 - 40 = 48$

$\therefore$ The required two digit number is 37 or 48

tens	unit
$x-4$	x



Application : 18. The unit digit of a two digit number exceeds the tens digit by 6 and the product of two digits is less by 12 than the number, let us write by calculating the possible unit digit of two digit number. **[Let me do it myself]**

Application : 19. In the Annual sport competition of Andul school, the students stood in a vacuum square with a depth of 6. Consequently, the number of students in the front line would be lesser by 24 if the students were stood in a solid square. By calculating, let us write the number of students.

Let the number of students in front line be x if they stood in a vacuum square

$\therefore$ The total number of students $= x^2 - (x-2 \times 6)^2 = x^2 - (x-12)^2$

Again, if they stood in a solid square, the total number of students $(x-24)^2$

According the given condition, $x^2 - (x-12)^2 = (x-24)^2$

$$\text{or, } x^2 - (x^2 + 144 - 24x) = x^2 - 48x + 576$$

$$\text{or, } -x^2 + 72x - 720 = 0$$

$$\text{or, } x^2 - 72x + 720 = 0$$

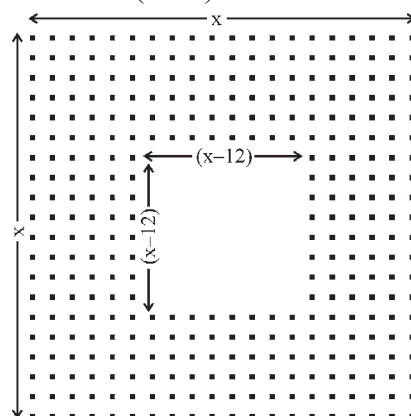
$$\text{or, } x^2 - 60x - 12x + 720 = 0$$

$$\text{or, } (x-60)(x-12) = 0$$

Either, $x-60 = 0$, $\therefore x = 60$

or, $x-12 = 0$, $\therefore x = 12$

$\therefore x = 60$ and $x = 12$



But, here $x \neq 12$ as the number of students in the front line of vacuum square with depth of 6 must be greater than 12. $\therefore x = 60$

$\therefore$ The required number of students $= (x-24)^2 = (60-24)^2 = 36^2 = 1296$

$\therefore$ The number of students is 1296.



Let us work out 1.3

1. The difference of two positive whole numbers is 3 and the sum of their squares is 117; by calculating, let us write the two numbers.
2. The base of a triangle is 18m. more than two times of its height, if the area of the triangle is 360 sq.m., then let us determine the height of it.
3. If 5 times of a positive whole number is less by 3 than twice of its square, then let us determine the number.
4. The distance between two places is 200 km.; the time taken by a motor car for going from one place to another is less by 2 hrs than the time taken by a jeep car. If the speed of the motor car is 5 kms/hr. more than the speed of the jeep car, then by calculating let us write the speed of the motor car.
5. The area of the Amita's rectangular land is 2000sq.m.and perimeter of it is 180 m. . By calculating, let us write the length and breadth of the Amita's land.
6. The tens digit of a two digital number is less by 3 than the units digit. If the product of the two digits is subtracted from the number, the result is 15. Let us write the unit digit of the number by calculation.
7. There are two pipes in a water reservoir of our school. Two pipes together take $11\frac{1}{9}$ minutes to fill the reservoir.If the two pipes are opened separately, then one pipe would take 5 minutes more time than that of the other pipe. Let us write by calculating, the time taken to fill the reservoir separately by each of the pipes.
8. Porna and Pijush together complete a work in 4 days. If they work separately, then the time taken by Porna would be 6 days more than the time taken by Pijush. Let us write, by calculating, the time taken by Porna alone to complete the work.
9. If the price of 1 dozen pen is reduced by `6, then 3more pens will be got for `30. Before the reduction of price, let us calculate the price of 1 dozen pen.

10. **V.S.A.**

(A) **M.C.Q.**

- (i) The number of roots of a quadratic equation is (a) one (b) two (c) three (d) none of them
- (ii) If $ax^2+bx+c=0$ is a quadratic equation, then (a) $b \neq 0$ (b) $c \neq 0$ (c) $a \neq 0$ (d) none of these
- (iii) The highest power of the variable in a quadratic equation is (a) 1 (b) 2 (c) 3 (d) none of these.
- (iv) The equation $4(5x^2-7x+2) = 5(4x^2-6x+3)$ is (a) linear (b) quadratic (c) 3rd degree (d) none of these.
- (v) The root/two roots of the equation $\frac{x^2}{x} = 6$ (a) 0 (b) 6 (c) 0 & 6 (d) -6

(B) **Let us identify the following statements as true or false :**

- (i) $(x-3)^2 = x^2-6x+9$ is a quadratic equation (ii) 5 is the only root of the equation $x^2=25$

(C) **Let us fill in the blanks :**

- (i) If $a = 0$ and $b \neq 0$ in the equation $ax^2+bx+c=0$, then the equation is a equation.
- (ii) If the two roots of a quadratic equation be 1 then the equation is
- (iii) The two roots of the equation $x^2=6x$ are &

11. **S.A.**

- (i) Let us find the value of a, if one root of the equation $x^2+ax+3=0$ is 1
- (ii) Let us write the value of the other root if one root of the equation $x^2-(2+b)x+6=0$ is 2.
- (iii) Let us write the value of the other root if one root of the equation $2x^2+kx+4=0$ is 2.
- (iv) Let us write the equation; if the difference of a proper fraction and its reciprocal is $\frac{9}{20}$.
- (v) Let us write the values of a and b, if the two roots of the equation $ax^2+bx+35=0$ are -5 and -7.

7 Let us see, what is the type of the roots of the quadratic equation $4x^2 + 9 = 0$

$$4x^2 + 9 = 0$$

$$\text{or, } 4x^2 = -9$$

$$\text{or, } x^2 = -\frac{9}{4}$$

But for any real value of x , $x^2 \neq -\frac{9}{4}$. Because, square of any real number can not be negative.

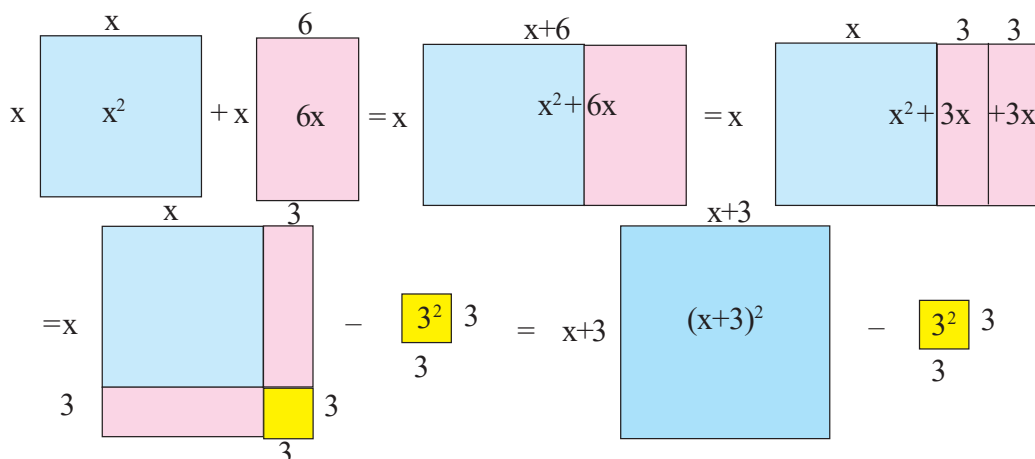
$\therefore$ We observe that, the quadratic equation $4x^2 + 9 = 0$ has no real root.

Now, how can we know when we will get the real roots of a quadratic equation?

At first, we express the quadratic equation $ax^2 + bx + c = 0$ [where a, b, c are real and $a \neq 0$] in the form of $(x+p)^2 - q^2 = 0$ [where p, q are real] and by taking square root we try to know the nature of the two roots.

I express the quadratic equation $x^2 + 6x + 5 = 0$ in the form of $(x+p)^2 - q^2 = 0$ [where p, q are real numbers]

At first, by hands on trial I express $(x^2 + 6x)$ as the difference of two squares.



By hands on trial, let me write what I have got.

$$x^2 + 6x = (x^2 + \frac{6x}{2}) + \frac{6x}{2}$$

$$= x^2 + 3x + 3x = (x+3)x + 3x = (x+3)x + 3x + 3 \times 3 - 3 \times 3 = (x+3)x + (x+3)3 - 3 \times 3 = (x+3)^2 - 3^2$$

$$\therefore x^2 + 6x + 5 = (x+3)^2 - 9 + 5 = (x+3)^2 - 4$$

8 What the method of writing $x^2 + 6x + 5 = 0$ in the form of $(x+3)^2 - 4 = 0$ is called?

It is called the Method of Completing the square.

The quadratic equation $x^2 + 6x + 5 = 0$ can be written as,

$$(x+3)^2 - 9 + 5 = 0$$

$$\text{or, } (x+3)^2 - 4 = 0$$

$$\text{or, } (x+3)^2 = 4$$

$$\text{or, } x+3 = \pm 2$$

$$\text{Either, } x+3 = 2 \quad \therefore x = -1$$

$$\text{or, } x+3 = -2 \quad \therefore x = -5$$

$\therefore$ The two roots of the quadratic equation $x^2 + 6x + 5 = 0$ are -1 and -5 .

9 Let us see, how shall we get the two roots of the quadratic equation $3x^2+x-10=0$ by the method of completing square.

The co-efficient of x^2 in the quadratic equation $3x^2+x-10=0$ is 3, which is not a perfect square number.

∴ By multiplying both sides of the equation $3x^2+x-10=0$ by 3, we get,

$$9x^2+3x-30=0$$

$$\text{Now, } 9x^2+3x-30 = (3x)^2 + 2 \cdot 3x \cdot \frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 30$$

$$= (3x + \frac{1}{2})^2 - \frac{1}{4} - 30 = (3x + \frac{1}{2})^2 - (\frac{1}{4} + 30) = (3x + \frac{1}{2})^2 - \frac{121}{4}$$

$$\therefore \text{ We can write } 9x^2+3x-30=0 \text{ as, } (3x + \frac{1}{2})^2 - \frac{121}{4} = 0$$

$$\text{or, } (3x + \frac{1}{2})^2 = \frac{121}{4} = (\frac{11}{2})^2$$

$$\text{or, } 3x + \frac{1}{2} = \pm \frac{11}{2}$$

$$\text{Either, } 3x + \frac{1}{2} = \frac{11}{2}$$

$$\text{or, } 3x = \frac{11}{2} - \frac{1}{2}$$

$$\text{or, } 3x = \frac{10}{2} \quad \therefore x = \frac{5}{3}$$

$$\text{or, } 3x + \frac{1}{2} = -\frac{11}{2}$$

$$\text{or, } 3x = -\frac{11}{2} - \frac{1}{2}$$

$$\text{or, } 3x = -\frac{12}{2} \quad \therefore x = -2$$

∴ -2 and $\frac{5}{3}$ are the two roots of the quadratic equation $3x^2+x-10=0$

10 By another method, I determine the two roots of the equation $3x^2+x-10=0$, dividing both sides of it by 3.

$$3x^2+x-10=0$$

$$\text{or, } x^2 + \frac{x}{3} - \frac{10}{3} = 0$$

$$\text{or, } x^2 + 2 \cdot x \cdot \frac{1}{2} \cdot \frac{1}{3} + \left(\frac{1}{2} \cdot \frac{1}{3}\right)^2 - \left(\frac{1}{2} \cdot \frac{1}{3}\right)^2 - \frac{10}{3} = 0$$

$$\text{or, } \left(x + \frac{1}{2} \cdot \frac{1}{3}\right)^2 - \left(\frac{1}{2} \cdot \frac{1}{3}\right)^2 - \frac{10}{3} = 0$$

$$\text{or, } \left(x + \frac{1}{6}\right)^2 - \left(\frac{1}{6}\right)^2 - \frac{10}{3} = 0$$

$$\text{or, } \left(x + \frac{1}{6}\right)^2 - \frac{1}{36} - \frac{10}{3} = 0$$

$$\text{or, } \left(x + \frac{1}{6}\right)^2 - \left[\frac{1+120}{36}\right] = 0$$

$$\text{or, } \left(x + \frac{1}{6}\right)^2 = \frac{121}{36}$$

$$\text{or, } \left(x + \frac{1}{6}\right)^2 = \left(\frac{11}{6}\right)^2$$

$$\text{or, } x + \frac{1}{6} = \pm \frac{11}{6}$$

$$\text{Either, } x + \frac{1}{6} = \frac{11}{6} \quad \text{or, } x + \frac{1}{6} = -\frac{11}{6}$$

$$\text{or, } x = \frac{11}{6} - \frac{1}{6} \quad \text{or, } x = -\frac{11}{6} - \frac{1}{6}$$

$$\therefore x = \frac{5}{3}$$

$$\therefore x = -2$$

∴ -2 and $\frac{5}{3}$ are the roots of the quadratic equation $3x^2+x-10=0$



Application : 20. I determine the two roots of the quadratic equation $5x^2+23x+12=0$ by the method of completing the square.

$$5x^2+23x+12=0$$

$$\text{or, } x^2+\frac{23}{5}x+\frac{12}{5}=0$$

$$\text{or, } \left\{x+\frac{1}{2}\left(\frac{23}{5}\right)\right\}^2 - \left\{\frac{1}{2}\left(\frac{23}{5}\right)\right\}^2 + \frac{12}{5}=0$$

$$\text{or, } \left(x+\frac{23}{10}\right)^2 - \left(\frac{23}{10}\right)^2 + \frac{12}{5}=0$$

$$\text{or, } \left(x+\frac{23}{10}\right)^2 - \frac{529}{100} + \frac{12}{5}=0$$

$$\text{or, } \left(x+\frac{23}{10}\right)^2 = \frac{529}{100} - \frac{12}{5} = \frac{529-240}{100} = \frac{289}{100} = \left(\frac{17}{10}\right)^2$$

$$\text{or, } x+\frac{23}{10} = \pm \frac{17}{10}$$

$$\text{Either, } x+\frac{23}{10} = \frac{17}{10} \quad \text{or, } x+\frac{23}{10} = -\frac{17}{10}$$

$$\text{i.e. Either, } x = \boxed{}$$

$$\text{or, } x = \boxed{}$$

[Let me do it myself]

$\therefore \boxed{}$ and $\boxed{}$ are the two roots of the quadratic equation $5x^2+23x+12=0$

Application : 21. By another method, I determine the two roots of the quadratic equation $5x^2+23x+12=0$, multiplying left hand side and right hand side of it by 5 with the help of the method of completing the square. **[Let me do it myself]**

Application : 22. I solve the quadratic equation $2x^2-6x+1=0$ by the method of completing the square.

Multiplying both sides of the equation $2x^2-6x+1=0$ by 2, I get,

$$4x^2-12x+2=0$$

$$\text{or, } (2x)^2-2\cdot 2x\cdot 3+3^2-3^2+2=0$$

$$\text{or, } (2x-3)^2-9+2=0$$

$$\text{or, } (2x-3)^2=7$$

$$\text{or, } 2x-3=\pm\sqrt{7}$$

$$\text{or, } 2x=3\pm\sqrt{7}$$

$$\therefore x = \frac{3\pm\sqrt{7}}{2}$$

$$\therefore \text{ I have got the roots } \frac{3+\sqrt{7}}{2}, \frac{3-\sqrt{7}}{2}$$

$$\therefore x = \frac{3-\sqrt{7}}{2} \text{ and } x = \frac{3+\sqrt{7}}{2} \text{ are the solutions of the given quadratic equation.}$$



Application : 23. I solve the quadratic equation $9x^2+30x+31=0$ by the method of completing the square.

$$9x^2+30x+31=0$$

$$\text{or, } (3x)^2+2\cdot 3x\cdot 5+(5)^2-(5)^2+31=0$$

$$\text{or, } (3x+5)^2-25+31=0$$

$$\text{or, } (3x+5)^2=-6$$

But for any real value of x , $(3x+5)^2$ can not be negative.

$\therefore$ The quadratic equation $9x^2+30x+31=0$ has no real root.



Through the determination of the roots of the quadratic equation by the method of completing the square, we have observed that some quadratic equations have real roots, again some other quadratic equations have no real root.

11 I determine the roots of the quadratic equation $ax^2+bx+c=0$ [a, b, c are real numbers and $a \neq 0$] by the method of completing the square and try to know the nature of the roots.

$$ax^2+bx+c=0 \text{ [} a, b, c \text{ are real numbers and } a \neq 0 \text{]}$$

Dividing both sides by a , I get,

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\text{or, } (x + \frac{b}{2a})^2 - (\frac{b}{2a})^2 + \frac{c}{a} = 0$$

$$\text{or, } (x + \frac{b}{2a})^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\text{or, } (x + \frac{b}{2a})^2 - (\frac{b^2-4ac}{4a^2}) = 0$$

$$\text{or, } (x + \frac{b}{2a})^2 = \frac{b^2-4ac}{4a^2}$$

If $b^2-4ac \geq 0$, then taking square root of both sides, I get,

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$\text{or, } x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\therefore x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$\therefore$ The two real roots of the quadratic equation $ax^2+bx+c=0$ [$a \neq 0$] are

$$\frac{-b + \sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2-4ac}}{2a} \text{ where } b^2-4ac \geq 0$$

But what will happen when $b^2-4ac < 0$

If $b^2-4ac < 0$, then the quadratic equation $ax^2+bx+c=0$ [a, b, c are real and $a \neq 0$] will have no real root.



In this phase, we shall solve those problems of quadratic equations, where $b^2 - 4ac \geq 0$;
We have got, two roots of the quadratic equation $ax^2 + bx + c = 0$ [a, b, c are real and $a \neq 0$], one is $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and other is $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$

But what the general formula of determining the roots of any quadratic equation in one variable is called?

This general formula of determining the roots of any quadratic equation in one variable is called Sridhara Acharyya's method.

A famous mathematician Sridhara Acharyya (cr. 750 approx) of Ancient India discovered this method. For this reason, we call the general formula of determination of the roots of the quadratic equation as Sridhara Acharyya's formula. He composed different books on arithmetic and algebra by considering them as separate subjects. His contributions to the method of determining square root and cube root and solving the problems by rule of three are found in different books. But sadly though some parts of his original arithmetic book have been found, his book on algebra is yet to be found. We have come to know about the formula of quadratic equation, discovered by him from the reference of this formula in the name of Sridhara Acharyya written in a book by another great mathematician of the next era, second Bhaskaracharya (cr. 1150).



Application : 24. My elder brother has written two such numbers in his copy that one is smaller than the other by 3 and their product is 70; I determine the two numbers which my elder brother has written by constructing the quadratic equation in one variable and with the help of Sridhara Acharyya's formula.

Let, one number be x

$\therefore$ Other number is $(x-3)$

According to the condition

$$x(x-3) = 70$$

$$\text{or, } x^2 - 3x - 70 = 0 \text{ ————— (I)}$$

For using Sridhara Acharyya's formula, comparing equation (I) with

$$ax^2 + bx + c = 0 \text{ [} a \neq 0 \text{]} \text{ I get,}$$

$$a = 1, b = -3 \text{ and } c = -70$$

$\therefore$ From Sridhara Acharyya's formula, I get,

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-3) \pm \sqrt{(-3)^2 - 4.1 \times (-70)}}{2.1} \\ &= \frac{3 \pm \sqrt{9 + 280}}{2} = \frac{3 \pm \sqrt{289}}{2} = \frac{3 \pm 17}{2} \end{aligned}$$



$$\text{Either, } x = \frac{3+17}{2} = \frac{20}{2} = 10$$

$$\text{or, } x = \frac{3-17}{2} = \frac{-14}{2} = -7$$

When $x = 10$, then other number will be $10 - 3 = 7$
and when $x = -7$, then other number will be $-7 - 3 = -10$
 $\therefore$ The two numbers will be 10 and 7 or -7 and -10 .

Application : 25. Let us check whether the two obtained values of x that is, $x = 10$ and $x = -7$ satisfy the quadratic equation (I). **[Let me do it myself]**

If the two values of x satisfy the quadratic equation in one variable, then it can be confirmly said that those values are the solutions or roots of that quadratic equation.

Application : 26. The product of two consecutive positive odd numbers is 143. Let me construct the equation and find out the two odd numbers by applying Sridhara Acharyya's formula.

[Let me do it myself]

Application : 27. There are `195 deposited to a group and each member of the group subscribes money equal to the number of members in that group and then the total money is divided equally among the members. So each one has got `28. Let us determine the number of members in that group by applying Sridhara Acharyya's formula.

Let, the number of members in that group be x .

∴ If each one subscribes `  $x$  then the amount of total money = ` $x \times x = x^2$

The deposit money is ` 195

∴ Total amount of money = ` $(x^2 + 195)$

According to the given condition, $x^2 + 195 = 28 \times x$

or, $x^2 - 28x + 195 = 0$ _____ (I)

For using Sridhara Acharyya's method, comparing (I) with $ax^2 + bx + c = 0$ ($a \neq 0$), we get, $a = 1$, $b = -28$ and $c = \square$

∴ From Sridhara Acharyya's formula, we get,

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-28) \pm \sqrt{(-28)^2 - 4.1 \times 195}}{2.1} \\ &= \frac{28 \pm \sqrt{\square - 780}}{2} \quad \text{[Let me do it myself]} \\ &= \frac{28 \pm 2}{2} \end{aligned}$$

Either, $x = \frac{28+2}{2} = \square$ or, $x = \frac{28-2}{2} = \square$

∴ The number of members may either be 15 or 13.

Let us justify whether $x = 15$ and $x = 13$ satisfy the equation (I) or not **[Let me do it myself]**

Application : 28. By applying Sridhara Acharyya's formula, let us write a positive number which is less than its square by 30.

Hints : Let, the number = x

∴ By the condition, $x^2 - x = 30$

With the help of Sridhara Acharyya's formula, we get $x = 6$ or -5 **[Let me do it myself]**

Since the number is positive, so the value -5 is not being accepted

∴ The required number = 6



Application : 29. Meher takes 5 days less than Pritam to complete a piece of work. If by working together they can do the work in 6 days, let us determine in how many days Pritam will alone complete the work by applying Sridhara Acharyya's formula.

Suppose, Pritam alone completes the work in x days.

∴ Meher alone completes the work in $(x-5)$ days.

∴ Pritam does $\frac{1}{x}$ part of work in 1 day.

Meher does $\frac{1}{x-5}$ part of work in 1 day.

Pritam and Meher together complete the work in 6 day.

∴ They together do $\frac{1}{6}$ part of the work in 1 day.

According to the condition, $\frac{1}{x} + \frac{1}{x-5} = \frac{1}{6}$ ————— (I)

$$\text{or, } \frac{x-5+x}{x(x-5)} = \frac{1}{6}$$

$$\text{or, } \frac{2x-5}{x^2-5x} = \frac{1}{6}$$

$$\text{or, } x^2-5x = 12x-30$$

$$\text{or, } x^2-17x+30 = 0 \text{ ————— (II)}$$

The quadratic equation (II) is the simple form of the equation (I). The solution of this equation (II) according to the Sridhara Acharyya's formula will be —

$$x = \frac{-(-17) \pm \sqrt{(-17)^2 - 4 \cdot 1 \cdot 30}}{2 \cdot 1} = \frac{17 \pm \sqrt{289-120}}{2} = \frac{17 \pm \square}{2}$$

$$\text{Either, } x = \frac{17+13}{2} = \square \text{ or, } x = \frac{17-13}{2} = \square$$

∴ $x = 15$ or, 2

Here, if $x = 2$, then Pritam will do the work in 2 days. Since Meher takes 5 days less than Pritam to complete the work, So, Meher will complete the work in $(2-5)$ days = -3 days. But the number of days can not be negative in anyway. So, $x \neq 2$.

∴ $x = 15$ i.e. Pritam alone complete the work in 15 days.

Application : 30. If the following quadratic equations have real roots, then let us determine the roots by applying Sridhara Acharyya's formula.

$$(i) x^2-6x+4 = 0 \quad (ii) 9x^2+7x-2 = 0 \quad (iii) x^2-6x+9 = 0 \quad (iv) 2x^2+x+1 = 0$$

$$(v) 1-x = 2x^2 \quad (vi) 2x^2-9x+7 = 0 \quad (vii) x^2-(\sqrt{2}+1)x+\sqrt{2} = 0$$

$$(i) x^2-6x+4 = 0 \text{ ————— (I)}$$

Comparing the quadratic equation (I) with the quadratic equation $ax^2+bx+c=0$ [$a \neq 0$], we get, $a=1$, $b=-6$ and $c=4$

$$\therefore b^2-4ac = (-6)^2-4 \cdot 1 \cdot 4 = 36-16 = 20 > 0$$



∴ The quadratic equation (I) has real roots.

$$\begin{aligned}\text{The roots} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 4}}{2 \cdot 1} \\ &= \frac{6 \pm \sqrt{36 - 16}}{2} \\ &= \frac{6 \pm \sqrt{20}}{2} = \frac{6 \pm \sqrt{4 \times 5}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = \frac{2(3 \pm \sqrt{5})}{2} = 3 \pm \sqrt{5}\end{aligned}$$



∴ $(3 + \sqrt{5})$ and $(3 - \sqrt{5})$ are the two roots of the equation (I).

(ii) $9x^2 + 7x - 2 = 0$ _____ (I)

Comparing the quadratic equation (I) with the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$], we get,
 $a = 9$, $b = 7$ and $c = -2$

$$\therefore b^2 - 4ac = \boxed{} > 0 \quad \text{[Let me do it myself]}$$

The quadratic equation (I) has real roots.

$$\text{The roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{(7)^2 - 4 \cdot 9 \cdot (-2)}}{2 \cdot 9} = \frac{-7 \pm \sqrt{121}}{18} = \frac{-7 \pm 11}{18}$$

$$\text{Either, } x = \frac{-7 + 11}{18} = \frac{4}{18} = \frac{2}{9} \quad \text{or, } x = \frac{-7 - 11}{18} = -1$$

∴ -1 and $\frac{2}{9}$ are the two roots of the equation (I)

(iii) $x^2 - 6x + 9 = 0$ _____ (I)

Comparing the quadratic equation (I) with the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$], we get,
 $a = 1$, $b = -6$ and $c = 9$

$$\therefore b^2 - 4ac = (-6)^2 - 4 \cdot 1 \cdot 9 = 36 - 36 = 0$$

∴ The quadratic equation (I) has real roots,

$$\text{The roots} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-7) \pm \sqrt{0}}{2 \cdot 1} = \frac{6 \pm 0}{2}$$

$$\text{Either, } x = \frac{6 + 0}{2} = 3 \quad \text{or, } x = \frac{6 - 0}{2} = 3$$

∴ The equation (I) has the two roots -3 and 3 .

(iv) $2x^2 + x + 1 = 0$ _____ (I)

Comparing the quadratic equation (I) with the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$], we get,
 $a = 2$, $b = 1$ and $c = 1$

$$\therefore b^2 - 4ac = (1)^2 - 4 \cdot 2 \cdot 1 = -7 < 0$$

∴ The quadratic equation (I) has no real root.

Understanding (v), (vi) & (vii) myself and by applying Sridhara Acharyya's formula, let me solve them and determine the roots.

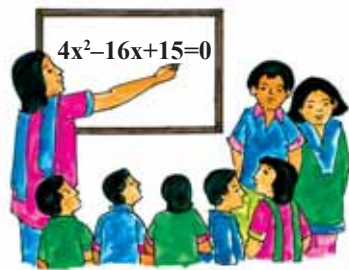


Let us work out 1.4

1. (i) Let us write whether Sridhara Acharyya's formula is applicable to solve the equation $4x^2 + (2x-1)(2x+1) = 4x(2x-1)$
- (ii) Let us write what type of equations can be solved with the help of Sridhara Acharyya's formula.
- (iii) By applying Sridhara Acharyya's formula in the equation $5x^2 + 2x - 7 = 0$, it is found that $x = \frac{k \pm 12}{10}$, let us write by calculating, what will be the value of k.
2. If the following quadratic equations have real roots, then let us determine them with the help of Sridhara Acharyya's formula.

(i) $3x^2 + 11x - 4 = 0$	(ii) $(x-2)(x+4) + 9 = 0$	(iii) $(4x-3)^2 - 2(x+3) = 0$
(iv) $3x^2 + 2x - 1 = 0$	(v) $3x^2 + 2x + 1 = 0$	(vi) $10x^2 - x - 3 = 0$
(vii) $10x^2 - x + 3 = 0$	(viii) $25x^2 - 30x + 7 = 0$	(ix) $(4x-2)^2 + 6x = 25$
3. Let us express the following mathematical problems in the form of quadratic equations with one variable and solve them by applying Sridhara Acharyya's formula or with the help of factorisation.
 - (i) Sathi has drawn a right-angled triangle whose length of the hypotenuse is 6cm. more than the twice of that of the shortest side. If the length of the third side is 2 cm. less than the length of the hypotenuse, then by calculating, let us write the lengths of three sides of the right-angled triangle drawn by Sathi.
 - (ii) If a two digit positive number is multiplied by its unit digit, then the product is 189 and if the tens digit is twice the unit digit, then let us calculate the unit digit.
 - (iii) The speed of Salma is 1m/second more than the speed of Anik. In a 180 m race, Salma reaches 2 seconds before than Anik. Let us write by calculating the speed of Anik in m/sec.
 - (iv) There is a square park in our locality. The area of a rectangular park is 78 sqm. less than the twice of the area of that square-shaped park whose length is 5m. more than the length of the side of that park and the breadth is 3m. less than the length of the side of that park. Let us write by calculating, the length of the side of the square-shaped park.
 - (v) In our village, Proloy babu bought 350 chilli plants for planting in his rectangular land. when he put the plants in rows, he noticed that, if he would put 24 plants more than the number of rows in each row, 10 plants would remain excess. Let us write by calculating the number of rows.
 - (vi) Joseph and Kuntal work in a factory. Joseph takes 5 minutes less time than Kuntal to make a product. Joseph makes 6 products more than Kuntal while working for 6 hours. Let us write by calculating, the number of products Kuntal makes during that time.
 - (vii) The speed of a boat in still water is 8 kms/hr. If the boat can go 15 kms. down stream and 22kms. up stream in 5 hours, then let us write by calculating, the speed of the stream.
 - (viii) A superfast train runs at the speed of 15kms/hr more than that of an express train. Leaving the same station the superfast train reached a station of 180kms. distance 1 hour before than the express train. Let us determine the speed of the superfast train in km/hr.
 - (ix) Rehana went to the market and saw that the price of dal of 1kg. is ` 20 and the price of rice of 1 kg. is ` 40 less than that of price of 1 kg fish. The total quantity of fish and that of dal each in ` 240 is equal to the quantity of rice in ` 280. Let us calculate the cost price of 1 kg. fish.

Today we play a funny game in our school. Each of us will write a quadratic equation on the black board and other friends will justify the existence of real roots of those quadratic equations and calculate the roots in due time and they will inform them.



12 At first, I write on the board $4x^2 - 16x + 15 = 0$

$$4x^2 - 16x + 15 = 0 \text{ (I)}$$

Comparing the quadratic equation (I) with the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$], we get $a=4$, $b=-16$ and $c=15$

$$\therefore b^2 - 4ac = (-16)^2 - 4 \cdot 4 \cdot 15 = 16 > 0$$

$\therefore$ The quadratic equation (I) has real roots.

$\therefore$ The real roots of the equation (I) are and [Let me do it myself]

$\therefore$ We have got that the roots of the equation (I) are real and unequal.



13 Now, Nibedita wrote on the board $4x^2 + 12x + 9 = 0$

$$4x^2 + 12x + 9 = 0 \text{ (II)}$$

Comparing the quadratic equation (II) with the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$], we get $a=4$, $b=12$ and $c=9$

$$\therefore b^2 - 4ac = (12)^2 - 4 \cdot 4 \cdot 9 = 144 - 144 = 0$$

$\therefore$ The quadratic equation (II) has real roots.

$$4x^2 + 12x + 9 = 4x^2 + 6x + 6x + 9 = 2x(2x+3) + 3(2x+3) = (2x+3)(2x+3)$$

$$4x^2 + 12x + 9 = 0$$

$$\therefore (2x+3)(2x+3) = 0$$

$$\therefore \text{The roots are } -\frac{3}{2} \text{ and } -\frac{3}{2}$$

$\therefore$ We have got, the equation (II) has real and equal roots.

14 Now, Prodip has written $4x^2 - 16x + 21 = 0$

$$4x^2 - 16x + 21 = 0 \text{ (III)}$$

Comparing the quadratic equation (III) with the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$], we get $a=4$, $b=-16$ and $c=21$

$$\therefore b^2 - 4ac = (-16)^2 - 4 \cdot 4 \cdot 21 = 256 - 336 = -80 < 0$$

$\therefore$ We will not get any real root of the quadratic equation (III).

15 I determine the roots of the quadratic equation $ax^2 + bx + c = 0$ [a, b, c are real numbers and $a \neq 0$] and try to know the nature of roots of it.

$$ax^2 + bx + c = 0 \text{ [a, b, c are real numbers and } a \neq 0]$$

The two roots of this quadratic equation are $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$



(I) If $b^2 - 4ac = 0$, then we get the two roots as $-\frac{b}{2a}$ and $-\frac{b}{2a}$
that is, the two roots are real and equal.

(II) If $b^2 - 4ac > 0$, then we get the two roots as $\left(-\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right)$ and $\left(-\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right)$

i.e. We get two real and unequal roots.

(III) If $b^2 - 4ac < 0$, then we will not get any real root.

We have understood that the nature of the roots of the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$] depends on the value of $(b^2 - 4ac)$.

16 What do we call $(b^2 - 4ac)$ for the quadratic equation $ax^2 + bx + c = 0$?

As $b^2 - 4ac$ of the quadratic equation $ax^2 + bx + c = 0$ [a, b, c are real numbers and $a \neq 0$] determines the nature of the roots, $b^2 - 4ac$ is called the **Discriminant** of that quadratic equation.

We have got, the two roots of the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$]

(I) real and equal if $b^2 - 4ac = 0$

(II) real and unequal if $b^2 - 4ac > 0$

(III) no real root if $b^2 - 4ac < 0$

The converse statements of (I), (II) and (III) are also true



Application : 31. I determine the nature of the two roots of the following quadratic equations :

(i) $3x^2 + x - 1 = 0$ (ii) $4x^2 - 4x + 1 = 0$ (iii) $x^2 + x + 1 = 0$ (iv) $2x^2 + x - 2 = 0$

(i) Comparing the quadratic equation $3x^2 + x - 1 = 0$ with the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$], we get, $a = 3$, $b = 1$ and $c = -1$

$$\therefore \text{The discriminant} = b^2 - 4ac = (1)^2 - 4 \cdot 3 \cdot (-1) \\ = 13 > 0$$

$\therefore$ The two roots of the given quadratic equation are real and unequal.

$$\begin{aligned} \text{(ii) The discriminant of the quadratic equation } 4x^2 - 4x + 1 = 0 \text{ is } b^2 - 4ac &= (-4)^2 - 4 \cdot 4 \cdot 1 \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

$\therefore$ The two roots of the given quadratic equation are real and equal.

$$\begin{aligned} \text{(iii) The discriminant of the quadratic equation } x^2 + x + 1 = 0 \text{ is } b^2 - 4ac &= (1)^2 - 4 \cdot 1 \cdot 1 \\ &= -3 < 0 \end{aligned}$$

$\therefore$ We will not get any real root of the given quadratic equation.



Let us write by understanding the nature of the roots of the quadratic equation (iv)

Application : 32. Let us write the value of k for which the two roots of the quadratic equation $9x^2+3kx+4=0$ would be real and equal.

Comparing the quadratic equation $9x^2+3kx+4=0$ with the quadratic equation $ax^2+bx+c=0$ [$a \neq 0$], we get, $a=9$, $b=3k$ and $c=4$

Since the two roots are real and equal,

$\therefore$ The discriminant $= 0$

$\therefore b^2-4ac = 0$

i.e. $(3k)^2-4 \cdot 9 \cdot 4 = 0$

or, $9k^2 = 4 \times 9 \times 4$

or, $k^2 = 4 \times 4 \quad \therefore k = \pm 4$

$\therefore$ For $k = \pm 4$, the two roots of the given quadratic equation are real and equal.



Application : 33. By understanding, let us write the value of k for which the two roots of the quadratic equation $2x^2-10x+k=0$ are real and equal. [Let me do it myself]

Application : 34. Let us prove that the quadratic equation $x^2(a^2+b^2)+2(ac+bd)x+(c^2+d^2)=0$ has no real root, when $ad \neq bc$.

$x^2(a^2+b^2)+2(ac+bd)x+(c^2+d^2)=0$ _____ (I)

The discriminant of the quadratic equation (I) $= [2(ac+bd)]^2-4(a^2+b^2)(c^2+d^2)$

$$= 4(ac+bd)^2-4(a^2c^2+b^2c^2+a^2d^2+b^2d^2)$$

$$= 4[a^2c^2+b^2d^2+2acbd-a^2c^2-b^2c^2-a^2d^2-b^2d^2]$$

$$= 4[-(b^2c^2-2acbd+a^2d^2)]$$

$$= -4(bc-ad)^2 < 0$$

[since, $ad \neq bc \Rightarrow bc-ad \neq 0$]

$\therefore$ The quadratic equation (I) has no real root, when $ad \neq bc$

Application : 34. If two roots of the quadratic equation $(1+m^2)x^2+2mcx+(c^2-a^2)=0$ are real and equal, then let us prove that, $c^2 = a^2(1+m^2)$

$(1+m^2)x^2+2mcx+(c^2-a^2)=0$ _____ (I)

The two roots of the quadratic equation (I) are equal.

$\therefore$ The discriminant $= 0$

$\therefore (2mc)^2-4(1+m^2)(c^2-a^2) = 0$

or, $4m^2c^2-4(c^2+c^2m^2-a^2-a^2m^2) = 0$

or, $4m^2c^2-4c^2-4c^2m^2+4a^2+4a^2m^2 = 0$

or, $4c^2 = 4a^2+4a^2m^2$

or, $c^2 = a^2 + a^2m^2$

$\therefore c^2 = a^2(1+m^2)$ [proved]



17 Let us see, what we shall get by addition and multiplication of two roots of the quadratic equation $ax^2+bx+c=0$ [$a \neq 0$]

$$ax^2+bx+c=0 \text{ [a, b, c are real numbers and } a \neq 0] \text{ _____ (I)}$$

Let, two roots of the quadratic equation be α and β

$$\therefore \alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\begin{aligned} \therefore \alpha + \beta &= \frac{-b+\sqrt{b^2-4ac}}{2a} + \frac{-b-\sqrt{b^2-4ac}}{2a} \\ &= \frac{-b+\sqrt{b^2-4ac} - b - \sqrt{b^2-4ac}}{2a} = -\frac{2b}{2a} = -\frac{b}{a} \end{aligned}$$

We have understood that, the sum of the two roots of the quadratic equation = $-\frac{\text{co-efficient of } x}{\text{co-efficient of } x^2}$

$$\begin{aligned} \alpha \times \beta &= \frac{-b+\sqrt{b^2-4ac}}{2a} \times \frac{-b-\sqrt{b^2-4ac}}{2a} \\ &= \frac{(-b)^2 + (\sqrt{b^2-4ac})^2}{4a^2} = \frac{b^2 - (b^2 - 4ac)}{4a^2} = \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a} \end{aligned}$$

We have understood

that, the product of the two roots of the quadratic equation = $\frac{\text{constant term of the equation}}{\text{co-efficient of } x^2}$

Ayan has written a quadratic equation $6x^2-19x-7=0$ on the board.

18 I determine two roots of the quadratic equation written by Ayan and see what I shall get by adding and multiplying two roots of the equation.

$$6x^2-19x-7=0 \text{ _____ (IV)}$$

Comparing the quadratic equation (IV) with the quadratic equation $ax^2+bx+c=0$ [$a \neq 0$], I get, $a=6$, $b=-19$ and $c=-7$

The discriminant of the quadratic equation (IV) = $b^2-4ac = (-19)^2 - 4 \cdot 6 \cdot (-7) > 0$

The quadratic equation (IV) has real roots and the two roots are and

[Let me calculate and write it myself]

$\therefore$ I have got two roots $\frac{7}{2}$ and $-\frac{1}{3}$

$$\therefore \text{ The sum of two roots } = \frac{7}{2} + \left(-\frac{1}{3}\right) = \frac{19}{6} = \frac{-(-19)}{6} = -\frac{\text{co-efficient of } x}{\text{co-efficient of } x^2}$$

$$\text{The product of two roots} = \frac{7}{2} \times \left(-\frac{1}{3}\right) = \frac{-7}{6} =$$

19 I find the sum and the product of two roots of any quadratic equation by determining its roots.

$$\text{The sum of two roots} = -\frac{\text{co-efficient of } x}{\text{co-efficient of } x^2}$$

$$\text{The product of two roots} = \frac{\text{constant term of the equation}}{\text{co-efficient of } x^2}$$

[Let me do it myself]



Application : 36. I determine the sum and product of two roots of the quadratic equations:

(i) $6x^2 - x - 2 = 0$ (ii) $4x^2 - 9x = 100$

(i) $6x^2 - x - 2 = 0$ _____ (I)

The sum of two roots of the equation (I) $= -\frac{-1}{6} = \frac{1}{6}$

The product of two roots $= \frac{-2}{6} = -\frac{1}{3}$



Let me write myself the sum and product of two roots of the quadratic equation (ii)

Application : 37. Let me write, by calculating the value of k for which the roots of the quadratic equation $5x^2 + 13x + k = 0$ are reciprocal to each other.

$5x^2 + 13x + k = 0$ _____ (I)

Let two roots of the equation (I) be α and $\frac{1}{\alpha}$

$$\therefore \alpha \times \frac{1}{\alpha} = \frac{k}{5}$$

$$\text{or, } \frac{k}{5} = 1, \therefore k = 5$$

Application : 38. If one of the roots of the quadratic equation $3x^2 - 10x + 3 = 0$ is $\frac{1}{3}$, then let me determine the other root of it. [Let me do it myself]

Application : 39. If the ratio of two roots of the quadratic equation $ax^2 + bx + c = 0$ [$a \neq 0$] is $1 : r$, then let us show that, $\frac{(r+1)^2}{r} = \frac{b^2}{ac}$

$ax^2 + bx + c = 0$ _____ (I)

Let, two roots of the equation (I) be α and $r\alpha$

$$\therefore \alpha + r\alpha = -\frac{b}{a}$$

$$\text{or, } \alpha(1+r) = -\frac{b}{a}$$

$$\text{or, } \alpha^2(1+r)^2 = \frac{b^2}{a^2} \text{ _____ (II)}$$

Again, $\alpha \times r\alpha = \frac{c}{a}$

$$\alpha^2 r = \frac{c}{a} \text{ _____ (III)}$$

Dividing (II) by (III), we get, $= \frac{a^2(1+r)^2}{a^2 r} = \frac{b^2}{\frac{c}{a}}$

$$\text{or, } \frac{(r+1)^2}{r} = \frac{b^2}{a^2} \times \frac{a}{c} \therefore \frac{(r+1)^2}{r} = \frac{b^2}{ac} \text{ [proved]}$$



Application : 40. If two roots of the quadratic equation $x^2+px+q=0$ are α and β , then let us express the value of $\alpha^3 + \beta^3$ and $\frac{1}{\alpha} + \frac{1}{\beta}$ in terms of p and q .

$$x^2+px+q=0 \text{ (I)}$$

The two roots of the equation (I) are α and β

$$\therefore \alpha + \beta = -p \text{ and } \alpha\beta = q$$

$$\therefore \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = (-p)^3 - 3q(-p)$$

$$= -p^3 + 3pq = 3pq - p^3$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-p}{q} = -\frac{p}{q}$$

Application : 41. If α and β are two roots of the quadratic equation $ax^2+bx+c=0$ [$a \neq 0$], then let us express the value of $\left(\frac{1}{\alpha^3} + \frac{1}{\beta^3}\right)$ in terms of a , b and c . [Let me do it myself]

My friend Shila has done a fun.

She has written on the board — "If two roots are 3 and 4, then what will be the quadratic equation?"

20 If two roots of a quadratic equation are known, then how shall we get that quadratic equation?

That is, we determine the quadratic equation if its roots are α and β .

Let the equation, which has roots α and β be

$$ax^2+bx+c=0 \text{ [} a \neq 0 \text{]} \text{ (I)}$$

$$\therefore ax^2+bx+c=0$$

$$\text{or, } x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \text{ [dividing both sides by } a \text{]}$$

$$\text{or, } x^2 - (\alpha + \beta)x + \alpha\beta = 0 \text{ [} \because \alpha, \beta \text{ are the roots of the equation (I)]}$$

The quadratic equation, which has two roots α and β is,

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

21 Now, I shall make the quadratic equation according to the condition written by Shila on the board.

The quadratic equation having roots 3 and 4 will be

$$x^2 - (3+4)x + 3 \times 4 = 0 \therefore x^2 - 7x + 12 = 0$$

Application : 42. By determining, we are observing that two roots of the quadratic equation $x^2 - 7x + 12 = 0$ are 3 and 4. [Let me do it myself]

Application : 43. If α and β are the roots of the equation $ax^2+bx+c=0$ [$a \neq 0$], then let us determine the quadratic equation which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.

$$ax^2+bx+c=0 \text{ (I)}$$

$$\alpha \text{ and } \beta \text{ are the roots of the quadratic equation (I), } \therefore \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{\left(-\frac{b}{a}\right)^2 - 2 \times \frac{c}{a}}{\frac{c}{a}}$$



$$= \frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}} = \frac{b^2 - 2ac}{a^2} \times \frac{a}{c} = \frac{b^2 - 2ac}{ac}$$



Again, $\frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 1$ ————— (III)

∴ The quadratic equation having roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is given by,

$$x^2 - \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)x + \frac{\alpha}{\beta} \times \frac{\beta}{\alpha} = 0$$

$$\text{or, } x^2 - \left(\frac{b^2 - 2ac}{ac}\right)x + 1 = 0 \quad \therefore acx^2 - (b^2 - 2ac)x + ac = 0$$

Let us work out 1.5

- Let us write the nature of two roots of the following quadratic equations :
 (i) $2x^2 + 7x + 3 = 0$ (ii) $3x^2 - 2\sqrt{6}x + 2 = 0$ (iii) $2x^2 - 7x + 9 = 0$ (iv) $\frac{2}{5}x^2 - \frac{2}{3}x + 1 = 0$
- By calculating, let us write the value(s) of k for which each of the following quadratic equations has real and equal roots —
 (i) $49x^2 + kx + 1 = 0$ (ii) $3x^2 - 5x + 2k = 0$ (iii) $9x^2 - 24x + k = 0$ (iv) $2x^2 + 3x + k = 0$
 (v) $x^2 - 2(5+2k)x + 3(7+10k) = 0$ (vi) $(3k+1)x^2 + 2(k+1)x + k = 0$
- Let us form the quadratic equations from two roots given below —
 (i) 4, 2 (ii) -4, -3 (iii) -4, 3 (iv) 5, -3
- Find m for which the two roots of the quadratic equation $4x^2 + 4(3m-1)x + (m+7) = 0$ are reciprocal to each other.
- If two roots of the quadratic equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, then let us prove that, $2b = a+c$
- If two roots of the quadratic equation $(a^2+b^2)x^2 - 2(ac+bd)x + (c^2+d^2) = 0$ are equal, then let us prove that, $\frac{a}{b} = \frac{c}{d}$
- Let us prove that, the quadratic equation $2(a^2+b^2)x^2 + 2(a+b)x + 1 = 0$ has no real root, if $a \neq b$.
- If two roots of the quadratic equation $5x^2 + 2x - 3 = 0$ are α and β , then let us determine the value of,
 (i) $\alpha^2 + \beta^2$ (ii) $\alpha^3 + \beta^3$ (iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (iv) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$
- If one root of the equation $ax^2 + bx + c = 0$ is twice of the other, then let us show that, $2b^2 = 9ac$.
- Let us form the equation whose roots are reciprocals to the roots of the equation $x^2 + px + 1 = 0$.
- Let us determine the equation whose roots are square of the roots of the equation $x^2 + x + 1 = 0$.

12. **V.S.A.**

(A) M.C.Q. :

- (i) The sum of the two roots of the equation $x^2 - 6x + 2 = 0$ is
 (a) 2 (b) -2 (c) 6 (d) -6
- (ii) If the product of the two roots of the equation $x^2 - 3x + k = 10$ is -2, then the value of k is
 (a) -2 (b) -8 (c) 8 (d) 12
- (iii) If the two roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) are real and unequal, then $b^2 - 4ac$ will be
 (a) > 0 (b) $= 0$ (c) < 0 (d) none of these
- (iv) If the two roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) be equal, then
 (a) $c = -\frac{b}{2a}$ (b) $c = \frac{b}{2a}$ (c) $c = \frac{-b^2}{4a}$ (d) $c = \frac{b^2}{4a}$
- (v) If the two roots of the equation $3x^2 + 8x + 2 = 0$ be α and β , then the value of $(\frac{1}{\alpha} + \frac{1}{\beta})$ is
 (a) $-\frac{3}{8}$ (b) $\frac{2}{3}$ (c) -4 (d) 4

(B) Let us identify the following statements as true or false :

- (i) The two roots of the equation $x^2 + x + 1 = 0$ are real.
- (ii) The two roots of the equation $x^2 - x + 2 = 0$ are not real.

(C) Let us fill in the blanks :

- (i) The ratio of the sum and the product of two roots of the equation $7x^2 - 12x + 18 = 0$ is _____.
- (ii) If two roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) are reciprocal to each other, then $c =$ _____.
- (iii) If two roots of the equation $ax^2 + bx + c = 0$ ($a \neq 0$) are reciprocal to each other and opposite in sign, then $a + c =$ _____.

13. **S.A.**

- (i) Let us write the quadratic equation if sum of its roots is 14 and the product of them is 24.
- (ii) If the sum and the product of the two roots of the equation $kx^2 + 2x + 3k = 0$ ($k \neq 0$) are equal, let us write the value of k.
- (iii) If the two roots of the equation $x^2 - 22x + 105 = 0$ are α and β , let us write the value of $(\alpha - \beta)$.
- (iv) If the sum of the two roots of the equation $x^2 - x = k(2x - 1)$ is zero, let us write the value of k.
- (v) If one of the roots of the two equations $x^2 + bx + 12 = 0$ and $x^2 + bx + q = 0$ is 2, let us write the value of q.

2

SIMPLE INTEREST

Today we have great fun. All students of our class will open bank account in their names. Each of us will have our own Bank pass book. We shall be able deposit or withdraw our money as we like.

Last year my elder brother opened an account in the bank. He deposited ₹ 800 there.

After 1 year, in his pass book I have seen that the amount has increased to ₹ 832 from ₹ 800. But why is it so?

For using my elder brother's money, bank has given him the extra ₹ $(832 - 800) = ₹ 32$

1 What is this extra money called?

This extra money is called **Interest**.

We shall call this type of interest as **Simple Interest**.

Here interest = ₹ $(832 - 800) = ₹ 32$

Principal = ₹ 800, Amount = Interest + Principal = ₹ $32 + ₹ 800$
= ₹ 832

and Time = 1 yr.

Principal— The money which is given or taken as loan or the money which is deposited.

Time— The time for which loan is given or taken or the time for which money is deposited.

Interest— In return of using the creditor's money for a certain period of time, according to condition, the debtor gives him some extra money. This sum of money is the interest.

The person or organisation who gives money as loan, is called creditor and the person or organisation who takes money as loan, is called debtor. When a person deposits money in the post office/bank, he becomes the creditor and that post office/bank is the debtor; therefore the post office or the bank gives interest on the deposited money.

Again, when a person takes a loan from a bank or co-operative organisation, then he is the debtor and the bank or cooperative organisation is the creditor; Therefore the person gives interest to the bank or cooperative organisation.

I have deposited ₹ 500 in that bank. But my friend Sajal has deposited ₹ 300 in the bank.

After, 1 yr., my ₹ 500 has increased to ₹ 520 and Sajal's ₹ 300 has increased to ₹ 312.

∴ In 1 yr., I have got an interest ₹ $(520 - 500) = ₹ 20$

But in 1 yr., Sajal has got an interest ₹ $(312 - 300) = ₹ 12$

2 Why have we got different interests for the same time?

If the time is fixed, the interest depends on principal. More the principal, more is the interest on it.



3 What interest shall we get by depositing an amount in that bank? How shall we calculate it easily?

At first, we have to determine the rate of interest of that bank.

What is "rate of interest"?

Interest is generally computed in respect to year. The interest on ₹ 100 given in 1 yr is the "rate of interest in percent per annum". For example, 10% simple interest per annum means the interest on ₹ 100 in each span of 1 year is ₹ 10. In some cases, the interest is calculated on half yearly, monthly or even daily basis.

In mathematical language, the problem is,

Principal (₹)	Time (yr.)	Interest (₹)
500	1	20
100	1	?

The interest of ₹ 500 in 1 yr. is ₹ 20

The interest of ₹ 1 in 1 yr is $\frac{20}{500}$

The interest of ₹ 100 in 1 yr is $\frac{20}{500} \times 100 = ₹ 4$

∴ In that bank, the rate of simple interest per annum is 4%

Application 1. If I deposit ₹ 1200 in that bank at the rate of 4% simple interest per annum, then after 1 yr. what interest I shall get-let us calculate.

In mathematical language, the problem is,

Principal (₹)	Time (Yr.)	Interest (₹)
100	1	4
1200	1	?

In that bank, the rate of simple interest is 4%

So, the interest of ₹ 100 in 1 yr. is ₹ 4

the interest of ₹ 1 in 1 yr. is $\frac{4}{100}$

the interest of ₹ 1200 in 1 yr. is $\frac{4 \times 1200}{100} = ₹ \boxed{}$

Application : 2. [Let me do it myself]

Principal	Time	Rate of simple interest per annum	Total interest
₹ 600	1 yr.	5%	
₹ 1800	1 yr.	$4\frac{1}{2}\%$	

Application : 3. Shraboni has got an interest of ` 45 by depositing a sum of money in a bank. If the rate of simple interest of the bank is 5% per annum, then by calculating let us write, the principal deposited by Shraboni in the bank.

In mathematical language, the problem is

Principal (`)	Time (year)	Interest (`)
100	1	5
?	1	45



The rate of simple interest of the bank is 5% per annum.

∴ In 1 yr. ` 5 will be the interest, when the principal is ` 100

In 1 yr. ` 1 will be the interest, when the principal is ` $\frac{100}{5}$

In 1 yr. ` 45 will be the interest when the principal is ` $\frac{100 \times 45}{5} = \text{` } \boxed{}$

∴ Shraboni deposited ` 900 in the bank.

Application : 4. In one year, if Shraboni would get ` 60 as interest at the rate of simple interest 5% per annum in the bank, how much principal she deposited— let us calculate and write it.

[Let me do it myself]

Application : 5.

Principal	Time	Rate of simple interest per annum	Total interest
	1 year	6%	` 90
	1 year	3.5%	` 59.50

[Let me do it myself]

Application : 6. Rahamatchacha took a loan of ` 750 for 3 years at the rate of simple interest 10% per annum from the rural cooperative bank. Let me write the total interest and principal along with the interest after calculating it.

But what is meant by "total interest"?

The interest given or received for a certain period of time on certain principal is called "Total interest".

Principal along with interest = Principal + total interest



In the cooperative bank, the rate of simple interest is 10% per annum.

In mathematical language, the problem is,

Principal (`)	Time (Year)	Interest (`)
100	1	10
750	3	?

Interest of 1 year on ` 100 is ` 10

Interest of 1 year on ` 1 is ` $\frac{10}{100}$

Interest of 1 year on ` 750 is ` $\frac{10 \times 750}{100}$

Interest of 3 years on ` 750 is ` $\frac{10 \times 750 \times 3}{100} = \text{` } \boxed{}$

The total interest given by him is ` $\boxed{}$

∴ In this case, principal along with interest = ` 750 + ` 225 = ` $\boxed{}$

4 I calculate it in another way and see what I shall get.

Let, the principal = ` p, time = t year

Rate of simple interest per annum = r% and total interest = ` I.

In another way, Interest of 1 year on ` 100 is ` r

$$\text{Interest of 1 year on ` 1 is } \frac{r}{100}$$

$$\text{Interest of 1 year on ` p is } \frac{pr}{100}$$

$$\text{Interest of t years on ` p is } \frac{prt}{100} \therefore I = \frac{prt}{100} \therefore \text{we have got, } I = \frac{prt}{100}$$



Here, p = 750, t = 3, r = 10 and I = ?

By unitary method of above calculation we have got, $I = \frac{10 \times 750 \times 3}{100} = 225$.

$\therefore$ total interest = ` I = ` 225

Application : 7. But, if Rahamatchacha would have lended at the same rate i.e. at the rate of 10% simple interest per annum he would lend ` 750 for 8yrs, then how much interest he would have given, let us calculate.

Interest of 1 yr. on ` 100 is ` 10

Interest of 1 yr. on ` 1 is ` $\frac{10}{100}$

$$\text{Interest of 8 yrs on ` 750 is } \frac{10 \times 750 \times 8}{100} = \boxed{}$$

$$\text{In another method, Interest (I) = } \frac{prt}{100} = \frac{750 \times 10 \times 8}{100} = \boxed{}$$

[Here, p = 750, r = 10 and t = 8]

We are observing that (i) if the principal and the rate of simple interest in percent per annum remain unchanged then the time and the total interest are in direct relation. i.e. if the time is increased, the total interest will be $\boxed{}$ (increased/decreased) and if time is $\boxed{}$ (increased/decreased), the total interest will be decreased.

Again, (ii) if the time and the rate of simple interest in percent per annum remain unchanged, then the principle and the total interest are in [direct/inverse] relation, i.e. if the principal is increased, the total interest will be increased and if the principal is decreased, the total interest is $\boxed{}$. [Let me write it myself by understanding it.]

But, if the principal and the time remain unchanged (or constant), then let us see the relation exists between the total interest and the rate of simple interest in percent per annum.

Application : 8. Prasantababu has deposited ` 580 each in bank and post office for 4 years. If the rates of simple interest per annum in the bank and in the post office are 5% and 6% respectively, then let us determine the total interest he would get in each of the cases.

In the bank, the rate of simple interest per annum is 5%,

$$\therefore \text{After 4 yrs. the total interest he will get} = \frac{prt}{100} = \frac{580 \times 5 \times 4}{100} = \boxed{}$$

In the post office, the rate of simple interest per annum is 6%

$$\therefore \text{After 4 yrs, the total interest he will get} = \frac{580 \times 6 \times 4}{100} = \text{` } 139.20$$

We are observing that, (iii) if the principal and the time are unchanged, the total interest and the rate of simple interest in percent per annum are in $\boxed{}$ (direct/inverse) relation i.e. if the rate of simple interest in percent per annum is increased, the total interest will be increased and if the rate of simple interest in percent per annum is decreased, the total interest will be $\boxed{}$.

Application : 9. If a loan of ₹ 5000 is taken at the rate of simple interest of 5% per annum from 1st January to 8th August 2003, then let us write, by calculating the interest and the amount.

Time = January 31 days + February 28 days + March 31 days + April 30 days + May 31 days + June 30 days + July 31 days + August 7 days = 219 days = $\frac{219}{365}$ yr. = $\frac{3}{5}$ yr. [2003 is not lip-year, February is a month of 28 days]

[For calculating time from 1st January to 8th August, either 1 day from January or 1 day from August will be deducted]

In mathematical language, the problem is,

Principal (₹)	Time (yr)	Interest (₹)
100	$\frac{1}{3}$	5
5000	$\frac{3}{5}$	?

∴ Interest for 1 yr. on ₹ 100 is ₹ 5

Interest for 1 yr on ₹ 1 is ₹ $\frac{5}{100}$

Interest for $\frac{3}{5}$ yr. on ₹ 5000 is ₹ $\frac{5}{100} \times 5000 \times \frac{3}{5}$ = ₹

In another method, Interest (I) = $\frac{prt}{100}$ [Where p (principal) = ₹ 5000, r (the rate of simple interest in percent per annum) = 5 and t (time in yr.) = $\frac{3}{5}$ yr.]

$$= ₹ \frac{500 \times 5 \times \frac{3}{5}}{100} = ₹ 150$$

The total amount = ₹ (5000+150) = ₹ 5150

Application : 10. [Let us do it myself]

Principal	Time	interest per annum	Total interest	principal along with interest
₹ 500	3 yr.	$6\frac{1}{4}$ %		
₹ 146	1 day	$2\frac{1}{2}$ %		
₹ 4565	2 yr. 6 months	4 %		

Application : 11. I got some money as interest by depositing ₹ 500 (in a bank) for 2yr. at the rate of simple interest 6% per annum. Let us find out, the period of time for which the same interest can be received by depositing ₹ 400 in that very bank.

By depositing ₹ 500 for 2 yrs. in the bank at the rate of 6% interest per annum, the interest received = ₹ , [Let me do it myself]

In mathematical language, the problem is

Principal (₹)	Time (yr.)	rate of interest in percent per annum	Total interest (₹)
500	2	6	60
400	?	6	60

I suppose, by depositing ₹ 400 in the bank for t yrs., I have got ₹ 60 as interest.

$$\therefore \frac{400 \times t \times 6}{100} = 60$$

$$\text{or, } 24t = 60$$

$$\text{or, } t = \frac{60}{24} \quad \therefore t = 2\frac{1}{2}$$

$\therefore$ In $2\frac{1}{2}$ yr. the total interest on ₹ 400 at the rate of simple interest of 6% per annum is ₹ 60.

In another method (for the 2nd part): The total interest on ₹ 500 is ₹ 60 in 2 yrs.

$\therefore$ The total interest on ₹ 1 is ₹ 60 in 2×500 yrs.

$\therefore$ The total interest on ₹ 400 is ₹ 60 in $\frac{2 \times 500}{400}$ yrs. = $2\frac{1}{2}$ yrs.

$\therefore$ We are observing that (iv) if the rate of simple interest be in percent per annum and the total interest remains unchanged, then the principal and the time are in (direct/inverse) relation, i.e, if principal is increased, time will be decreased and if principal is decreased, time will be .

Application : 12. Ashadebi deposited a sum of money in the bank for 4 yr. at the rate of simple interest of 6% per annum. After that time she got interest ₹ 240 in total.

Let us calculate and see what sum of money Ashadebi deposited in the bank.

In mathematical language, the problem is

Principal (₹)	Time (yr.)	Interest (₹)
100	1	6
?	4	240



In 1 yr. the interest is ₹ 6 when principal is ₹ 100

In 1 yr. the interest is ₹ 1 when principal is ₹ $\frac{100}{6}$

In 4 yrs. the interest is ₹ 1 when principal is ₹ $\frac{100}{4 \times 6}$

In 4 yrs. the interest is ₹ 240 when principal is ₹ $\frac{100 \times 240}{4 \times 6} = ₹$

In another method, I suppose, she deposited ₹ p in the bank

$$\therefore \text{Interest} = ₹ \frac{p \times 6 \times 4}{100}$$

$$\text{According to the condition, } \frac{p \times 6 \times 4}{100} = 240$$

$$\therefore p = \frac{240 \times 100}{6 \times 4} = 1000$$

$\therefore$ Ashadebi deposited ₹ 1000 in the bank.



Application : 13. Let us write, in the blank by calculating it. [let me do it myself]

Principal	Time	Rate of simple interest per annum	Total Interest
	4 yr.	$4\frac{1}{2}\%$	₹ 72
	1 day	5 %	₹ 1

Application : 14. By depositing ` 700 in a bank for a certain period of time at the fixed rate of simple interest per annum, I got ` 900 as the amount. Let us write, by calculating, the amount to be deposited for which I would get ` 1350 for the same time and at the same rate.

In mathematical language, the problem is

Principal (`)	Principal with interest (`)
700	900
?	1350



∴ The amount is ` 900 when Principal is ` 700

The amount is ` 1 when the principal is ` $\frac{700}{900}$

The amount is ` 1350 when the principal is ` $\frac{700 \times 1350}{900} = \text{` } \boxed{}$

Application : 15. Let us write by calculating, the principal for which the amount will be ` 5160 when the rate of simple interest per annum is $7\frac{1}{2}\%$ and the time is 8yr.

In mathematical language, the problem is

Principal (`)	Time (yr.)	Total interest (`)
100	1	$7\frac{1}{2}$
100	8	?

The interest on ` 100 in 1 yr. is ` $7\frac{1}{2} = \frac{15}{2}$

The interest on ` 100 in 8 yrs. is ` $\frac{15}{2} \times 8 = \text{` } 60$

∴ In this case, amount = ` 100 + ` 60 = ` 160

∴ Newly, the problem is,

Principal (`)	Amount (`)
100	160
?	5160



∴ If amount is ` 160, the principal is ` 100

If amount is ` 1, the principal is ` $\frac{100}{160}$

If amount is ` 5160, the principal is ` $\frac{100 \times 5160}{160} = \text{` } \boxed{}$

Application : 16. Let us write in the blank by calculation. [Let me do it myself]

Principal	Time	Rate of simple interest per annum	Amount
	5 yrs	3 %	` 966
	6 yrs	6 %	` 13,600

Application : 17. I got the interest of ₹ 900 after 3 yrs. by depositing ₹ 5000 in a bank at the rate of simple interest of 6% per annum. If the rate of interest in that bank would be 7% per annum, then let us write by calculating, the time for which I would get ₹ 900 as interest.

I suppose, at the rate of simple interest of 7% per annum, in yrs. the interest of ₹ 5000 is ₹ 900.

$$\therefore \text{Interest (I)} = \frac{prt}{100} \quad [\text{here } p = ₹ 5000, r = 7, I = ₹ 900]$$

$$\therefore 900 = \frac{5000 \times 7 \times t}{100} \therefore t = \frac{900}{350} = 2\frac{4}{7} \therefore \text{In } 2\frac{4}{7} \text{ yrs. I shall get the interest ₹ 900. } 2\frac{4}{7} < 3$$

[We are observing that (v) if the principal and the total interest remain unchanged, then the rate of simple interest in percent per annum are in (direct /inverse) relation with the time, i.e, if the rate of simple interest in percent annum is increased, the time will be decreased and if the rate of simple interest in percent per annum is decreased, the time will be increased] [Let me write and justify it by taking any other examples]

Application : 18. Ramu Pradhan has deposited ₹ 12500 in a bank at the rate of simple interest of $5\frac{1}{2}\%$ per annum. After a certain time, he got ₹ 2750 as interest. Let us write, by calculating, the period of time, for which he has deposited the money in the bank.

In mathematical language, the problem is, Principal (₹) Time (yr.) Total interest (₹)

100	1	$5\frac{1}{2} = \frac{11}{2}$
12500	?	2750

For, ₹ 100 the interest is ₹ $\frac{11}{2}$ in 1 yr.

For, ₹ 1, the interest is ₹ $\frac{11}{2}$ in 1×100 yr.

For, ₹ 1, the interest is ₹ 1 in $\frac{1 \times 100}{\frac{11}{2}}$ yr. = $\frac{1 \times 2 \times 100}{11}$ yr.

For ₹ 12500 the interest is ₹ 1 in $\frac{1 \times 2 \times 100}{11 \times 12500}$ yr.

For ₹ 12500, the interest is ₹ 2750 in $\frac{1 \times 2 \times 100 \times 2750}{11 \times 12500}$ yr. = in yr.

$\therefore$ Ramu Pradhan has deposited money in the bank for 4 yrs.

In another method, I suppose that Ramu Pradhan has deposited money in the bank for t yrs.

$\therefore$ He got interest ₹ 2750 in t yrs. for ₹ 12500 at the rate of $5\frac{1}{2}\%$ interest per annum.

$\therefore 2750 = \frac{prt}{100}$ [Here, p (Principal) = ₹ 12500, $r = \frac{11}{2}$ (rate of simple interest in percent per annum), I = ₹ 2750 (Total interest)]

$$\text{or, } 2750 = \frac{12500 \times \frac{11}{2} \times t}{100} \quad \text{or, } t = \frac{2750 \times 100 \times 2}{11 \times 12500} = 4$$

$\therefore$ Ramu Pradhan has deposited money in the bank for 4 yrs

Application : 19.

Principal	Time	Rate of simple interest per annum	Total Interest
₹ 6400		$4\frac{1}{2}\%$	₹ 1008
₹ 500		5 %	₹ 50

[Let me do it myself]

Application : 20. Saheli gave the total interest by taking a loan of ₹ 700 for 5 yrs at the rate of simple interest of 4% per annum which is equal to the total interest if she would take a loan of ₹ 900 for the same time, let us write by calculating, the rate of simple interest in percent per annum.

The interest of ₹ 700 at the rate of simple interest of 4% per annum for 5yrs. = $\frac{700 \times 5 \times 4}{100}$
= ₹ 140

Let us determine the rate of simple interest in percent per annum if Saheli gives the total interest ₹ 140 by taking the loan of ₹ 900 for 5yrs.

In mathematical language, the problem is, Principal (₹) Time (yr) Total Interest (₹)

900	5	140
100	1	?

The interest on ₹ 900 for 5 yrs is ₹ 140

The interest on ₹ 1 for 1 yr. is $\frac{140}{900 \times 5}$

The interest on ₹ 100 for 1 yr. is $\frac{140 \times 100}{900 \times 5} = ₹ 3\frac{1}{9}$

∴ The required rate of simple interest per annum is $3\frac{1}{9}\%$

[We are observing that (vi) if the time and the total interest remain unchanged, the principal and the rate of simple interest in percent per annum are in (direct/inverse) relation i.e, if the principal is increased, the rate of interest will be decreased and if the principal is decreased, the rate of interest will be increased]

Application : 21. What is the rate of simple interest in percent per annum if the total interest of ₹ 5000 for 8yrs is ₹ 4800?—By calculating, let us write it.

In mathematical language, Principal (₹) Time (yr.) Total interest (₹)

5000	8	4800
100	1	?

The interest on ₹ 5000 for 8yrs. is ₹ 4800

The interest on ₹ 1 for 1 yr is $\frac{4800}{5000 \times 8}$

The interest on ₹ 100 for 1 yr is $\frac{4800 \times 100}{5000 \times 8} = \text{₹ } \text{ } \text{yr.}$

∴ The rate of simple interest per annum is = %

In another method, let the rate of simple interest per annum be $r\%$

∴ The interest of ₹ 5000 at the rate of simple interest of $r\%$ per annum in 8yrs = $\frac{5000 \times r \times 8}{100}$

According to the condition, $\frac{5000 \times r \times 8}{100} = 4800$

$$\text{or, } r = \frac{4800 \times 100}{5000 \times 8} \therefore r = 12$$

∴ The required rate of simple interest per annum is 12%.



Application : 22. What is the rate of simple interest in percent per annum if the amount is ₹ 73001 for ₹ 73000 in 1 day – by calculating, let us write it.

$$\text{Total interest} = ₹ 73001 - ₹ 73000 = ₹ 1$$

In mathematical language, the problem is	Principal (₹)	Time (day)	Total interest (₹)
	73000	1	1
	100	365	?

∴ The interest of ₹ 73000 in 1 day is ₹ 1

The interest of ₹ 1 in 1 day is $\frac{1}{73000}$

The interest of ₹ 100 in 365 days is $\frac{100 \times 365}{73000} = ₹ 0.5$

∴ The required rate of simple interest per annum is 0.5%



In another method, Total interest (I) = ₹ 73001 – ₹ 73000 = ₹ 1

Principal (p) = ₹ 73000, t = 1 day = $\frac{1}{365}$ yr.

Let, the rate of simple interest per annum be r%

$$I = \frac{prt}{100} \text{ or, } 1 = \frac{73000 \times r \times 1}{100 \times 365} \text{ or, } r = \frac{36500}{73000} \text{ or, } r = \frac{5}{10} \therefore r = 0.5$$

∴ The rate of simple interest per annum is 0.5%

Application : 23. (i) Let us determine the rate of simple interest in percent per annum, if the interest on ₹ 500 in 4 yrs. is 100.

(ii) By calculating, let us write the rate of simple interest in percent per annum when the principal with interest on ₹ 910 in 2yrs 6 months will be ₹ 955.50

[Let me do it myself]

Application : 24. (i) Rabeya deposited ₹ 750 in the bank for 6 yrs at the rate of simple interest of 8% per annum, let us write by calculating, the amount she has got.

(ii) But if she would get ₹ 1200 as the amount in the same time from that money, then what would be the rate of simple interest in percent per annum—let us determine it.

(iii) If the rate of simple interest in percent per annum would remain the same as in the first case, then in how many yrs. she would get ₹ 1170 as the principal along with interest from the money of the first case – let us write by calculating it.

(i) In mathematical language, the problem is	Principal (₹)	Time (yr.)	Total interest (₹)
	100	1	8
	750	6	?

The interest of ₹ 100 in 1 yr. is ₹ 8

The interest of ₹ 1 in 1 yr. is $\frac{8}{100}$

The interest of ₹ 750 in 6 yrs is $\frac{8 \times 750 \times 6}{100} = \boxed{}$

In another method, the interest of ₹ 750 at the rate of 8% per annum in 6 yrs = $\frac{75 \times 8 \times 6 \times 8}{100} = ₹ 360$

[By the help of the formula $I = \frac{prt}{100}$]

∴ Rabeya has got in total the principal along with interest ₹ 750 + ₹ 360 = ₹ 1110



(ii) In mathematical language, the problem is	Principal (₹)	Time (yr)	Total Interest (₹)
	750	6	$(1200 - 750) = 450$
	100	1	?

The interest on ₹ 750 in 6 yrs is ₹ 450

The interest on ₹ 1 in 6 yrs is ₹ $\frac{450}{750}$

The interest on ₹ 1 in 1 yr. is ₹ $\frac{450}{750 \times 6}$

The interest on ₹ 100 in 1 yr. is ₹ $\frac{450 \times 100}{750 \times 6} = \boxed{}$

∴ The required rate of simple interest per annum is 10%

In another method, let the required rate of interest per annum be $r\%$

$I = \frac{prt}{100}$, where, I = Total interest, p = Principal, r = rate of simple interest in percent per annum, t = time (in yr.)

$$\therefore \text{Interest} = \frac{750 \times r \times 6}{100}$$

$$\text{According to condition, } 450 = \frac{750 \times r \times 6}{100} \therefore r = \frac{450 \times 100}{750 \times 6} = 10$$

∴ The required rate of simple interest per annum is 10%.

(iii) In mathematical language, the problem is,	Principal (₹)	Time (yr.)	Total interest (₹)
	100	1	8
	750	?	$(1170 - 750) = 420$

The interest on ₹ 100 is ₹ 8 in 1 yr.

The interest on ₹ 1 is ₹ 8 in 1×100 yr.

The interest on ₹ 1 is ₹ 1 in ₹ $\frac{1 \times 100}{8}$ yr.

The interest on ₹ 750 is ₹ 1 in $\frac{1 \times 100}{750 \times 8}$ yr.

The interest on ₹ 750 is ₹ 420 in $= \frac{1 \times 100 \times 420}{750 \times 8}$ yr. = 7 yr.

In another method, let in t yrs the total interest of ₹ 750 at the rate of simple interest of 8% per annum be ₹ 420.

$I = \frac{prt}{100}$ where, I = Total interest, p = Principal, r = rate of simple interest in percent per annum, t = Time (in yr.)

$$\therefore 420 = \frac{750 \times t \times 8}{100}$$

$$\therefore t = \frac{420 \times 100}{750 \times 8} = 7$$

∴ The required time is 7 yrs.



Application : 25. If a principal amounts ₹ 560 in 3 yrs and ₹ 600 in 5yrs. at the same rate of simple interest in percent per annum, let us determine the principal and the rate of simple interest in percent per annum.

Analysing the given data, we get,

$$\begin{array}{rcl} \text{Principal} + & 5 \text{ yrs. interest} & = ₹ 600 \\ \text{Principal} + & 3 \text{ yrs. interest} & = ₹ 560 \\ \hline (\text{Subtracting we get}) & 2 \text{ yrs' interest} & = ₹ 40 \end{array}$$

The interest of 2 yrs. is ₹ 40

The interest of 1 yr is ₹ $\frac{40}{2}$

The interest of 3 yrs is ₹ $\frac{40 \times 3}{2} = ₹ 60$

∴ Principal = ₹ 560 – ₹ 60 = ₹ 500,

In mathematical language, the problem is,

Principal (₹)	Time (yr.)	Total interest (₹)
500	3	60
100	1	?

The interest of ₹ 500 in 3yrs. is ₹ 60

The interest of ₹ 1 in 3yrs. is ₹ $\frac{60}{500}$

The interest of ₹ 1 in 1 yrs. is ₹ $\frac{60}{500 \times 3}$

The interest of ₹ 100 in 1 yr. is ₹ $\frac{60 \times 100}{500 \times 3} = ₹ 4$

∴ The rate of simple interest per annum is 4%

So, the principal is ₹ 500 and the rate of simple interest per annum is 4%

Application: 26. A principal amount ₹ 496 in 3 yr. and ₹ 560 in 5 yr. at the same rate of simple interest in percent per annum. By calculating, let us write the principal and the rate of simple interest in percent per annum. [Let me do it myself]

Application: 27. At the time of retirement, Subirbabu got ₹ 6,00,000 from provident fund and gratuity all together at a time. He wants to deposit this money in the Post Office and in the bank so that he would get ₹ 34,000 as interest per year. If the rates of simple interest per annum in the post office and bank are 6% and 5% respectively, then let us write by calculating the sum of money deposited by him separately in the post office and in the bank.

If Subirbabu would deposite all of his money in the bank at the rate of simple interest 5% per annum, then he would get the interest = ₹ $600000 \times \frac{5}{100} = ₹ 30000$

But he wants to get (₹ 34000 – ₹ 30000) = ₹ 4000 more.

In post office he gets (6% – 5%) = 1% more interest in 1 yr.

∴ He would deposit that amount of money in the post office so that from these it would be 1% more interest = ₹ 4000



In mathematical language, the problem is,	Deposited in the post office (₹)	More interest that will be received (₹)
	100	1
	?	4000

∴ ₹ 1 more interest will be received for the deposit of ₹ 100.

So, ₹ 4000 more interest will be received for the deposit of ₹ $100 \times 4000 = ₹ 400000$

∴ Subirbabu deposited ₹ 4,00,000 in post office and ₹ $(600000 - 400000)$

In another method, = ₹ 200000 in the bank.

I suppose, Subirbabu deposited ₹ x in the bank and ₹ $(600000 - x)$ in the post office.

∴ From the bank, he will get the interest, ₹ $\frac{x \times 5 \times 1}{100}$

From the post office, he will get the interest, ₹ $\frac{(600000 - x) \times 6 \times 1}{100}$

According to the given condition,, $\frac{5x}{100} + \frac{6(600000 - x)}{100} = 34000$

$$\text{or, } 5x + 3600000 - 6x = 34000 \times 100$$

$$\text{or, } -x = 3400000 - 3600000$$

$$\text{or, } -x = -200000$$

$$\therefore x = 200000$$

∴ Subirbabu deposited ₹ 2,00,000 in the bank and ₹ $(600000 - 200000)$

= ₹ 4,00,000 in the post office

Application: 28. A Cooperative Society of weavers took a loan at the time of purchasing a power loom from central Cooperative bank subject to the condition, that they would repay the interest at the rate of simple interest of 9% per annum and $\frac{1}{5}$ th part of the principal at every span of 2yr. of interval of time.

After two years, if the society repays ₹ 19,000 as first instalment, then let us write, by calculating, then the amount of loan taken by the society.

Let, the Cooperative society took ₹ x loan.

∴ Interest of 2yrs. = ₹ $\frac{x \times 2 \times 9}{100} = ₹ \frac{9x}{50}$

According to the condition, $\frac{9x}{50} + \frac{x}{5} = 19000$

$$\text{or, } \frac{9x + 10x}{50} = 19000$$

$$\text{or, } 19x = 19000 \times 50$$

$$\text{or, } x = \frac{19000 \times 50}{19}$$

$$\therefore x = 50000$$

∴ The Cooperative society took a loan of ₹ 50000.



Application : 29. My aunt will make a will for ₹ 56,000 in the names of her two sons of the ages 13 yrs and 15 yrs such that, each of them will receive equal amount, having the rate of simple interest of 10% per annum, at the time of attaining 18 years of age. Let us determine the sum of money she had allotted for each her sons in her will.

Let, the money allotted for younger son be ₹ x and for the older son be ₹ $(56000 - x)$

... At the age of 18 yrs. the principal along with interest for the younger son will be

$$₹ \left\{ x + \frac{x \times (18-13) \times 10}{100} \right\} = ₹ \frac{3x}{2}$$

At the age of 18 yrs. the principal along with interest for the older son will be

$$= ₹ \left\{ (56000 - x) + \frac{(56000 - x) \times (18 - 15) \times 10}{100} \right\}$$

$$= ₹ \frac{13}{10} (56000 - x)$$

According to the condition, $\frac{3x}{2} = \frac{13}{10} (56000 - x)$

$$\text{or, } 30x = 26(56000 - x)$$

$$\text{or, } 15x = 13(56000 - x)$$

$$\text{or, } 28x = 13 \times 56000 \quad \therefore x = \boxed{}$$

$\therefore$ The principal allotted for younger son is ₹ 26,000 and the principal allotted for older son is ₹ $(56000 - 26000) = ₹ 30000$

In another method, let the amount allotted for younger son be ₹ x

and the amount allotted for older son be ₹ y

According to the condition, $x + y = 56000$ _____ (i)

$\therefore$ At 18 yrs. age, the principal along with interest for the younger son will be

$$₹ \left\{ y + \frac{y(18-15) \times 10}{100} \right\} = ₹ \frac{13y}{10}$$

long with interest for the older son will be

$$= ₹ \left\{ x + \frac{y(18-15) \times 10}{100} \right\} = ₹ \left\{ x + \frac{3y}{10} \right\} = ₹ \frac{13y}{10}$$

According to the condition, $\frac{3x}{2} = \frac{13y}{10}$ or, $30x = 26y \quad \therefore x = \frac{13y}{15}$ _____ (ii)

From the equation (i), we get, $x + y = 56000$

$$\text{or, } \frac{13y}{15} + y = 56000 \quad \text{or, } \frac{28y}{15} = 56000 \quad \text{or, } y = 56000 \times \frac{15}{28} \quad \therefore y = 30000$$

$$x = 56000 - 30000 = 26000$$

$\therefore$ My aunt will allot ₹ 26000 for her younger son and ₹ 30,000 for her older son.

Application : 30. Bimalkaku deposited ₹ 1,87,500 for his son of 12 yr. of age and daughter of 14 yr. of age in the bank at the rate of simple interest 5% per annum in such away that, both of them will get equal principal at their 18 yrs. of age. Let us calculate the money he had deposited in the bank for each of his son and daughter. **[Let me do it myself]**



Application : 31. Fatimabibi deposits ` 100 on the first day of every month in a monthly savings scheme. In this way, she has deposited money for one yr. If the rate of simple interest is 6% per annum; then let us determine the amount she will get at the end of the yr.

Fatimabibi deposits money on the 1st, 2nd, 3rd,, last month for 12 months, 11 months, 10 months, ..., 1 month, respectively.

So, the total interest of 1 yr.

$$= \left(\frac{100 \times \frac{12}{12} \times 6}{100} + \frac{100 \times \frac{11}{12} \times 6}{100} + \frac{100 \times \frac{10}{12} \times 6}{100} + \dots + \frac{100 \times \frac{1}{12} \times 6}{100} \right)$$

$$= \frac{100 \times 6}{12 \times 100} (12 + 11 + 10 + \dots + 1) = \frac{1}{2} \times 78 = ` 39$$

∴ She will get the amount after 1 yr. = ` (100 × 12 + 39) = ` 1239

Application : 32. Jayanta deposits ` 1000 on the first day of every month in a monthly savings scheme. In the bank, if the rate of simple interest is 5% per annum, then let us calculate the amount Jayanta will get at the end of 6 months. **[Let me do it myself]**

Application : 33. Ramenbabu deposits ` 3,70,000 in total in three banks, The rates of simple interest per annum in three banks are 4%, 5% and 6% respectively; after 1 yr., the total interests in three banks are equal. Let us calculate the principals he has deposited in each of the three banks.

I suppose, he deposits ` x in the 1st bank, ` y in the 2nd bank and ` z in the 3rd bank.

After 1 yr, the total interest of the 1st bank = ` $\frac{x \times 4 \times 1}{100} = \frac{4x}{100}$

After 1 yr., the total interest of the 2nd bank = ` $\frac{y \times 5 \times 1}{100} = \frac{5y}{100}$

After 1 yr., the total interest of the 3rd bank = ` $\frac{z \times 6 \times 1}{100} = \frac{6z}{100}$



According to the condition, $x + y + z = 370000$ (i)

$$\frac{4x}{100} = \frac{5y}{100} = \frac{6z}{100} \text{ (ii)}$$

So, $4x = 5y = 6z = k$ (I suppose), where $k > 0$

$$\therefore x = \frac{k}{4}, y = \frac{k}{5}, z = \frac{k}{6}$$

Again, $x + y + z = 370000$

$$\text{So, } \frac{k}{4} + \frac{k}{5} + \frac{k}{6} = 370000$$

$$\text{or, } \frac{15k + 12k + 10k}{60} = 370000$$

$$\text{or, } 37k = 370000 \times 60$$

$$\text{So, } x = \frac{600000}{4} = 150000, y = \frac{600000}{5} = 120000 \text{ and } z = \frac{600000}{6} = 100000$$

∴ He deposits ` 150000, ` 120000 and ` 100000 respectively in the three banks.

Application : 34. Soma aunt deposits ` 6,20,000 in such a way in three banks at the rate of simple interest of 5% per annum for 2yrs., 3 yrs. and 5yrs. respectively so that the total interests in the 3 banks are equal. Let us calculate the sums of money deposited by Soma aunt in each of the three banks. [Let me do it myself]

Let us work out 2

1. Two friends together took a loan of ` 15,000 from a bank to run a business at the rate of simple interest of 12% per annum. Let us write, by calculating, the interest they have to pay after 4yrs.
2. Let us determine the interest on ` 2000 at the rate of simple interest of 6% per annum from 1st January to 26th May, 2005.
3. Let us determine the amount (Principal along with interest) on ` 960 at the rate of simple interest of $8\frac{1}{3}\%$ per annum for 1yr. 3months.
4. Utpalbabu took a loan of ` 3200 for 2yrs. from a Cooperative bank for cultivation of his land at the rate of simple interest of 6% per annum. Let us write by calculating, the amount he has to repay after 2yrs.
5. Sovadebi deposited a sum of money in a bank at the rate of simple interest of 5.25% per annum. After 2yrs. she has got ` 840 as interest. Let us write by calculating, the money she has deposited in the bank.
6. Goutam took a loan from a Cooperative bank for opening a poultry farm at the rate of simple interest of 12% per annum. Every month he has to repay ` 378 as interest. Let us determine the loan amount taken by him.
7. Let us calculate, period of time for which a sum of money becomes twice having the rate of simple interest of 6% per annum.
8. Mannan Miyan observed, after 6 years of taking a loan that the interest to be paid had become $\frac{3}{8}$ th of its principal. Let us determine the rate of simple interest in percent per annum.
9. An agricultural Co-operative society gives agricultural loan to its members at the rate of simple interest of 4% per annum. But an interest is to be given at the rate of simple interest of 7.4% per annum for a loan taken from the bank. If a farmer being a member of the Co-operative society takes a loan of ` 5000 from it instead of taking loan from the bank, then let us write, by calculating the money to be saved as interest per annum.
10. If the interest of ` 292 in 1 day be 1 paisa, then let us write by calculating, the rate of simple interest in percent per annum.
11. Let us calculate the number of yrs. for which the interest on ` 600 at the rate of simple interest of 8% per annum will be ` 168.
12. If I get ` 1200 as amount (principal along with interest) by depositing ` 800 in the bank at the rate of simple interest of 10% per annum, then let us write by calculating, the time for which the money was deposited in the bank.
13. At the same rate of simple interest per annum, if a principal yields the amount of ` 7100 in 7 yrs. and of ` 6200 in 4 yrs., let us determine the principal and rate of simple interest per annum.
14. Amal Roy deposits ` 2000 in a bank and Poshupoti Ghosh deposits ` 2000 in a post office at the same time. After 3yrs. they get the return amounts as ` 2360 and ` 2480 respectively. Let us write by calculating, the ratio of the rates of simple interest per annum in the bank and in the post office.

15. A weaver Cooperative society takes a loan of ₹ 15,000 for buying a power loom. After 5 yr. the society has to repay ₹ 22125 for recovering the loan. Let us determine the rate of simple interest per annum.
16. Aslamchacha got ₹ 1,00,00 when he retired from his service. He deposited a part of that money in a bank and rest of his money in a post office and got ₹ 5400 in total per year as interest. If the rates of simple interest per annum in the bank and in the post office are 5% and 6%, respectively, then let us write by calculating, the sums of money he had deposited in the bank and post office.
17. Rekhadidi deposited ₹ 10,000 of her savings in two separate banks at the same time. The rate of simple interest per annum is 6% in one bank and 7% in other bank; after 2 yr., if she gets ₹ 1280 in total as interest, then let us write by calculating, the sums of money she had deposited separately in each of the two banks.
18. A bank gives 5% simple interest per annum. In that bank, Dipubabu deposits ₹ 15, 000 at the beginning of the year, but withdraws ₹ 3000 after 3 months and then again, after 3 months he deposits ₹ 8000. Let us determine the amount (Principal along with interest) that Dipubabu will get at the end of the year.
19. Rahamatchacha takes a loan of ₹ 2,40,000 from a bank for constructing a building at the rate of simple interest of 12% per annum. After 1yr. of taking the loan he rents the house at the rate of ₹ 5200 per month. Let us determine the number of yrs. he would take to repay his loan along with interest from the income of the house rent.
20. Rothinbabu deposits money for each of his two daughters in such a way that when the ages of each of his daughters will be 18yrs. each one will get ₹ 1,20,000. The rate of simple interest per annum in the concerned bank is 10% and the present ages of his daughters are 13yr. and 8 yr. respectively. Let us determine the sums of money, he had deposited separately in the bank in favour of each of his daughters.

21. **V.S.A.**

M.C.Q

- (i) If the interest of ₹ p at the rate of simple interest of r% per annum in t years is I, then
 (a) $I = prt$ (b) $prtI = 100$ (c) $prt = 100 \times I$ (d) none of these.
- (ii) A principal becomes twice as its amount in 20 yrs at a certain rate of simple interest. At the same rate of simple interest, that very principal becomes thrice as its amount in
 (a) 30 yrs. (b) 35 yrs. (c) 40 yrs. (d) 45 yrs.
- (iii) If a principal becomes twice as its amount in 10 yrs., the rate of simple interest per annum is
 (a) 5% (b) 10% (c) 15% (d) 20%
- (iv) If the total interest comes upto ₹ x for a principal for the rate of simple interest of x% per annum in x years then the principal will be
 (a) ₹ x (b) ₹ 100x (c) ₹ $\frac{100}{x}$ (d) ₹ $\frac{100}{x^2}$

- (v) The total interest of a principal in n yrs. at the rate of simple interest of $r\%$ per annum is $\frac{Pnr}{25}$, the principal will be
 (a) ` 2p (b) ` 4p (c) `  $\frac{p}{2}$  (d) ` $\frac{p}{4}$

(B) Let us write, whether the following statements are true or false:

- (i) A man takes a loan is called debtor.
 (ii) If the principal and the rate of simple interest in percent per annum be constants, then the total interest and the time are in inverse relation.

(C) Let us fill in the blanks :

- (i) A man who gives loan is called _____.
 (ii) The amount of ` 2p in  $t$  yr. at the rate of simple interest of  $\frac{r}{2}\%$  per annum is `(2p + _____).
 (iii) The ratio of the principal and the amount (principal along with interest) in 1 yr. is 8 : 9, the rate of simple interest per annum is _____.

22. S.A.

- (i) Write the number of yrs. in which the amount becomes twice of the principal having the rate of simple interest $6\frac{1}{4}\%$ per annum.
 (ii) The rate of simple interest per annum reduces to $3\frac{3}{4}\%$ from 4% and for this, Amal babu's annual income decreases by ` 60. Let us determine Amal babu's principal.
 (iii) What is the rate of simple interest per annum, when the interest of a principal in 4 yrs. will be $\frac{8}{25}$ part of its principal – let us determine it.
 (iv) What is the rate of simple interest per annum, when the interest of a sum of money in 10 yrs. will be $\frac{2}{5}$ part of its amount (principal along with interest) – Let us determine it.
 (v) Calculate the principal whose monthly interest is ` 1 having the rate of simple interest of 5% per annum.

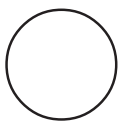
3

THEOREMS RELATED TO CIRCLE

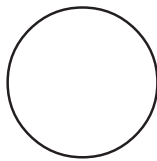
Every sunday we, brothers and sisters, remain busy in our household work. In the last few days we cannot find the keys of our house. They have been hapazardly dispersed here and there.

So, we have decided today that the keys will be arranged properly in the separate rings.

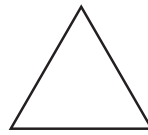
My brother has many circular and triangular key rings. These are,



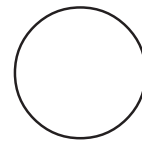
(i)



(ii)



(iii)

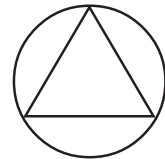


(iv)

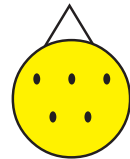


We observe that the triangular key ring has got fixed in a circular ring as in the adjoining figure.

∴ The circular ring is approximately like the [Circumcircle/Incircle]-of the triangular key ring.



Now, we have made circular disc of hard cardboard and hung it on the wall with a hook. By fixing pins in this cardboard, we shall hang the key-rings.

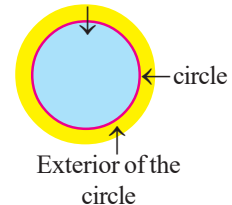


We have also made some more circular copper key rings.

I draw the circular rings on my copy and observe what I shall get. I am observing that each circle drawn on the copy divides the surface of the copy into three parts (i) Interior of the circle (ii) circle and (iii) exterior of the circle |

The circle and the interior of circle together form the **Circular region**.

Interior of the circle

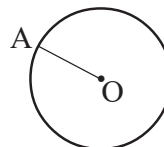


1 I draw a circle on my copy with a scale and pencil compass and try to know some matters related to it.

I have drawn a circle with centre O and radius OA.

The circle is formed with infinite number of points.

I am observing that each point is from its centre |



∴ The circle is a collection of infinite number of points lying on a plane, all the points are equidistant from a fixed point on that plane.

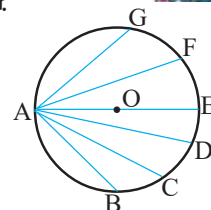
That fixed point is and the line segment drawn from the centre to any point of the circle is .

2 Let us see what type of line segment I shall get by joining any two points of the circle.



On the circle with the centre O, I took the points A, B, C, D, E, F and G.

Now, by joining the points B, C, D, E, F and G with the point A, I got the line segments AB, AC, AD, AE, AF and AG respectively.

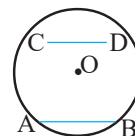


3 What these line segments AB, AC, AD, AE, AF and AG are called.

The line segments AB, AC, AD, AE, AF and AG are called **Chord** of the circle with the centre O.

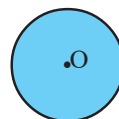
ie. The line segment joining any two points of the circle is called a **Chord** of that very circle.

In the adjoining figure, we are observing that the line segment AB is a , but the line segment CD is not a .



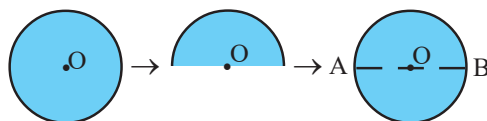
A greatest chord of the circle is a [diameter/radius] **[Let me do it myself]**

∴ Diameter is a of a circle. But any chord is not a of a circle.

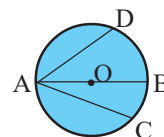


Hands on trial

- By drawing a circle on the copy I cut out that circular region whose centre is O.
- Now the circular region is folded along any line through the point O in such a way that one part is overlapped with the other part.
- Now by unfolding it I got the diameter AB as a 'crease' that passes through O.



- Except the point B I took the other two points C and D of the circle. And by folding AC and AD and subsequently unfolding I got two chords AC and AD which do not pass through O.



- By folding and matching the two chords AC and AD with the diameter AB, we are observing that

AB AC [putting >/<], AB AD [putting >/<]

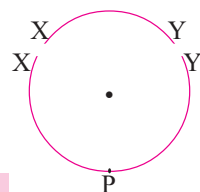
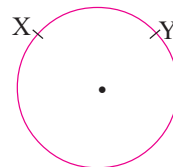
∴ By hands on trial, we have got that the diameter of the circle is the greatest chord of that circle,

My younger brother's friend Rabeya joined with us in this funny work. She drew a few circles with the help of her pair of bangles. But one of these has been broken into two pieces.



I try to get the form of the original circular bangle by joining the broken two pieces or keeping them side by side.

As the bangle has been broken at the points of X and Y, I have got the part of a circle from X to Y.



4 What this part of the circle is called?

The part of the circle from X to Y is called an **Arc** and it is written as $\widehat{XY}$.

This smaller arc is called Minor Arc and the greater arc is called **Major Arc**. The name of **Minor Arc** is $\widehat{XY}$ and the name of greater arc is $\widehat{XPY}$.



If these very two arcs are equal in length, then what shall we get?

If the two arcs are equal in length, then the two points X and Y will be the end points of a diameter of the circle and then we shall get **Semi circle**.

Again the total length of the circle is **Circumference**.

My brother has drawn a chord AB in the circle drawn by Rabeya and AB is not a diameter of the circle. As a result the circular region has been divided into two parts and each of which has been coloured by him.

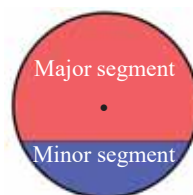
5 The chord AB divides the circular region into two parts. What each of the parts is called?

Each of the part is called **Segment**.

i.e., The region formed by the chord and arc of the circle is called **Segment**.

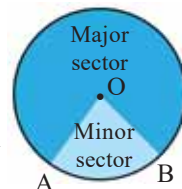
We have got two types of segments in the picture. One is greater segment and other is smaller segment. The greater segment is Major Segment and smaller segment is **Minor Segment**.

I have drawn a circle whose centre is O and joining the two points A and B on the circle with the centre O, I have got two radii OA and .



6 What is called the region bounded by these radii OA and OB and the arc $\widehat{AB}$?

The region bounded by the radii OA and OB and the arc $\widehat{AB}$ of the circle with centre O is called **Sector**.



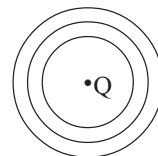
That is, the region bounded by the two radii and the arc of any circle is called a **Sector**. From this we shall get two sectors. One is **Major Sector** and the other is **Minor Sector**.



If the said two sectors are equal, then we shall get semi circular region and in that case, let me write, by drawing, the types of two radii and the arc.

My friend Ruma drew many circles on the board by taking a certain point Q as the centre like the adjoining figure.

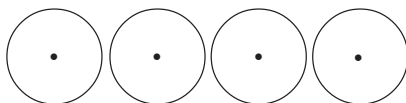
- 7 What are called the infinite number of circles drawn with a fixed point as the common centre having the radii of different lengths?



These circles are called **Concentric Circles**.

We are observing that the radii of concentric circles are not fixed.

- 8 What shall we get if we draw the circles with a radius of fixed length but having the centres at different points? – Let us see.

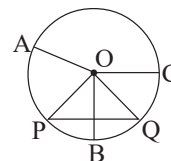


These are called **Equal or congruent circles**.

Note : Any circle is congruent to itself also.

Let us work out 3.1

- Let us see the adjoining figure of the circle with centre O and write the radii which are situated in the segment PAQ.
- Let us write in the following** **by understanding it.**
 - In a circle, there are number of points.
 - A greatest chord of a circle is a of it.
 - Any chord divides the circular region into two .
 - All diameters of the circle pass through the .
 - If two segments are equal, then their two arcs are in length.
 - Any sector of a circular region is the region enclosed by an arc and the two .
 - The length of the line segment joining a point outside the circle and the centre is than the length of radius.
- With the help of scale and pencil compass let us draw a circle and indicate the centre, a chord, a diameter, a radius, a major arc, a minor arc on it.
- Let us write true or false :**
 - Circle is a plane figure.
 - Segment of a circle is a plane region.
 - Sector of a circle is a plane region.
 - A chord is a straight line segment.
 - An arc is a straight line segment.
 - There are finite number of chords of same length in a circle.
 - One and only one circle can be drawn by taking a fixed point as its centre.
 - The lengths of the radii of two congruent circles are equal.

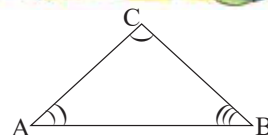


Keeping the keys of the house in separate key-rings and fixing them on a circular hard cardboard, they hung it in one corner of our reading room. But many extra long and hard copper wires of different lengths have been spread out here and there.

My brother Tirtha has formed a triangle ABC by fixing a copper wire with other two copper wires.



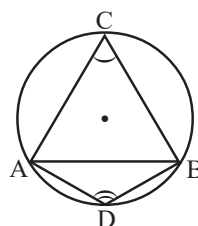
We are observing that the line segment AB forms $\angle ACB$ at the point C. **This angle formed by AB at the point C is subtended angle.**



In similar way, the line segment BC forms at the point . $\angle BAC$, is a subtended angle formed by BC at the point and the line segment CA forms at the point . $\angle ABC$, is a angle formed by AC at the point B.

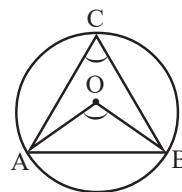
My friend Puja did a fun. She fixed some sticks inside a circular ring as in the adjoining figure.

We are observing that, in the circular key-ring, the chord AB of the circle has formed subtended angle $\angle ACB$ at the point C and subtended angle at the point D.



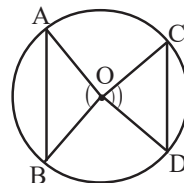
But $\angle ACB$ $\angle ADB$ [Putting = / $\neq$]

I draw a circle with its centre at O as in the adjoining figure. In this circle, I draw a chord AB other than a diameter. The chord AB will make the two subtended angles at the centre and at any point C on the circle – Let us measure them and write their relations with the help of a protractor. [Let me do it myself]



I draw a circle and by drawing many chords in this circle, let me see the types of subtended angles formed by them.

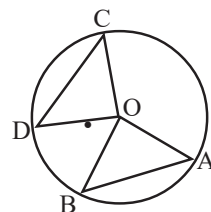
I have drawn two chords AB and CD in the circle with its centre at O, where $AB > CD$; the chord AB and the chord CD have subtended $\angle AOB$ and respectively at O. Measuring with the help of protractor, I am observing that $\angle AOB$ $\angle COD$, [Let me put $>$ / $<$]



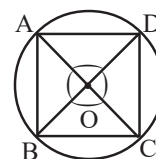
But if I draw some chords of equal lengths in a circle, then whether they will make equal subtended angles at the centre-let me see it by drawing and measuring with a protractor.

I have drawn two chords AB and CD of equal lengths in a circle with its centre at O, which have produced subtended angles at the centres and respectively.

Measuring with the help of the protractor, we are observing that, $\angle AOB$ $\angle COD$ [Let me put = / $\neq$]



In the adjoining figure, let us measure the subtended angles formed by the sides of the square ABCD at the centre O with the help of a protractor and determine the relation among them.



Hands on trial

- I have drawn a circle with its centre at O.
- In that circle, I have drawn two chords AB and CD of equal lengths.
- The two chords AB and CD have intercepted the two arcs $\widehat{APB}$ and $\widehat{CQD}$ respectively and have formed subtended angles $\angle AOB$ and $\angle COD$ at the centre respectively.
- Cutting the two sectors OAPB and OCQD with the scissors and putting them one upon another I am observing that the two sectors are overlapping completely.



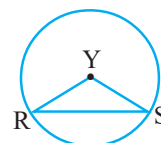
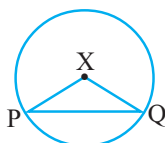
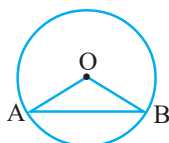
$\therefore$ I have got : the two arcs $\widehat{APB}$ and $\widehat{CQD}$ are equal
and $\angle AOB = \angle COD$



$\therefore$ By hands on trial, I have got, in a circle the chords of equal lengths cut off arcs of equal lengths and produce equal subtended angles at the centre.

Rebeya has drawn many congruent circles in her copy. i.e. the length of the radii of the circles are (equal / unequal)

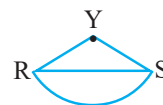
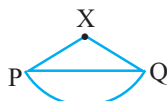
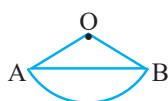
By drawing the chords of equal length in the congruent circles drawn by Rabeya, let us see what shall I get?



Measuring the subtended angles of the congruent circles with centres O, X and Y, produced by the chords AB, PQ and RS respectively of equal lengths, we are observing that, $\angle AOB$ $\angle PXQ$ $\angle RYS$ [Let us put = / $\neq$]



By hands on trial, I draw the chords of equal lengths in more than one circle with radii of equal lengths and cutting the angles with arcs which are formed by chords at the centres and placing them by one overlapping upon another, we are observing that.



The chords of equal lengths in the congruent circles cut the equal arcs and produce equal subtended angles at the centres. [Let me do it by hands on trial]

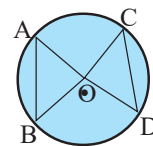


And in reverse, i.e. if more than one chord of a circle, produce equal subtended angles at the centre, then let us check the type of relation exists among the chords.

I have drawn a circle whose centre is O; by cutting this circular region with centre O and folding it to form the angles $\angle AOB$ and $\angle COD$ at the centre in such a way that $\angle AOB = \angle COD$.

By fixing strings or sticks at the points A and B and at the points C and D, and then by measuring the distances of AB and CD, we are observing that AB

CD [Let us put $=$ / $\neq$]

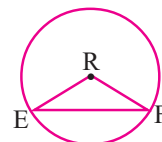
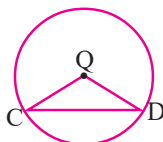
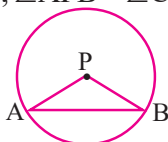


$\therefore$ By hands on trial, we are observing that if more than one chords of a circle produce equal angles at the centre then the chords are of equal lengths.

I draw another circle by taking different radius and in that circle I also draw more than one chords which make equal angles at the centre. Now measuring the chords with scale I am observing that the lengths of the chords are . [Let me write it by measuring]

$\therefore$ We have got, the chords of the circle, making equal subtended angles at the centre, are equal in length.

Tirtha has drawn many equal circles in his copy, whose centres are P, Q and R; I draw few chords viz AB, CD and EF in the circles drawn by Tritha which make equal subtended angles at the centre i.e., $\angle APB = \angle CQD = \angle ERF$



Measuring with scale, we are observing that, AB CD EF [Let us Put $=$ / $\neq$] [By drawing and measuring, let me do it, myself.]

$\therefore$ We have got, the chords of equal circles making equal subtended angles at the centres are equal in length.

Since they are equal in length, so the arcs intercepted by them are also (equal/unequal) in length.

In a circle, if the two equal subtended angles formed by two arcs, the lengths of two arcs are equal i.e., the lengths of the two chords are equal. [By hands on trial, let us check it by cutting papers]

Application : 1. In a circle, the chords are PQ, QR, RS and ST. If $PQ = QR = RS = ST$, then let us prove that, $PR = QS = RT$

Given : In the circle with its centre at O, the chords are PQ, QR, RS and ST and $PQ = QR = RS = ST$

To prove : $PR = QS = RT$

Proof : $PQ = QR$ So $\widehat{PQ} = \widehat{QR}$ (i) (Since equal chords of same circle intercept equal arcs)

Again, $QR = RS$ So $\widehat{QR} = \widehat{RS}$ (ii)

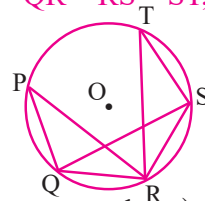
Adding (i) & (ii) we get, $\widehat{PQ} + \widehat{QR} = \widehat{QR} + \widehat{RS}$

$\therefore \widehat{PR} = \widehat{QS}$, $\therefore PR = QS$ (Since two arcs of same circle are equal in length so the two chords are equal in length)

Again, $RS = ST$, So $\widehat{RS} = \widehat{ST}$ (iii)

Adding (ii) & (iii) we get, $\widehat{QR} + \widehat{RS} = \widehat{RS} + \widehat{ST}$ So $\widehat{QS} = \widehat{RT}$ $\therefore QS = RT$

$\therefore PR = QS = RT$ (proved)



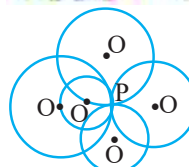
Today, some of my friends have decided to play a funny game. Some of our friends Rita, Masud, Diptarka and Tapas will draw some points on the board. We shall try to draw circles passing through these points and try to know their nature.



At first, Masud drew only one fixed point P on the board.

9 I draw circles passing through the fixed point P and see what I shall get.

Taking any point 'O' as the centre on the board, the circle, drawn with the length OP as radius, passes through the point P.

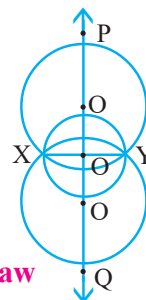


We are observing that we are getting [1/2/3/infinite] circles passing through the fixed point P.

Now Rita drew two points X and Y on the board.

10 I draw the circles passing through the points X and Y and see what I shall get.

Each point on the perpendicular bisector of the line segment joining the points X and Y is equidistant from X and Y. Taking any point O on the straight line PQ as the centre and the radius equal to the length of OX or OY, if a circle be drawn, it will pass through the points X and Y, where O is any point on the straight line PQ.



$\therefore$ We can draw circles passing through the points X and Y. **[Let me draw and write it myself]**

11 Now, Diptarka drew three non-collinear points X, Y and Z on the board.

I try to draw a circle through those three non-collinear points X, Y and Z.

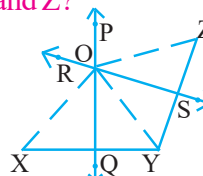
At first, by joining X, Y and Y, Z I got the line segments XY and YZ. For drawing the circle passing through the points X, Y and Z I determine the centre i.e. I determine the point which is equidistant from the points X, Y & Z.



12 But how shall I get the point which is equidistant from the points X, Y and Z?

I draw the perpendicular bisector PQ of the line segment XY. Each of the points on this line PQ is equidistant from the two points X and Y.

Again, I draw the perpendicular bisector RS of the line segment ZY. Each of the points on this line RS is equidistant from the two points Y and Z.



$\therefore$ The perpendicular bisectors of the line segments XY and YZ are PQ and RS respectively and since the points X, Y and Z are non-collinear, the line segments PQ and RS intersect each other at some point O (say).

$\therefore$ The point O is from the points X, Y and Z.

$\therefore$ If the circle is drawn by taking the point O as the centre and the length OX or OY or OZ as radius it will pass through the points X, Y and Z.

13 Is it possible to draw one and only one circle through the three non-collinear points X, Y and Z?

Since the three non-collinear points X, Y, Z are fixed, so, the two line segments XY and YZ are fixed and the two perpendicular bisectors PQ and RS are fixed. Therefore, the intersecting point of PQ and RS is 'O' i.e., the centre is fixed and the length of the radius OX is fixed. The only one circle drawn is possible by taking fixed centre and fixed length of radius.

$\therefore$ We got that drawing of one and only one circle is possible through the three non-collinear points X, Y & Z.

I am observing that by drawing a circle in the same way through any three non-collinear points, one and only one circle can be drawn through given three non-collinear points. **[Let me draw it myself]**

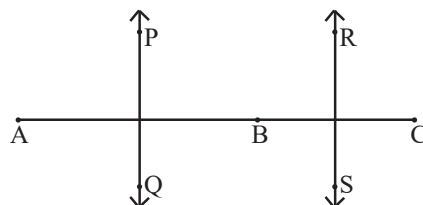
Now, Tapas drew three collinear points A, B & C on the board.

14 Let me see whether any circle can be drawn or not through three collinear points A, B and C.

I took three collinear points A, B and C. The perpendicular bisector of AB is PQ and the perpendicular bisector of BC is RS.

Since AB and BC are parts of same line-segment, So $PQ \parallel RS$. $\therefore$ It is not possible to get the point of intersection of PQ & RS.

$\therefore$ No circle can be drawn through three collinear points.



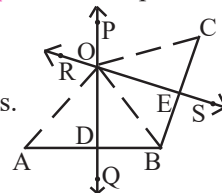
Theorem : 31. Only one circle can be drawn through three non-collinear points. (The proof is not included in the evaluation)

Given : A, B, C are three non-collinear points.

To prove : Only one circle can be drawn through three non-collinear points.

Construction : The two points A, B and the two points B and C are joined. The perpendicular bisectors of AB and BC are drawn and they are PQ & RS respectively.

Since the two line-segments AB and BC are not parallel, so their two perpendicular bisectors PQ and RS intersect at the point O. The straight lines PQ and RS intersect the two line-segments AB and BC respectively at the points D and E, O, A; O, B and O, C are joined.



Proof : In $\triangle OAD$ and $\triangle OBD$

$AD = BD$ ($\because$ OD is perpendicular bisector of the line-segment AB)

$\angle ODA = \angle ODB$ ($\because$ Each is right angle)

OD is common side $\therefore \triangle OAD \cong \triangle OBD$ (By S - A - S axiom of congruency)

$\therefore OA = OB$ (Corresponding sides of congruency)

Similarly, $\triangle OBE \cong \triangle OCE$

$\therefore OB = OC$

$\therefore OA = OB = OC$

$\therefore$ If the circle is drawn with its centre at O and the radius equal to the length of OA, it will pass through the points A, B, C.



So, only one circle can be drawn through three non-collinear points.

The three points A, B, C are fixed. So the two line-segments AB and BC are fixed.

The perpendicular bisectors of the two line-segments AB and BC are the straight lines PQ and RS respectively. PQ and RS are also fixed.

So their point of intersection O is fixed.

Again since the two points O and A are fixed,

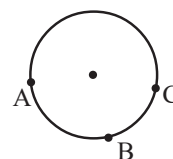
So the length of the line-segment OA is fixed. Therefore, only one circle can be drawn by taking the fixed point 'O' at the centre with a fixed length of radius.

$\therefore$ Only one circle can be drawn through three non-collinear points A, B, C.

15 My sister has drawn a circle with the help of her circular bangle.

I find out the centre of this circle.

I take any three points A, B and C on the circle. Now I find out the centre of the circle by determining the point of intersection of the perpendicular bisectors of the line-segments AB and BC. [Let me do it myself].



16 If an arc of a circle is given, then let us see, how the circle can be drawn.

I took a point R on the arc PQ. I draw a circle passing through the points P, R and Q by determining its centre as before. **[Let me do it myself]**



We are observing that one and only one circle can be drawn through three non-collinear points. But is it possible to draw a circle with more than three points?

It may not be possible to draw a circle passing through more than three points. If it is possible, then the points are called **Concyclic**.

17 What is called the quadrilateral whose vertices are situated on the circle?

The quadrilateral whose vertices lie on a circle is called **Cyclic Quadrilateral**.

18 What is Isosceles Trapezium?

The quadrilateral of which one pair of opposite sides is parallel and other pair of opposite sides is unparallel and equal i.e., the two transversal sides are equal is called **Isosceles Trapezium**.

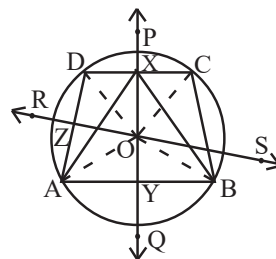
The parallelogram is a trapezium. But is it an isosceles trapezium? **[Let me write it myself.]**

Application : 2. I prove with reason that the vertices of an isosceles trapezium are concyclic.

Given : ABCD is an isosceles trapezium. $AB \parallel DC$ and $AD = BC$

To prove : The vertices of the trapezium are concyclic.

Construction : I draw the perpendicular bisector PQ of the side DC and the perpendicular bisector RS of the side AD which intersect DC at the point X and AD at the point Z respectively. PQ intersects AB at the point Y. A, X and B, X are joined. PQ intersects RS at the point O. I join A, B, C, and D with the point O.



Proof : In $\triangle ADX$ and $\triangle BCX$, $DX = CX$ [$\because$ the perpendicular bisector of CD is PQ]

$$\angle ADX = \angle BCX$$

[$\because$ In the isosceles trapezium, the angles associated with a side of the two parallel sides are equal]

$$AD = BC \text{ [Given]}$$

$\therefore \triangle ADX \cong \triangle BCX$ [By S-A-S axiom of congruency]

$\therefore AX = BX$ [Corresponding sides of congruent triangles.]

In right angled triangle $\triangle AXY$ and right-angled triangle $\triangle BXY$, $\angle AYX = \angle BYX$

[$\because AB \parallel CD$ and $CD \perp PQ$, $\therefore AB \perp PQ$]

Hypotenuse $AX =$ Hypotenuse BX [**proved it before**] and XY is common side of both triangles.

$\therefore \triangle AXY \cong \triangle BXY$ [by R-H-S axiom of congruency]

$\therefore AY = BY$ [Corresponding sides of congruent triangles]

$\therefore PQ$ is the perpendicular bisector of AB .

$\therefore O$ is situated on the perpendicular bisector PQ of DC .

$\therefore DO = CO$; similarly $DO = AO$ and $AO = BO$

$\therefore CO = DO = AO = BO$

$\therefore$ By taking O as centre with the radius of the equal length of OC or OD or OA or OB , if a circle is drawn, then it will pass through the points A, B, C and D . i.e., The vertices of the isosceles trapezium are concyclic. (**proved**)

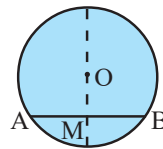




We have cut off circular region by drawing many circles. My brother Rana has drawn some chords which are not diameters.

Hands on trial

- (i) I took a circular region with its centre 'O' and took the chord AB which is not a diameter.
- (ii) I have folded the circular region with its centre 'O' in such a way that the fold passes through the point O and a part of the straight line remains on the other part. The fold intersects the line-segment AB at the point M; OM is joined.
- (iii) By measuring, I observed that $\angle OMA = \angle OMB = 1$ right angle and $AM = BM$.

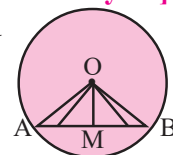


$\therefore$ By hands on trial, I got that the perpendicular drawn from the centre of a circle to the chord which is not a diameter, bisects the chord.

By cutting out another circular region, I am observing by hands on trial that a perpendicular to a chord which is not a diameter from the centre of the circle bisects the chord. **[Let me do it myself]**

We took a circular region whose centre is O and chord is AB; the centre O is joined with different points of the chord and got many line-segments of different lengths.

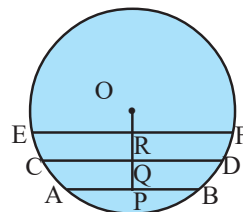
- 19** But among these different distances which will we call "the distance of the chord AB from the centre O?"



Among these, the length of the line-segment which is perpendicular on AB from the centre O is called the distance of the chord AB from the centre O or it is called the **perpendicular distance**.

- 20** I measure the perpendicular distances of the chords from the centre by taking another circular region with more than one chords drawing in it and what I shall get-let me see.

I have got that the perpendicular distances of the chords AB, CD and EF from the centre O of the circle, with its centre at O are OP, OQ and respectively. [Let me write it by observing the adjoining figure]



I am observing that, if the lengths of chords of a circle will increase, the perpendicular distances from the centre will [increase / decrease].

I am also observing that, the greatest chord of a circle i.e., passes through the centre.

$\therefore$ In that case, the perpendicular distance from the centre is Zero. $\therefore$ I have got that the perpendicular distance of the diameter from the centre of a circle is .

Let us prove with reason

Theorem : 32. The perpendicular drawn to a chord, which is not a diameter, from the centre of the circle, bisects the chord.

Given : AB is a chord of the circle with its centre at O, which is not a diameter, and OD, is perpendicular on the chord AB.

To prove : OD, bisects the chord AB i.e., $AD = DB$.

Construction : O, A and O, B are joined.

Proof : OD is perpendicular on the chord AB.

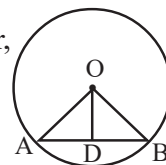
$\therefore \triangle ODA$ and $\triangle ODB$ are right-angled triangles.

$\therefore$ In right-angled $\triangle ODA$ and $\triangle ODB$, $\angle ODA = \angle ODB$ (Each is right-angle)

Hypotenuse $OA =$ Hypotenuse OB [radii of same circle] and OD is common side.

$\therefore \triangle ODA \cong \triangle ODB$ [By S-A-S axiom of congruency]

$\therefore AD = DB$ [Corresponding sides of the congruent triangles] **[proved]**



Application : 3. I draw a chord PQ of the circle with the centre A, which is not a diameter. I draw a perpendicular AM on PQ from A. I prove with reason that $PM = MQ$. [Let me prove it by drawing]

We have proved that if a perpendicular is drawn on the chord of a circle, which is not a diameter, from the centre, then it will bisect the chord.

21 But can the converse theorem of this theorem be possible? i.e., will the line-segment joining the mid-point of the chord, which is not a diameter, and the centre of the circle be a perpendicular on that chord?

Hands on trial

- At first, I cut out the circular region with its centre at O after drawing it. I draw a chord AB, which is not a diameter of that circle.
- For determining the mid-point of the chord AB, folding the papers by hands on trial, the chord of the circular paper AB is folded in such a way that the point A is mixing with the point B. I unfold the paper and the fold intersects AB at the point D. i.e., the line-segment joining the centre O and the mid-point of the chord AB is perpendicular on the chord AB.

$\therefore OD \perp AB$

$\therefore$ By hands on trial, I have got that if any straight line passing through the centre of the circle bisects any chord, which is not a diameter then the straight line will be perpendicular on that chord.



Let us prove with reason

Theorem : 33. Let us prove that if any straight line passing through the centre of a circle bisects any chord, which is not a diameter, then the straight line will be perpendicular on that chord.

Given : Let, AB be a chord of the circle with its centre at O, which is not a diameter and D, is the mid-point of AB i.e., $AD = DB$.

To prove : $OD \perp AB$ i.e., OD, is perpendicular on the chord AB.

Construction : I join O, A and O, B.

Proof : In $\triangle OAD$ and $\triangle OBD$

$OA = OB$ [radii of same circle]

$AD = DB$ [given] [D, is the mid-point AB]

and OD is common side

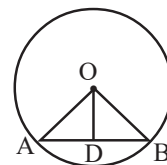
$\therefore \triangle OAD \cong \triangle ODB$ [By S-S-S axiom of congruency]

$\therefore \angle ODA = \angle ODB$ [Corresponding parts of the congruent triangles]

Since, OD, makes equal angles on the chord AB

So, $\angle ODA = \angle ODB = 1$ right-angle

$\therefore OD \perp AB$ [proved]



Application : 4. I prove the theorem-33 by the proof of congruency of $\triangle OAD$ and $\triangle OBD$ with the help of S-A-S axiom of congruency. [Let me do it myself]

Application : 5. Niyamat has drawn a circle whose radius is 13 cm. in length. I have drawn a chord AB with the length of 10 cm on it. Let us write by calculating, the distance of the chord AB from the centre of the circle.

Let, the centre of the circle be O; the perpendicular OM, drawn on the chord AB from O, intersects AB at the point M.

$\therefore AM = \frac{1}{2} AB = \square$ cm. [$\because$ The perpendicular drawn on the chord, which is not a diameter, from the centre of circle, bisects the chord.]

In right-angled triangle AMO,

$$OA^2 = AM^2 + OM^2 \text{ (by Pythagoras theorem)}$$

$$OM^2 = OA^2 - AM^2 = (13^2 - 5^2) \text{ sq. cm.} = \square \text{ sq. cm.}$$

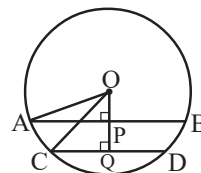
$$\therefore OM = 12 \text{ cm.}$$

$\therefore$ We have got that the perpendicular distance of the chord with length of 10 cm. from the centre of the circle having the of radius of 13 cm. is 12 cm.

Application : 6. The perpendicular distance of a chord from the centre of a circle, having the radius of 17 cm. is 8 cm. in length. Let us write by calculating, the length of this chord. [Let me do it myself]

Application: 7. The lengths of two parallel chords of a circle with the radius 10 cm. in length are 16 cm. and 12 cm. Let us see by calculating, the distance between these two chords, if they are in (i) same side (ii) opposite side of the centre.

- (i) Let, the length of the radius of the circle with its centre O be 10 cm. and the two chords AB and CD are in the same side of the centre. The lengths of AB and CD are 16 cm. and 12 cm. respectively. $AB \parallel CD$.



From the point O, a perpendicular OQ is drawn on the chord CD which intersects AB at the point P.

Since $AB \parallel CD$ and $OQ \perp CD$, so, $OP \perp AB$.

$\angle OQD = \text{Corresponding } \angle OPB. \therefore \angle OQD = 90^\circ, \therefore \angle OPB = 90^\circ$

$$\therefore AP = \frac{1}{2} AB = \frac{1}{2} \times 16 \text{ cm.} = 8 \text{ cm.}$$

Again, $OA = 10 \text{ cm.}$

$\therefore$ In right-angled $\triangle APO$,

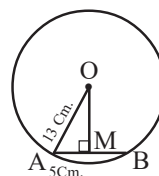
$$OP^2 = OA^2 - AP^2 = (10^2 - 8^2) \text{ sq. cm.} = \square \text{ sq. cm.}$$

$$\therefore OP = 6 \text{ cm.}$$

$\therefore OQ \perp CD \therefore CQ = \square$ [Let me write it myself]

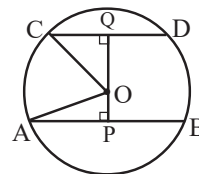
$\therefore$ From right-angled $\triangle OCQ$, we got, $OQ = \square$ [Let me write it myself]

$\therefore$ The distance between the chords AB and CD is $PQ = OQ - OP$
 $= (8 - 6) \text{ cm.} = 2 \text{ cm.}$



(ii) But if the two chords AB and CD would remain in opposite side of the centre of the circle.

In that case, the distance of two chords AB and CD = PQ
 $= OP + OQ = (6 + 8) \text{ cm.}$
 $= 14 \text{ cm.}$



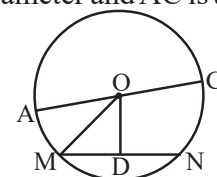
Application : 8. In a circle with the radius of 5 cm in length, the two parallel chords of length 8cm. and 6 cm. are situated on opposite sides of the centre. Let us write by calculating, the distance between two chords. **[Let me do it myself]**

Application : 9. Let us prove that the greatest chord of a circle is its diameter.

Given : In a circle with its centre at O, MN is any chord which is not a diameter and AC is a diameter.

To prove : $AC > MN$ i.e., the diameter is the greatest chord.

Construction : From the centre O, the perpendicular OD is drawn on the chord MN. O, M are joined.



Proof: $OM > MD$ [$\because$ OMD is a right-angled triangle and OM is its hypotenuse.]

or, $OA > MD$ [$\because$ $OA = OM$, the radii of same circle]

or, $\frac{1}{2} AC > \frac{1}{2} MN$

or, $AC > MN$

$\therefore$ Diameter is greatest chord in a circle. **(proved)**

The sum of the lengths of any two sides of a triangle is greater than the length of the third side. With the help of this theorem, Let us try to prove the application – 9

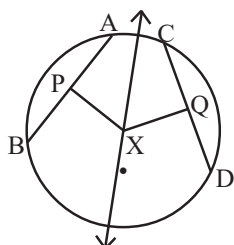
My friend Merri, has drawn two chords AB and CD of same length in a circle with its centre X.

I drew two perpendiculars XP and XQ on two chords AB and CD of same lengths from the centre X.



By hands on trial, I find out the relation between XP and XQ by folding the paper.

Hands on trial



(i) In a tracing-paper, like above, two chords of same lengths AB and CD are drawn in a circle with its centre at X. From X two perpendiculars XP and XQ are drawn.

(ii) Now, by cutting out the circular region of the tracing paper I fold it thrice in such a way that the point A is mixed with the point C and the point B is mixed with the point D.

I am observing that, the point P is mixed with the point Q and by unfolding it I observe that the fold passes through the point X.

$\therefore$ We have got $XP = XQ$

$\therefore$ By hands on trial, we have got that two chords of same length are equidistant from the centre.

Application : 10. I prove with reason that two equal chords of any circle are equidistant from its centre.

Given : In a circle with its centre O, two equal chords are AB and CD. The distances of AB and CD from the centre O are OE and OF respectively i.e., $OE \perp AB$ and $OF \perp CD$.

To prove : $OE = OF$

Construction : O, A and O, C are joined.

Proof : $OE \perp AB$ and $OF \perp CD$ [Given]

$\therefore AE = \frac{1}{2} AB$ and $CF = \frac{1}{2} CD$ [Since, the perpendicular drawn from the centre to a chord, which is not a diameter, bisects the chord]

Again, $AB = CD$ [Given]

$\therefore AE = CF$ (i)

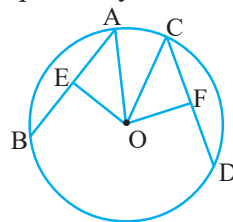
$\therefore$ In right-angled $\triangle AEO$ and in right-angled $\triangle CFO$, $\angle OEA = \angle OFC$ [Each is right-angle]

Hypotenuse $OA =$ Hypotenuse OC , [radii of same circle]

$AE = CF$ [From (i)]

$\triangle AEO \cong \triangle CFO$ [By R-H-S axiom of congruency]

$\therefore OE = OF$ [proved]



I justify by hands on trial and prove, with reason that, two equal chords of a circle are equidistant from its centre.

But is the converse of it true? i.e. if two chords of a circle are equidistant from its centre, then will they be equal? Let us justify it by hands on trial and prove it with reason.

Hands on trial

(i) On a tracing paper, I drew two equal line-segments OP and OQ in a circle with its centre O. Now I drew two chords AB and CD so that $AB \perp OP$ and $CD \perp OQ$.

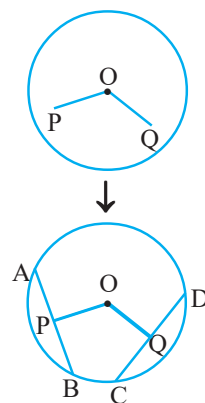
(ii) Now, I cut out the paper with circular region and fold it twice along the point O so that the point P coincides with the point Q. But I am observing that the chord is overlapped on the chord CD.

$\therefore$ We have got $AB = CD$

$\therefore$ By hands on trial I have got that if the distances of two chords of a circle from the centre are equal, then the lengths of two chords are also equal. **(I prove it myself with reason)**

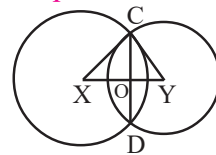
Application : 11. Let us prove that the perpendicular bisector of a chord of a circle passes through its centre. [Let me prove it myself]

Application : 12. Let us prove that a straight line can not intersect a circle at more than two points. [Let me prove it myself]



Application : 13. If two circles intersect each other at two points, then let us prove that their two centres lie on the perpendicular bisector of their common chord.

Given : The two circles with the centres at X and Y intersect each other at the points C and D. So CD is their common chord.



To prove : The two points X and Y lie on the perpendicular bisector of the common chord CD.

Construction : From the point X, the perpendicular XO is drawn on CD, O and Y are joined.

Proof : CD is a chord of the circle with its centre X and $XO \perp CD$.

$\therefore$ O, is the mid-point of CD.

Again, CD is a chord of the circle with its centre Y and O, is the mid-point of CD.

$\therefore$ $OY \perp CD$

Since only one perpendicular can be drawn at a point on a straight line,

So, XO and OY are situated on the same line.

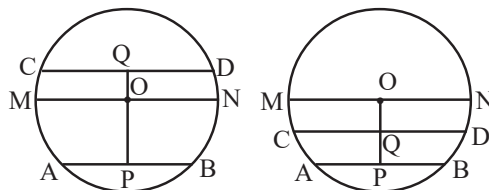
$\therefore$ The XY is the perpendicular bisector of the common chord CD.

$\therefore$ The two centres X and Y of the two circles lie on the perpendicular bisector of their common chord CD. **[proved]**



Application : 14. I prove with reason that the line joining two mid-points of two parallel chords of a circle passes through its centre.

Given : Let, two chords AB and CD of a circle with its centre O, are parallel to each other and the mid-points of AB and CD are P and Q respectively.



To prove : PQ, passes through O.

Construction : O, P and O, Q are joined and through O the line-segment MN is drawn parallel to AB and CD.

Proof : P, is the mid-point of the chord AB. $\therefore$ $OP \perp AB$

Again, $AB \parallel MN$, $\therefore$ $OP \perp MN$

Similarly, $OQ \perp CD$ [$\because$ Q, is the mid point of CD]

$\therefore$ $OQ \perp MN$ [$\because$ $MN \parallel CD$]

$\therefore$ Both OP & OQ are perpendicular on MN at the point O.

Since, only one perpendicular can be drawn on a straight line at a point.

So, P, O and Q are collinear.

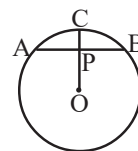
$\therefore$ PQ passes through the centre O of the circle. **[proved]**



Let us work out

3.2

1. The length of a radius of a circle with its centre O is 5 cm. and the length of its chord AB is 8 cm. Let us write by calculating, the distance of the chord AB from the centre O.
2. The length of a diameter of a circle with its centre at O is 26 cm. The distance of the chord PQ from the point O is 5 cm. Let us write by calculating, the length of the chord PQ.
3. The length of a chord PQ of a circle with its centre O is 4 cm. and the distance of PQ from the point O is 2.1 cm. Let us write by calculating, the length of its diameter.
4. The lengths of two chords of a circle with its centre O are 6 cm. and 8 cm. If the distance of smaller chord from centre is 4 cm., then let us write by calculating, the distance of other chord from the centre.
5. If the length of a chord of a circle is 48 cm and the distance of it from the centre is 7 cm. then let us write by calculating, the length of the chord which is 20 cm. distance away from the centre of the circle.
6. In the circle of adjoining figure with its centre at O, $OP \perp AB$; if $AB = 6$ cm. and $PC = 2$ cm., then let us write by calculating, the length of radius of the circle.
7. A straight line intersects one of the two concentric circles at the points A and B and the other at the point C and D. I prove with reason that $AC = DB$.
8. I prove that, the two intersecting chords of any circle can not bisect each other unless both of them are diameters of the circle.
9. The two circles with centres X and Y intersect each other at the points A and B. A is joined with the mid-point 'S' of XY and the perpendicular on SA through the point A is drawn which intersects the two circles at the points P and Q. Let us prove that $PA = AQ$.
10. The two parallel chords AB and CD with the lengths of 10 cm and 24 cm in a circle are situated on the opposite sides of the centre. If the distance between two chords AB and CD is 17 cm., then let us write by calculating, the length of the radius of the circle.



Hints : let the length of the radius of the circle be r cm. and the distance of the chord AB from the centre be x cm. $\therefore$ The distance of the chord CD from the centre is $(17 - x)$ cm.
 $\therefore r^2 = x^2 + 5^2$ & $r^2 = (17 - x)^2 + (12)^2$, So, $x^2 + 5^2 = (17 - x)^2 + 12^2 \therefore x = 12$

11. The centres of two circles are P and Q; they intersect at the points A and B. The straight line parallel to the line-segment PQ through the point A intersects the two circles at the points C and D. I prove that, $CD = 2PQ$.
12. The two chords AB and AC of a circle are equal. I prove that, the bisector of $\angle BAC$ passes through the centre.
13. If the angle -bisector of two intersecting chords of a circle passes through its centre, then let me prove that the two chords are equal.
14. I prove that, among two chords of a circle the length of the chord nearer to centre is greater than the length of the other.
15. Let us write by proving which chord with the least length through any point in a circle.

16. **V.S.A.**

(A) **M.C.Q. :**

- (i) The lengths of two chords AB and CD of a circle with centre O are equal. If $\angle AOB = 60^\circ$, then the value of $\angle COD$ is
(a) 40° (b) 30° (c) 60° (d) 90°
- (ii) The length of a radius of a circle is 13 cm. and the length of a chord of a circle is 10 cm., the distance of the chord from the centre of the circle is (a) 12.5 cm. (b) 12 cm. (c) $\sqrt{69}$ cm. (d) 24 cm.
- (iii) AB and CD are two equal chords of a circle with its centre O. If the distance of the chord AB from the point O is 4 cm. then the distance of the chord CD from the centre O of the circle is
(a) 2 cm. (b) 4 cm. (c) 6 cm. (d) 8 cm.
- (iv) The length of each of two parallel chords AB and CD is 16 cm. If the length of the radius of the circle is 10 cm., then the distance between two chords is (a) 12cm. (b) 16 cm. (c) 20 cm. (d) 5 cm.
- (v) The centre of two concentric circles is O; a straight line intersects a circle at the points A and B and other circle at the points C and D. If $AC = 5$ cm., then the length of BD is (a) 2.5 cm. (b) 5 cm. (c) 10 cm. (d) none of these

(B) **Let us write True/False :**

- (i) Only one circle can be drawn through three collinear points.
- (ii) The two circles ABCDA and ABCEA are same circle.
- (iii) If two chords AB and AC of a circle with its centre O are situated on the opposite sides of the radius OA, then $\angle OAB = \angle OAC$.

(C) **Let us fill in the blanks :**

- (i) If the ratio of two chords PQ and RS of a circle with its centre O is 1:1, then, $\angle POQ : \angle ROS = \underline{\hspace{2cm}}$.
- (ii) The perpendicular bisector of any chord of a circle is _____ of that circle.

17. **S.A. :**

- (i) Two equal circles of radius 10 cm. intersect each other and the length of their common chord is 12 cm. Let us determine the distance between the two centres of two circles.
- (ii) AB and AC are two equal chords of a circle having the radius of 5 cm. The centre of the circle is situated at the outside of the triangle ABC. If $AB = AC = 6$ cm., then let us calculate the length of the chord BC.
- (iii) The lengths of two chords AB and CD of a circle with its centre O are equal. If $\angle AOB = 60^\circ$ and $CD = 6$ cm., then let us calculate the length of the radius of the circle.
- (iv) P is any point in a circle with its centre O. If the length of the radius is 5 cm. and $OP = 3$ cm., then let us determine the least length of the chord passing through the point P.
- (v) The two circles with their centres at P and Q intersect each other. at the points A and B. Through the point A, a straight line parallel to PQ intersects the two circles at the points C and D respectively. If $PQ = 5$ cm., then let us determine the length of CD.

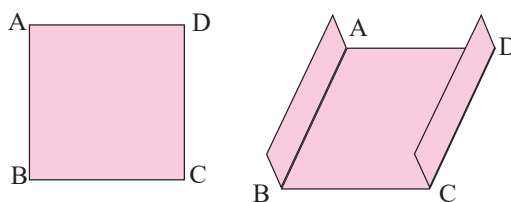
4

RECTANGULAR PARALLELOPIPED OR CUBOID

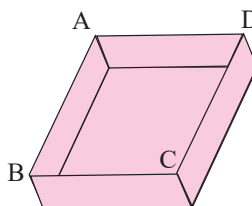
Our 'First Aid' box has been damaged. A new 'First Aid' box has to be made. So, today we all brothers and sisters, have assembled on the roof of our house in the holiday afternoon. At first, we will try to make a box.



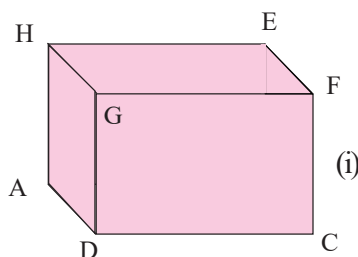
At first, we cut out a piece of rectangular cardboard ABCD. Sathi has fixed rectangular pieces like adjoining figure by cutting out two other rectangular pieces of same measures i.e the length AB of any breadth.



Shakil too like Sathi fixed two other rectangular pieces as in the adjoining figure with the same length of AD and with previous breadth.



I cut out another rectangular piece HEFG similar to the rectangular piece ABCD with same length and breadth and by fixing it on the opposite surface of ABCD, I have made a box like the adjoining figure.



1 What these solids with rectangular surfaces of which the opposite surfaces are of equal measures and adjacent surfaces are perpendicular to each other, are called?

These solids, of which each surface is rectangular and the length and breadth of opposite surfaces are equal and adjacent surfaces are perpendicular to each other, are called **Rectangular parallelepiped or Cuboid**.

I am observing that, the rectangular parallelepiped is situated on the rectangular surface ABCD.

2 In this position, what the surface ABCD is called?

In this position, the surface ABCD is called base of the rectangular parallelepiped and one of the two sides of the rectangular **base** is length and the other one is breadth. I have understood that the **length** and **breadth** of the rectangular parallelepiped (i) are AB and BC.

3 But what is called BE of the rectangular parallelepiped (i)?

BE is called the height of the rectangular parallelepiped (i)

The length, breadth and height are the **dimensions** of rectangular parallelepiped.

The rectangular parallelepiped has [1 / 2 / 3] dimensions

Again, I am observing that the rectangular parallelepiped has [4 / 5 / 6] surfaces



$\therefore$ Whole surface area of a rectangular parallelopiped
 $=$ Sum of area of 6 surfaces
 $= 2 (AB \times BC + AB \times BE + BC \times BE)$
 $= 2 (\text{length} \times \text{breadth} + \text{length} \times \text{height} + \text{breadth} \times \text{height})$



4 Two surfaces of a rectangular parallelopiped intersect each other at a straight line. What this intersecting line-segment is called?

The intersecting line segment of two surfaces of rectangular parallelopiped is called **edge**.

We are observing that, a rectangular parallelopiped has 12 **edges**.

Let me write the edges of the rectangular parallelopiped (i) by observing the figure. **[Let me write it myself]**

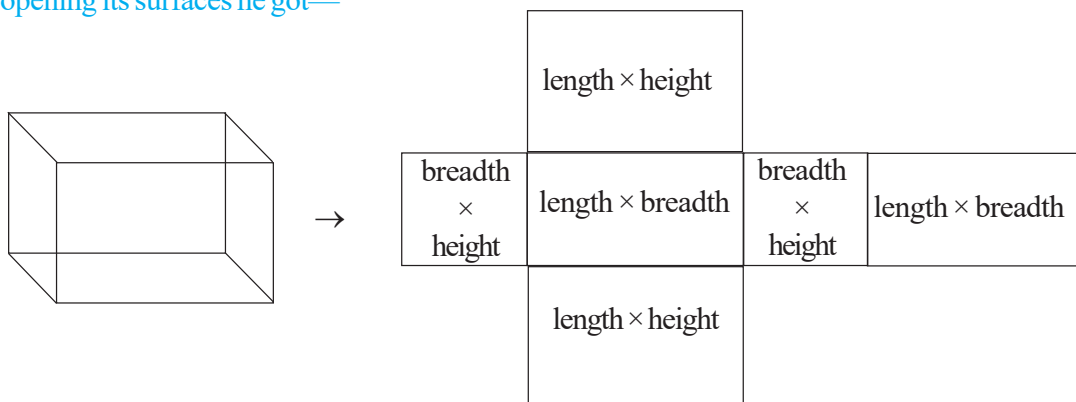
5 What the meet points of the edges of rectangular parallelopiped are called?

The meet points of the edges of a rectangular parallelopiped are called **Vertex**.

I am observing that, a rectangular parallelopiped has [6 / 7 / 8] vertices.

Let me write by understanding, the vertices of the rectangular parallelopiped (i) **[Let me write it myself]**

My brother has done a fun. He brought his rectangular parallelopiped box of toys and by opening its surfaces he got—



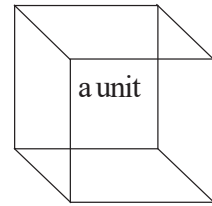
Application : 1 I am observing that the length, breadth and height of the rectangular parallelopiped box, brought by my brother, are 40cm., 25cm. and 15 cm. respectively.

$\therefore$ The total surface area of this box
 $= 2 (40 \times 25 + 40 \times 15 + 25 \times 15) \text{ cm}^2$.
 $=$ cm^2 .



Application : 2. The length, breadth and height of a rectangular parallelopiped box are 15cm., 12cm. and 20cm. respectively Let me write its total surface area. **[Let me do it myself]**

My friend Rajia has brought a cardboard box from her house. I am observing that each surface of it is a square i.e the length, breadth and height of the box are equal.



6 What this type of rectangular parallelopiped is called ?

The rectangular parallelopiped, whose length, breadth and height are equal, is called **Cube**.

The total surface area of a cube = $2 \times \{a \times a + a \times a + a \times a\}$ sq. unit

$$= 2 \times 3a^2 \text{ sq. unit}$$

$$= 6a^2 \text{ sq. unit}$$

[Where length of one side of a cube = a unit]

Application : 3 By measuring, I see that the length of one side of the cube, brought by Rajia is 27 cm.

$\therefore$ The total surface area of the cube = $6 \cdot (27)^2$ sq. unit = sq. unit [Let me write it myself]

Application : 4 Let us write by calculating, the quantity of coloured papers required to cover the whole surface of a cube whose length of one side is 12 cm. [Let me write it myself]

Application : 5 The length, breadth and height of our living room are 7m., 5m. and 4m respectively. The shape of the room is [cube/rectangular parallelopiped]

Let us calculate the total area to colour four walls of the room.

We will colour four walls of our room but we shall not colour the floor and the roof of it

$\therefore$ We must have to colour = $2 \times \text{length} \times \text{height} + 2 \times \text{breadth} \times \text{height}$

$$= 2 \times 7 \times 4 \text{ sq.m} + 2 \times 5 \times 4 \text{ sq.m.}$$

$$= \text{ sq.m.}$$



Area of four walls of a room = $2 \times (\text{length} + \text{breadth}) \times \text{height}$ = perimeter of the base $\times$ height

Application : 6 Let us write the length of one side of a cube whose total surface area is 150 sq.cm.

Let, the length of one side of the cube be a cm.

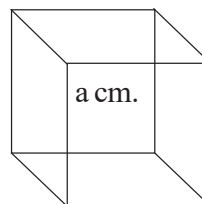
$\therefore$ The total surface area of the cube is $6a^2$ sq.cm.

$\therefore$ According to the given condition,

$$6a^2 = 150$$

$$\text{or, } a^2 = \frac{150}{6} = \text{$$

$$\therefore a = \pm 5$$



But $a \neq -5$, as length is always positive, $\therefore a = 5$; So, the length of one side of the cube is 5 cm.

Application : 7 Let us calculate the length of one side of cube whose total surface area is 486 sq.m. [Let me do it myself]

We have made a "First Aid" box and we have arranged rectangular parallelopiped and cube-shaped boxes by covering them with coloured papers. We have decided that we shall keep our necessary things in it.

My brother is putting his wooden scale in a green cuboid box.

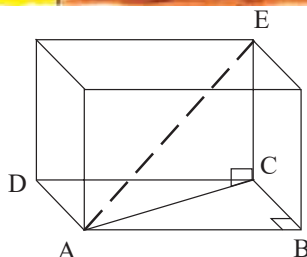
Let us see, the longest scale that can be put in this box.



7 How shall we get the length of a diagonal of a cuboid?

By drawing a picture, we try to determine the length of a diagonal of a cuboid.

Let, in the adjoining figure,
length = AB unit
breadth = BC unit
height = CE unit



In right angled triangle ABC, $AC^2 = AB^2 + BC^2$ _____ (i)

Again, in right angled triangle ACE, $AE^2 = AC^2 + CE^2 = AB^2 + BC^2 + CE^2$ [from (i)]

[If a straight line is perpendicular on a plane, then it will be perpendicular on any straight line on the plane passing through the point at which the perpendicular straight line intersects the plane. In the figure, the line-segment EC is perpendicular on the plane ABCD. So, the line-segment EC is perpendicular on CB, CD and CA at the point C. So, $\angle ACE = 90^\circ$]

$$\therefore AE = \sqrt{AB^2 + BC^2 + CE^2} \text{ unit}$$

$$\therefore \text{The length of any diagonal of a cuboid} = \sqrt{(\text{length})^2 + (\text{breadth})^2 + (\text{height})^2}$$



Application : 8 If the length, breadth and height of a cuboid box are 20 cm., 15cm. and 10 cm. respectively. What is the length of its diagonal?

$$\text{The length of the diagonal will be} = \sqrt{20^2 + 15^2 + 10^2} \text{ cm.} = 5\sqrt{29} \text{ cm.}$$

i.e., in that case, the longest scale of length $5\sqrt{29}$ cm. can be put in that cuboid box.

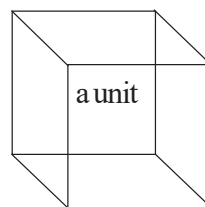
8 But what will be the length of a diagonal of a cube— Let us see.

Let, the length of one side of the cube be a unit.

$$\therefore \text{The length of the diagonal of the cube} = \sqrt{a^2 + a^2 + a^2} \text{ unit} = \sqrt{3} \text{ a unit}$$

$$\therefore \text{The length of any diagonal of a cube} = \sqrt{3} \times \text{length of one side}$$

$$\therefore \text{The length of the diagonal of a cube with its side of 5 cm. length is} = \sqrt{3} \times 5 \text{ cm.} = 5\sqrt{3} \text{ cm.}$$



Application : 9 A cuboidal room has its length, breadth and height as a , b and c unit respectively and if $a+b+c = 25$ units, $ab+bc+ca = 240.5$ units then let us write the length of the longest rod that can be kept in this room.

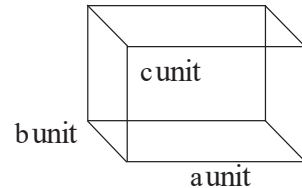
If the length = a unit, breadth = b unit and height = c unit of the room, then the longest rod that can be kept in the room = the length of the diagonal of the room = $\sqrt{a^2+b^2+c^2}$ unit

$$(a+b+c)^2 = a^2+b^2+c^2 + 2 \times \boxed{} \quad \text{[Let me write it myself]}$$

$$\text{or, } 25^2 = a^2+b^2+c^2 + 2 \times 240.5$$

$$\therefore a^2+b^2+c^2 = 625 - 481 = \boxed{}$$

$$\therefore \text{The length of the diagonal of the room} = \sqrt{a^2+b^2+c^2} \text{ unit} = \sqrt{144} \text{ unit} = \boxed{} \text{ unit}$$



Application : 10 I have made a rectangular parallelopiped by joining two cubes side by side made by Mita, whose edge is 8 cm in length. Let us calculate the total surface area and the length of the diagonal of the rectangular parallelopiped which is made in this way. [Let me do it myself]

Hints : The length of the rectangular parallelopiped which is made = $(8+8)$ cm. = 16 cm.,
breadth = 8 cm., height = 8 cm.

My friend Tathagata has coloured the surfaces of a cuboidal box of tin whose one face is open. He has decided that he will keep this box by filling sand in it to a corner of the roof. Sand is necessary for different works of the house.



9 But how shall I get the quantity of the sand that can be filled in that cuboidal box?

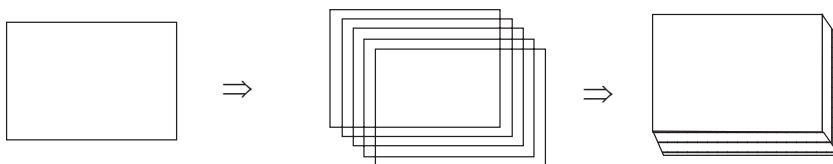
The quantity of the sand will be equal to the volume of the cuboidal box.

10 What is this volume? How shall we measure the volume of the cuboid?

The place which a solid occupies is the volume of that solid.

The volume of a cuboid = length $\times$ breadth $\times$ height

I have collected many surfaces of rectangular cardboard of same size and I have tried to make cuboid by putting them one after another and observed that as the height of the cuboid is increasing its volume is $\boxed{}$ [increasing / decreasing] |



Area of a rectangular cardboard = length $\times$ breadth

$\therefore$ We can write, **Volume of a cuboid = area of the base $\times$ height**

Application : 11 By measuring the cuboidal box, I am observing that the length is 32 cm. breadth is 21 cm. and height is 15 cm.

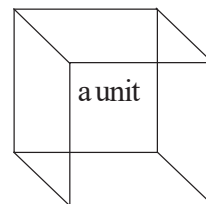
∴ The quantity of sand that can be filled in that box is = $(32 \times 21 \times 15)$ c.c = c.c

11 But how shall we measure the volume of that box if each of its edges is of equal length i.e., how shall we get the volume of a cube?

Let the length of one side of the cube be a unit

∴ Volume of the cube = $(a \times a \times a)$ unit³ = a^3 unit³

∴ Volume of a cube = (length of one side)³



Application : 12 The length of one side of a cube is 5 cm., its volume is c.c = c.c
[Let me do it myself]

Application : 13 If the numerical values of total surface area and volume of a cube are equal, then let us calculate the length of its diagonal.

Let the length of one side of the cube = a unit

∴ The total surface area of the cube = $6a^2$ sq.unit and its volume = a^3 cubic unit.

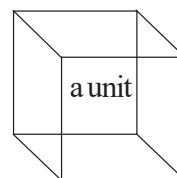
According to the condition, $a^3 = 6a^2$

$$\text{or, } a^3 - 6a^2 = 0$$

$$\text{or, } a^2 (a - 6) = 0$$

$$\text{or, } a - 6 = 0 \quad [\because a \neq 0] \quad \therefore a = 6$$

∴ The length of the diagonal of the cube = $\sqrt{3} \times \text{length of an edge} = 6\sqrt{3}$ unit.



Application : 14 The dimensions of a cuboid are 12 cm., 6 cm. and 3 cm respectively. Let us calculate the length of each edge of a cube whose volume is equal to that cuboid.

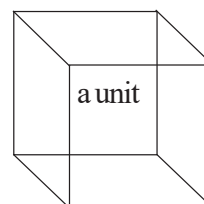
The volume of the cuboid is = $(12 \times 6 \times 3)$ c.c = $(6 \times 2 \times 6 \times 3)$ c.c = 6^3 c.c

Let the length of one edge of the cube be a cm.

∴ Volume of the cube = a^3 c.c

According to the condition, $a^3 = 6^3 \therefore a = 6$

∴ The length of each edge of the cube is 6 cm.



Application : 15 If the length, breadth and volume of a cuboidal room are 8 m., 6m. and 192 cubic m. respectively, then let us calculate the height of the room and the area of four walls of the room.

Let, the height of the room be h m.

∴ Volume of the room = $(8 \times 6 \times h)$ cubic m.

According to the given condition, $8 \times 6 \times h = 192$

∴ $h = \text{}$ [Let me write it myself]

∴ The height of the room is 4 m.

∴ Area of four walls of the room = $2 \times (\text{length} + \text{breadth}) \times \text{height} = \text{}$ [Let me write it myself]



Application : 16 If the area of one surface of cube be 4 times more than that of another cube, then how many times will be the volume of the first cube than that of the second cube— Let us write by calculating it.

Let the length of one edge of 1st cube be x unit and the length of one edge of 2nd cube be y unit.

∴ Area of one surface of 1st cube = x^2 sq.unit and Area of one surface of 2nd cube = y^2 sq.unit

According to the given condition, $x^2 = 4y^2$ ∴ $x = 2y$ [$\because x \neq -2y$]

$$\therefore \frac{\text{Volume of 1st cube}}{\text{Volume of 2nd cube}} = \frac{x^3}{y^3} = \frac{(2y)^3}{y^3} = \frac{8y^3}{y^3} = 8 \quad [\because y \neq 0]$$

∴ Volume of 1st cube = $8 \times$ volume of 2nd cube.

Volume of 1st cube is 8 times than that of 2nd cube.



Application : 17 The length and breadth of a cuboidal water land in neighbouring village are 18m. and 11m. respectively. In this water land, water is being irrigated from a nearby pond with a pump. If the pump can irrigate 39,600 lit. water per hour then for how much time is required to raise the height of the water level by 3.5 dcm of the water land – let us write by calculating it. [1lit = 1 cubic dcm.]

If the depth of water in cuboidal water land be 3.5 dcm., then the valume of this water will be $(180 \times 110 \times 3.5)$ dcm.³ [$\because$ 18 m. = 180 dcm., 11 m = 110 dcm.]

= $(180 \times 110 \times 3.5)$ lit, [$\because$ 1 lit = 1 dcm.³]

The pump can fill 39600 lit. water in one hour.

$$\therefore \text{The pump is to be run for } \frac{180 \times 110 \times 3.5}{39600} \text{ hr} = \boxed{} \text{ hr } \boxed{} \text{ min.}$$



Application : 18 If the pump can fill 37,400 lit. water in 1 hour, then what time will be required to raise the height of water 17 dcm. of a cuboidal water land whose length is 18m and breadth is 11m. Let us write by calculating it. [Let me do it myself]

Application : 19 From a wooden log with the length of 5 dcm., the breadth of 5 dcm. and the thickness of 3dcm., 40 planks are of 2m. length and 2dcm. breadth are cloven. For cleaving wood has been destroyed. But still now 108 cubic dcm. wood is remained in the log. What is the thickness of each plank that is cloven, let us calculate and write it.

In the log the volume of wood = $(\boxed{} \times \boxed{} \times \boxed{})$ dcm.³ = 600 dcm.³

The wood is destroyed = $600 \times \frac{2}{100}$ dcm.³ = 12 dcm.³

Let the thickness of each plank be x d.cm.

∴ In 1 plank the quantity of wood is $(20 \times 2 \times x)$ dcm.³

∴ In 40 planks the quantity of wood is $40 \times (20 \times 2 \times x)$ dcm.³ = $1600x$ dcm.³

After cleaving off, the remaining wood in the log is 108 dcm.³

According to the given condition, $1600x + 108 + 12 = 600$

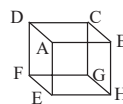
$$\therefore x = \boxed{} \quad [\text{Let me write it myself}]$$

∴ Each plank had been cloven with the thickness of each is 0.3 dcm. or 3cm.



Let us work out

4



- Let us write the names of 4 cuboid and 4 cube shaped solid things in our environment.
- Let us write the names of surfaces, edges and vertices of the adjoining cuboidal figure.
- The length, breadth and height of a cuboidal room are 5m., 4m. and 3m. respectively, Let us write the length of the longest rod which can be kept in that room.
- The area of one surface of a cube is 64 sq.m., let us calculate the volume of the cube.
- In our Bokultala village, a canal is cut whose breadth is of 2m. and depth is of 8 dcm. If the total quantity of soil extraeted is 240 cubic metre, then let us calculate the length of the canal.
- If the length of the diagonal of a cube is $4\sqrt{3}$ cm., then let us calculate the total surface area of the cube.
- The sum of the length of the edges of a cube is 60 cm., let us find out the volume of the cube.
- If the sum of areas of 6 surfaces of a cube be 216 sq.cm., then let us calculate the volume of the cube.
- The volume of a rectangular parallelopiped is 432 sq.cm. If it is converted into two cubes of equal volumes, then let us calculate the length of each edge of each cube.
- Each side of a cube is decreased by 50%. Let us calculate the ratio of the volumes original cube and changed cube.
- If the ratio of length, breadth and height of a cuboidal box is 3 : 2 : 1 and its volume is 384 cc. then let us calculate the total surface area of the box.
- The inner length, breadth and height of a box of tea are 7.5 dcm., 6 dcm. and 5.4 dcm. respectively, if the weight of the box filled with tea is 52 kg. 350gm., but in empty state, its weight is 3.75 kg. then let us write by calculating, the weight of 1 cubic dcm. tea.
- The length, breadth and weight of a brass plate with squared base are x cm., 1 mm. and 4725 gm. respectively, if the weight of 1 cubic cm. brass is 8.4 gm., then let us write by calculating, the value of x.
- The height of Chandmari Road is to be raised. So, 30 cuboidal holes with equal depth and of equal measure are dug out on both sides of the road and with this soil the road is elevated. If the length and breadth of each hole are 14m. and 8m. respectively and if the total quantity of soil required to make the road be 2520 cubic metre then let us calculate the depth of each hole.
- If the water of 64 water filled buckets of equal measure are taken out from the water of a cubical water filled tank, then $\frac{1}{3}$ rd of water remains in the tank. If the length of one edge of the tank is 1.2m, then let us calculate and write the quantity of water that can be hold in each bucket?
- If the length, breadth and height of one packet of one gross match box are 2.8 dcm., 1.5 dcm. and 0.9 dcm. respectively, then let us calculate the volume of one match box. [one gross = 12 dozen]. But if the length and breadth of one match box be 5cm. and 3.5 cm., then let us calculate and the height of it.

17. Half of a cuboidal water tank with length of 2.1 m. and breadth of 1.5 m. is filled with water. If 630 litre water is poured more into the tank, then let us calculate and write the depth that will be increased by.
18. The length and breadth of a rectangular field of the village are 20m. and 15m. respectively. For construction of pillars in the 4 corners of that field 4 cubic holes having length of 4 m. are dug out and the soils removed are dispersed on the remaining land. Let us calculate and write the height of the surfaces of the field that is increased by.
19. For elevating 6.5 dcm of a low land with length of 48 m. and breadth of 31.5 m, it is decided that the soil will be collected by scooping a hole in a nearby land with length of 27m. and breadth of 18.2m., let us calculate the depth of the hole, in metre.
20. There were 800lit, 725 lit and 575 lit. Kerosine oil in three kerosine oil drums of the house. The oil of these three drums is poured into a cuboidal pot and for this, the depth of oil in drums becomes 7dcm. If the ratio of the length and breadth of the cuboidal pot is 4 : 3, then let us write by calculating, the length and breadth of the pot.
If the depth of the cuboidal pot would be 5 dcm, then let us calculate whether 1620 lit. oil can be kept or not in that pot.
21. The daily requirements of water of three families in our three-storyed flat are 1200 lit, 1050 lit. and 950 lit respectively. After fulfilling these requirements in order to put up a tank again and to deposit to store 25% of the required water, only a land having the length of 2.5 m and breadth of 1.6m has been procured. Let us calculate the depth of the tank in metre that should be made.
If the breadth of the land would be more by 4 dcm, then let us calculate the depth of the tank to be made.
22. The weight of a wooden box with 5cm thickness. along with its covering is 115.5 kg. But the weight of the box filled with rice is 880.5 kg. The length and breadth of inner side of the box are 12 dcm. and 8.5 dcm. respectively and the weight of 1 cubic dcm. rice is 1.5 kg. Let us write the inner height of the box after calculation. Let us calculate the total expenditure to colour the outside of the box, if the rate is ` 1.50 per sq.dcm.
23. The depth of water of a cuboidal pond with length of 20m. and breadth of 18.5 m is 3.2 m., let us write by calculating, the time required to irrigate whole water of the pond with a pump having the capacity to irrigate 160 kilo lit. water per hour. If that quantity of water is poured on a paddy field with a ridge having the length of 59.2 m and breadth of 40m., then what is the depth of water in that land– let us write by calculating it, [1 cubic metre = 1 kilo litre]
24. **V. S. A. :**
(A) M.C.Q. :
 - (i) The inner volume of a cuboidal box is 440 cc. and the area of inner base is 88 sq.cm., the inner height of the box is
(a) 4 cm. (b) 5 cm. (c) 3 cm. (d) 6 cm.

- (ii) The length, breadth and height of a cuboidal hole are 40m., 12m. and 16m. respectively. The number of planks having the height of 5 m. the breadth of 4m. and the thickness of 2m., can be kept in that hole is
(a) 190 (b) 192 (c) 184 (d) 180
- (iii) The surface area of a cube is 256 sq.m., the volume of the cube is
(a) 64 cubic metre (b) 216 cubic metre (c) 256 cubic metre (d) 512 cubic metre
[Hints : The number of surfaces is 6]
- (iv) The ratio of the volumes of two cubes is 1 : 27, the ratio of total surface areas of two cubes is
(a) 1 : 3 (b) 1 : 8 (c) 1 : 9 (d) 1 : 18
- (v) If total surface area of a cube is s sq. unit and the length of the diagonal is d unit, then the relation between s and d is
(a) $s = 6d^2$ (b) $3s = 7d$ (c) $s^3 = d^2$ (d) $d^2 = \frac{s}{2}$

(B) Let us write True or False of the following statements :

- (i) If the length of each edge of a cube is twice of that 1st cube then the volume of this cube is 4 times more than that of the 1st cube.
- (ii) In rainy season, the height of rain fall in 2 hectre land is 5 cm, the volume of rain water is 1000 cubic metre.

[Hints : 1 are = 100 sq.m., 1 hectre = 100 are]

(c) Let us fill in the blanks :

- (i) The number of diagonals of a cuboid is _____.
- (ii) The length of the diagonal on a surface of a cube = _____ $\times$ the length of one edge.
- (iii) If the length, breadth and height of a rectangular parallelopiped are equal, then the special name of this solid is _____.

25. S.A. :

- (i) If the number of surfaces of a cuboid is x, the number of edges is y, the number of vertices is z and the number of diagonals is p, let us write the value of $x - y + z + p$.
- (ii) The lengths of the dimensions of two cuboids are 4, 6, 4 units and 8, $(2h - 1)$, 2 units respectively. If the volumes of two cuboids are equal, then let us write the value of h.
- (iii) If each edge of a cube is increased by 50%, then how much the total surface area of the cube will be increased in percent – let us write by calculating it.
- (iv) The lengths of each edge of three solid cubes are 3cm., 4cm. and 5cm. respectively, a new solid is made by melting these three solid cubes. Let us write the length of one edge of the new cube.
- (v) The lengths of two adjacent walls of a room are 12m. and 8m. respectively. If the height of the room is 4m., then let us write by calculating, the area of the floor of the room.

Today woman football match is going on in the big field of village of Tentultala. At first a football match between women team of Bharati Sangha club and women team of Netaji Sangha club is going on. A large number of students from different schools have come to watch the match.



We see there, 60% are boys and 40% are girls present in this foot ball match.

I and Sujoy form the ratio of students present in the field and compare with equivalent quantity
Number of boys : Number of girls = 60% : 40% = $3 \times 20 : 2 \times 20 = 3 : 2$

i.e. How many times or division of another same quantity in comparison to a quantity is called ratio.

A ratio does not alter, if its first and 2nd terms are multiplied or divided by the same non zero number.

We understand that, if number of boy students : number of girl students present in the field = $a : b$

$$a : b = \frac{a}{b} = \frac{ak}{bk} = ak : bk \ [k \neq 0], \text{ and } a : b = \frac{a}{b} = \frac{\frac{a}{k}}{\frac{b}{k}} = \frac{a}{k} : \frac{b}{k} \ [k \neq 0]$$



The value of ratio of two real numbers a and b ($b \neq 0$) is $a : b$ or $\frac{a}{b}$; this $a : b$ is read as "a is to b" a is called the antecedent and b is called consequent of the ratio.

1 What is called 3 and 2 of the ratio of 3:2?

3 is and 2 is of the ratio 3 : 2



2 What is called if the antecedent and consequent are equal in any ratio?

If $a=b$ in the ratio $a : b$, i.e. antecedent and consequent are equal, that ratio is called ratio of equality and if $a \neq b$ i.e. antecedent and consequent are not equal in that ratio is called ratio of inequality.

We understand that, the ratio 3:2 is ratio of inequality, but 2 : 2 is a ratio of equality.

3 But what is called the two ratios when the values of $\frac{a}{b} > 1$ and $\frac{a}{b} < 1$

If the value of ratio $\frac{a}{b} > 1$, it is called a ratio of greater inequality and if $\frac{a}{b} < 1$, the ratio is called ratio of less inequality.

We understand that 3 : 2 is a ratio of greater inequality as $\frac{3}{2} > 1$,

But in the ratio 2:3, $\frac{2}{3} < 1$;

$\therefore$ 2 : 3 is a ratio of .

- 4 A new ratio is formed when the antecedent and consequent of any ratio interchange their positions. What is called this new ratio with respect to the previous ratio ?

If the antecedent and consequent of a ratio be interchanged, the changed ratio is called the inverse ratio of the previous ratio.

$b : a$ is the inverse ratio of $a : b$

We understand that $3 : 2$ is the inverse ratio of $2 : 3$



My friend Sujoy did a funny work. He wrote on his copy three ratios of numbers of boys and girls present in the field as viewers of football match of different times.

He wrote $5 : 2, 4 : 3, 1 : 2$

- 5 But if we take antecedent as product of antecedents and consequent as product of consequents of the ratios written by Sanjay, what is called this ratio which we get ?

From two or more given ratios, if we take antecedent as product of antecedents of the given ratios and consequent as product of consequents of the given ratio, then the ratio thus formed is called mixed or compound ratio.

Example : Compound ratio of $a : b$ and $c : d$ is $ac : bd$

We understand that, compound ratio of $5 : 2, 4 : 3$ and $1 : 2$ is $(5 \times 4 \times 1) : (2 \times 3 \times 2) = 20 : 12 = 5 : 3$

Application 1 : Let us see the ratios are given below and let us write in the blank places.

Ratio	Ratio of equality/ Ratio of inequality	Ratio of greater inequality/ Ratio of less inequality	inverse ratio
$7 : 5$	Ratio of inequality	Ratio of greater inequality	$5 : 7$
$6 : 6$			
$1 : 4$			
$9 : 2$			
compound ratio of $7 : 5, 1 : 4$ & $9 : 2$ is			

Application 2 : Let us write under which condition $x : y$ becomes ratio of less inequality and under which condition it becomes ratio of greater inequality.

The ratio $x : y$ is a less inequality when $\frac{x}{y} < 1$ and greater inequality when $\frac{x}{y} > 1$

The two equivalent ratios of $x : y$ are $xk : yk$ and $\frac{x}{k} : \frac{y}{k}$ [when $k \neq 0$]



Application 3 : Let us write the inverse ratio of $pr : qr$ in lowest term.

$$pr : qr = \frac{pr}{qr} = \frac{p}{q} = p : q$$

$\therefore$ Lowest term of ratio $pr : qr$ is $p : q$

$\therefore$ Inverse ratio of lowest term of ratio $pr : qr$ is $q : p$.

Application 4 : Let us write the lowest term of inverse ratio of $x^2yp : xy^2p$ [Let me do it yourself]

Application 5 : If the lowest term of ratio of two positive real numbers is $p : q$ and their H.C.F is r , let us write the two positive numbers.

The two positive numbers are pr and qr .

Application 6 : If the ratio of two numbers is $2 : 3$ and their H.C.F is 7 , let us write two numbers. [Let me do it myself]

Application 7 : Let us write the compound ratios of the following ratios —

(i) $a : b, p : q$ and $x : y$ (ii) $a : bc, b : ca, c : ab$

(i) The compound or mixed ratio of $a : b, p : q$ and $x : y$ is

$$= a \times p \times x : b \times q \times y$$

$$= apx : bqy$$

(ii) The compound or mixed ratio of $a : bc, b : ca$ and $c : ab$ is

$$= a \times b \times c : bc \times ca \times ab$$

$$= abc : a^2b^2c^2 = 1 : abc \text{ [Dividing both the antecedent and consequent by } abc]$$

Application 8 : Let us find the compound ratio of $pq : r$ and $r : pq$ and let us write the type of ratio it is called ?

The Compound ratio of $pq : r$ and $r : pq$ is $= pq \times r : r \times pq$

$$= pqr : pqr$$

$$= 1 : 1 \text{ [divided both the antecedent and consequent by } pqr]$$

$\therefore$ The compound ratio of $pq : r$ and $r : pq$ is called a ratio of equality.

Application 9 : Let us write the inverse ratio of the mixed ratio of three ratios $p^2q : r, q^2r : p$ and $r^2p : q$ [Let me do it myself]

Application 10 : If $A : B = 4 : 5$ and $B : C = 6 : 7$, let us find $A : C$

$$A : B = 4 : 5 \text{ and } B : C = 6 : 7$$

$$\frac{A}{B} = \frac{4}{5} \text{ and } \frac{B}{C} = \frac{6}{7}$$

$$\therefore \frac{A}{B} \times \frac{B}{C} = \frac{4}{5} \times \frac{6}{7}$$

$$\frac{A}{C} = \frac{24}{35}$$

$$\therefore A : C = 24 : 35$$

Application 11 : If $A : B = 3 : 7$ and $B : C = 8 : 5$, let us find $A : C$ [Let me do it myself]



Application 12 : If $A : B = 6 : 7$ and $B : C = 8 : 9$, let us write by calculating the value of $A : B : C$

$$A : B = 6 : 7 \text{ and } B : C = 8 : 9$$

$$B : C = 8 : 9 = 1 : \frac{9}{8} = 7 : \frac{63}{8} \quad \therefore A : B : C = 6 : 7 : \frac{63}{8} = 48 : 56 : 63$$

Alternately, At the time of finding $A : B : C$, at first we make the values of B in both cases as equal.

Two values of B are 7 and 8 and their L.C.M is 56

$$A : B = 6 : 7 = 6 \times 8 : 7 \times 8 = 48 : 56$$

$$B : C = 8 : 9 = 8 \times 7 : 9 \times 7 = 56 : 63 \quad \therefore A : B : C = 48 : 56 : 63$$

Application 13 : If $A : B = 5 : 9$ and $B : C = 4 : 5$, let us write the value of $A : B : C$ [Let me do it myself]

Application 14 : If $x : y = 2 : 3$, let us write by calculating the value of $(4x - y) : (2x + 3y)$

$$x : y = 2 : 3$$

Let $x = 2p$ and $y = 3p$ [where p is a real number and $p \neq 0$]

$$\therefore (4x - y) : (2x + 3y) = \frac{4x - y}{2x + 3y} = \frac{4 \times 2p - 3p}{2 \times 2p + 3 \times 3p} = \frac{8p - 3p}{4p + 9p} = \frac{5p}{13p} = \frac{5}{13} = 5 : 13$$

$$\therefore (4x - y) : (2x + 3y) = 5 : 13$$



Alternative process, $(4x - y) : (2x + 3y) = \frac{4x - y}{2x + 3y} = \frac{\frac{4x - y}{y}}{\frac{2x + 3y}{y}}$ [dividing numerator and denominator by y]

$$= \frac{4 \frac{x}{y} - 1}{2 \frac{x}{y} + 3} = \frac{4 \times \frac{2}{3} - 1}{2 \times \frac{2}{3} + 3} \left[\text{putting } \frac{x}{y} = \frac{2}{3} \right]$$

$$= \frac{8 - 3}{4 + 9} = \frac{5}{13} = 5 : 13$$

Application 15 : If $x : y = 7 : 4$, let us show that $(5x - 6y) : (3x + 11y) = 11 : 65$ [Let me do it myself]

Application 16 : If $(3x + 5y) : (7x - 4y) = 7 : 4$, let us find the value of $x : y$

$$(3x + 5y) : (7x - 4y) = 7 : 4$$

$$\text{or, } \frac{3x + 5y}{7x - 4y} = \frac{7}{4}$$

$$\text{or, } 4(3x + 5y) = 7(7x - 4y)$$

$$\text{or, } 12x + 20y = 49x - 28y$$

$$\text{or, } 12x - 49x = -28y - 20y$$

$$\text{or, } -37x = -48y$$

$$\text{or, } \frac{x}{y} = \frac{48}{37} \quad \therefore x : y = 48 : 37$$



Application 17 : If $(2x+5y):(5x-7y) = 5:3$, let us find $x:y$. [Let me do it myself]

Application 18 : If $(3x-2y):(x+3y) = 5:6$, let us find the value of $(2x-5y):(3x+4y)$

$$\frac{3x-2y}{x+3y} = \frac{5}{6}$$

$$\text{or, } 6(3x-2y) = 5(x+3y)$$

$$\text{or, } 18x-12y = 5x+15y$$

$$\text{or, } 13x = 27y$$

$$\text{or, } \frac{x}{y} = \frac{27}{13} \quad \therefore x:y = 27:13$$

If $x = 27k$ and $y = 13k$ [k is non zero real number]

$$\therefore (2x-5y):(3x+4y) = \frac{2x-5y}{3x+4y} = \frac{2 \times 27k - 5 \times 13k}{3 \times 27k + 4 \times 13k} = \frac{54k - 65k}{81k + 52k} = \frac{-11k}{133k} = \frac{-11}{133} = -11:133$$

$$\therefore (2x-5y):(3x+4y) = -11:133$$



But here we see that one term of ratio is negative. Is it possible in real life.

Shyamal buys an article at ` 100 and sell it at ` 130. So, profit of Shyamal is ` 30. Rafikul buys an article for ` 100 and sells it at ` 80. So, loss of Rafikul is ` 20. i.e. profit of Rafikul is ` 20. Ratio of profit of Shyamal & Rafikul is 30 : -20 or, 3 : -2.

Application 19 : If $(7x-5y):(3x+4y) = 7:11$, let us find the value of $(5x-3y):(6x+5y)$ [Let me do it myself]

Application 20 : Let us write what should be added to each term of the ratio $x:y$ to make the ratio $p:q$

Let k is to be added to each term of $x:y$ to get the ratio $p:q$

$$\text{So, } \frac{x+k}{y+k} = \frac{p}{q}$$

$$\text{or, } q(x+k) = p(y+k)$$

$$\text{or, } qx + qk = py + pk$$

$$\text{or, } qk - pk = py - qx$$

$$\text{or, } k(q-p) = py - qx \quad \therefore k = \frac{py - qx}{q-p} \quad (\because p:q \text{ is a ratio of inequality } \therefore p \neq q; \therefore q - p \neq 0)$$

$\therefore$ Required number $\frac{py - qx}{q-p}$ is to be added to each term.



Application 21 : Let us write what should be added to each term of the ratio $5:3$ to make the ratio $7:6$. [Let me do it myself]

Application 22 : Let us write by calculating, what should be subtracted from each term of the ratio $x:y$ to make the ratio $p:q$

Let r is to be subtracted from each term of $x:y$ to get the ratio $p:q$

$$\text{So, } \frac{x-r}{y-r} = \frac{p}{q}$$

$$\text{or, } qx - qr = py - pr$$

$$\text{or, } pr - qr = py - qx$$

$$\text{or, } r(p-q) = py - qx$$

$$\therefore r = \frac{py - qx}{p-q} \quad \therefore \text{Required number } \frac{py - qx}{p-q} \text{ is to be subtracted from each term.}$$



Let us work-out 5.1

- Let us express the followings as ratios and let us write by understanding ratio of equality, ratio of less inequality, ratio of greater inequality in each case :
 - 4 months and 1 year 6 months (ii) 75 paise and ` 1 and 25 paise
 - 60 cm. and 0.6 metre (iv) 1.2 kg and 60 gram.
- Let us write the ratio of p kg and q gram.
 - Let us write when it is possible to find ratio of x days z months.
 - Let us write what type of mixed ratio of a ratio and its inverse ratio.
 - Let us find mixed ratio of $\frac{a}{b} : c, \frac{b}{c} : a, \frac{c}{a} : b$.
 - Let us write by calculating what ratio and $x^2 : yz$ will form the mixed ratio $xy : z^2$
 - Let us calculate compound ratio of inverse ratio of $x^2 : \frac{yz}{x}, y^2 : \frac{zx}{y}, z^2 : \frac{yx}{z}$
- Let us find mixed ratio or compound ratio of the following ratios
 - 4 : 5, 5 : 7 and 9 : 11 (ii) $(x+y) : (x-y), (x^2+y^2) : (x+y)^2$ and $(x^2-y^2)^2 : (x^4-y^4)$
- Of $A : B = 6 : 7$ and $B : C = 8 : 7$, let us find $A : C$
 - If $A : B = 2 : 3, B : C = 4 : 5$ and $C : D = 6 : 7$, let us find $A : D$.
 - If $A : B = 3 : 4$ and $B : C = 2 : 3$, let us find $A : B : C$
 - If $x : y = 2 : 3$ and $y : z = 4 : 7$ let us find $x : y : z$
- If $x : y = 3 : 4$, let us find $(3y-x) : (2x+y)$.
 - If $a : b = 8 : 7$, let us show that $(7a-3b) : (11a-9b) = 7 : 5$
 - If $p : q = 5 : 7$ and $p-q = -4$, let us find the value of $3p+4q$.
- If $(5x-3y) : (2x+4y) = 11 : 12$, let us find $x : y$.
 - If $(3a+7b) : (5a-3b) = 5 : 3$, let us find $a : b$.
- If $(7x-5y) : (3x+4y) = 7 : 11$, let us show that, $(3x-2y) : (3x+4y) = 137 : 473$
 - If $(10x+3y) : (5x+2y) = 9 : 5$, let us show that, $(2x+y) : (x+2y) = 11 : 13$
- Let us calculate what term should be added to both terms of the ratio 2 : 5 to make the ratio 6 : 11
 - Let us calculate what term should be subtracted from each term of the ratio $a : b$ to make the ratio $m : n$.
 - What term should be added to antecedent and subtracted from consequent of ratio 4 : 7 to make a compound ratio of 2 : 3 and 5 : 4.

Next month a football match of teenagers will be arranged in that field of village of Tentultala. Bharati Sangha has decided to buy jerseys for each teenager to play football.

Soumen uncle has bought 7 jerseys at ₹ 560. We accompanied with Sharmistha didi have bought 15 jerseys at ₹ 1200 at the same rate.



- 6 I make different ratios of number of jerseys bought by Soumen uncle and us and the price of jerseys, let us see what we get.

The number of jerseys bought by Soumen uncle : the number of jersey bought by us = 7 : 15

The price of jerseys bought by Soumen uncle : the price of jerseys bought by us = 560 : 1200 = $7 \times 80 : 15 \times 80 = 7 : 15$

- 7 But after expressing simplest form of two ratios we see that the two ratios are equal, what this type of numbers is called that formed ratio of equality?

If four real numbers are such that the ratio of the first two is equal to ratio of the last two, then the four numbers in that order are said to be proportional or said to be in proportion.

If four real numbers a, b, c and d ($b \neq 0$ and $d \neq 0$) be in proportion, we write $a : b :: c : d$ where a and b are called extreme terms and b, c are called middle term and ' d ' is called the fourth term or fourth proportion.

We understand, four numbers 7, 15, 560, 1200 are proportional i.e. $7 : 15 :: 560 : 1200$; here 7 and 1200 are extreme terms and 15, 560 are middle terms, 1200 is fourth term or fourth proportion.

- 8 If four numbers a, b, c and d are proportional, let us try to write relation among them.

a, b, c and d are proportional

$$\therefore a : b :: c : d$$

$$\text{i.e., } \frac{a}{b} = \frac{c}{d} \quad \therefore ad = bc$$



We observe that, if four numbers are in proportion, then the product of the extreme terms is equal to the product of the middle terms.

We understand on $7 : 15 :: 560 : 1200$

$$7 \times 1200 = \boxed{} = 15 \times 560 \quad \text{[Let me do it myself]}$$

Application 23 : Let us see whether four numbers 2, 3, 4 and 6 are in proportion or not.

$$2 \times 6 = \boxed{} \text{ and } 3 \times 4 = \boxed{} \quad \therefore 2 : 3 :: 4 : 6$$

Application 24 : Let us see whether four numbers 2.5, -2, -5 and 4 are in proportion or not.

$$2.5 \times 4 = \boxed{} \text{ and } (-2) \times (-5) = \boxed{} \quad \therefore 2.5 : (-2) :: (-5) : 4$$

Application 25 : Let us see whether four numbers 2, 7, 12 and 42 are in proportion or not.

[let me do it myself]

Application 26 : Let us see whether four terms $-\sqrt{2}$, 6, 1 and $-\sqrt{18}$ are in proportion or not.

[let me do it myself]

Application 27 : Let us find which pair of numbers given below are proportional to $5pq, 3q$

(a) $15pt, 3q$ (b) $15pt, 9t$ (c) $15pr, 9t$

$$5pq:3q = \frac{5pq}{3q} = \frac{5p}{3} = \frac{5p \times 3t}{3 \times 3t} = \frac{15pt}{9t}$$

$\therefore$ Required two numbers are $15pt$ and $9t$.



Application 28 : Let us see whether $2a, 3b, 6ac$ and $9bc$ are proportional or not.

$$2a \times 9bc = \boxed{} \text{ and } 3b \times 6ac = \boxed{}$$

We get product of the extreme terms = product of the middle terms.

$\therefore 2a, 3b, 6ac$ and $9bc$ are proportional.

Application 29 : Let us see whether $8x, 5yz, 40qx$ and $25qyz$ are proportional or not.

[Let me do it myself]

Application 30 : If $6:x::2:13$, then let us write by calculating the value of x .

$$6:x::2:13$$

$$\frac{6}{x} = \frac{2}{13}$$

$$\text{or, } 2x = 6 \times 13$$

$$\text{or, } x = \frac{78}{2} \quad \therefore x = 39$$

Application 31 : If $8:y::2:21$, let us write by calculating, the value of y . [Let me do it myself]

Application 32 : Let us find the fourth proportional of $6, 9, 12$

Let the fourth proportional be x

$$6:9::12:x$$

$$\text{or, } \frac{6}{9} = \frac{12}{x}$$

$$\text{or, } 6x = 9 \times 12 \quad \therefore x = \boxed{}$$



Application 33 : Let us write by calculating the fourth proportional of $5, 4, 25$

[Let me do it myself]

Let us see how many different proportions can be obtained by any four numbers in proportion.

If four numbers a, b, c and d are in proportion

$$a:b::c:d \quad \therefore \frac{a}{b} = \frac{c}{d}$$

$$(i) \frac{a}{b} = \frac{c}{d} \text{ or, } \frac{a}{c} = \frac{b}{d} \quad \therefore a:c::b:d \quad (ii) \frac{a}{b} = \frac{c}{d} \text{ or, } \frac{b}{a} = \frac{d}{c} \quad \therefore b:a::d:c$$

$$(iii) \frac{a}{b} = \frac{c}{d} \text{ or, } \frac{c}{a} = \frac{d}{b} \quad \therefore c:a::d:b$$

$\therefore$ We get if a, b, c and d are in proportion, then (i) a, c, b, d are in proportion (ii) b, a, d, c are in proportion (iii) c, a, d, b are in proportion

We understand that, it is possible to make the independent proportions out of the four numbers $2, 3, 4$ and 6 in proportion are (i) $2:4::3:6$ (ii) $3:2::6:4$ (iii) $4:2::6:3$

Application 34 : Let us write by calculating how many independent proportions can be obtained from the proportional numbers 5, 6, 10 and 12 and mention those proportions. [Let me do it myself]

Application 35 : Let us write by calculating which number is to be added to each of 2, 4, 6 and 10 to make the sums proportional.

Let x is to be added to each.

$\therefore (2+x), (4+x), (6+x)$ and $(10+x)$ will be proportional,

$$\therefore (2+x):(4+x)::(6+x):(10+x)$$

$$\text{or, } \frac{2+x}{4+x} = \frac{6+x}{10+x}$$

$$\text{or, } (2+x)(10+x) = (6+x)(4+x)$$

$$\text{or, } 20+10x+2x+x^2 = 24+4x+6x+x^2$$

$$\text{or, } 2x = 4 \quad \therefore x = 2$$

$\therefore$ Required number = 2.

Application 36 : Let us write by calculating which number is to be added to each of 12, 22, 42 and 72 to make the sums proportional. [Let me do it myself]

Application 37 : Let us write by calculating which number is to be added to each of a, b, c, d to make the sums proportional.

Let x is to be added to each number.

$\therefore (a+x):(b+x)::(c+x):(d+x)$

$$\therefore \frac{a+x}{b+x} = \frac{c+x}{d+x}$$

$$\text{or, } (a+x)(d+x) = (c+x)(b+x)$$

$$\text{or, } ad+dx+ax+x^2 = cb+bx+cx+x^2$$

$$\text{or, } x(d+a-b-c) = cb-ad \quad \therefore x = \frac{cb-ad}{a+d-b-c} \quad (\text{when } a+d \neq b+c)$$

$\therefore \frac{cb-ad}{a+d-b-c}$ is to be added to each of a, b, c and d to make the sums proportional.

Application 38 : Let us write by calculating which number is to be added to each of 3, 6, 7 and 10 to make the sums proportional. [Let me do it myself]

Sharmistha didi wrote three numbers on the blackboard of the room of club.

She wrote 4, 8 and 16

Let us see whether the three numbers 4, 8 and 16 can be written as proportion or not.

$$4:8 = 1:2$$

$$\text{and } 8:16 = 1:2$$

$\therefore$ We can write $4:8::8:16$

9 But if three quantities are in proportion like wise, what this proportion is called.

Among the three quantities of the same class, if ratio of first and second term be equal to ratio of second and 3rd term, then the three quantities are called continued proportion.

We understand that, the numbers 4, 8 and 16 which we write on the board are in continued proportion.



10 If three real numbers a , b and c ($b \neq 0$, $c \neq 0$) are in continued proportion, what relation among them we shall get. – Let us see.

a , b and c are in continued proportion

$$\therefore a : b :: b : c$$

$$\text{i.e. } \frac{a}{b} = \frac{b}{c}$$

$$\text{or, } b^2 = ac \quad \therefore b = \pm \sqrt{ac}$$

$$\therefore \text{We get if } a, b \text{ and } c \text{ are in continued proportion, } b^2 = ac \text{ or } b = \pm \sqrt{ac}$$



b will be of positive sign if a and c are both of positive sign and b will be of negative sign if a and c are both of negative sign.

If a and c be of opposite sign, then b will be undefined.

11 If a , b and c are in continued proportion what ' b ' is called ?

If a , b and c are in continued proportion, $\frac{a}{b} = \frac{b}{c} \therefore b^2 = ac$

Here b is called the mean proportional of a and c and c is the third proportional.

If a , b , c , d , e are continued proportional, then $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e}$

We understand that 8 is mean proportional and 16 is the third proportional of three continued proportional numbers 4, 8 and 16.

Application 39 : Let us find the 3rd proportional of 9 and 15.

Let the 3rd proportional be x

$\therefore 9, 15$ and x are in continued proportion.

$$\text{So, } \frac{9}{15} = \frac{15}{x}$$

$$\text{or, } 9x = 15 \times 15$$

$$\text{or, } x = \frac{15 \times 15}{9} \quad \therefore x = \boxed{}$$

$\therefore$ Required 3rd proportional is 25.



Application 40 : Let us find 3rd proportional of ` 3 and ` 12. [Let me do it myself]

Application 41 : Let us find 3rd proportional of $2a^2$ and $3ab$.

Let the 3rd proportional be x .

$\therefore 2a^2, 3ab$ and x are in continued proportion.

$$\therefore \frac{2a^2}{3ab} = \frac{3ab}{x}$$

$$\text{or, } 2a^2x = 3ab \times 3ab$$

$$\text{or, } x = \frac{3ab \times 3ab}{2a^2}$$

$$\therefore x = \frac{9b^2}{2}$$

$\therefore$ Required 3rd proportional is $\frac{9b^2}{2}$



Application 42 : Let us find 3rd proportional of $9pq$, $12pq^2$ [Let me do it myself]

Application 43 : Let us find mean proportional of $\frac{1}{12}$ and $\frac{1}{75}$

Let the mean proportional of $\frac{1}{12}$ and $\frac{1}{75}$ be x

$$\text{So, } \frac{\frac{1}{12}}{x} = \frac{x}{\frac{1}{75}}$$

$$\text{or, } x^2 = \frac{1}{12} \times \frac{1}{75}$$

$$\text{or, } x = \sqrt{\frac{1}{12 \times 75}}$$

$$\therefore x = \frac{1}{30}$$

$\therefore$ Required mean proportional is $\frac{1}{30}$



Application 44 : Let us write by calculating the mean proportional of 0.5 and 4.5.

[Let me do it myself]

Application 45 : If the extreme term of three positive continued proportional numbers are pqr , $\frac{pr}{q}$, let us find the mean proportional.

Let the mean proportional be x

$\therefore pqr, x$ and $\frac{pr}{q}$ are in continued proportion.

$$\therefore \frac{pqr}{x} = \frac{x}{\frac{pr}{q}}$$

$$\text{or, } x^2 = pqr \times \frac{pr}{q} = p^2r^2$$

$$\text{or, } x = \sqrt{p^2r^2}$$

$\therefore x = pr$ ($\because$ extreme terms are positive)

$\therefore$ Required mean proportional is pr .



Application 46 : Let us find the mean proportional of positive numbers xy^2 and xz^2

Let us see by calculating

5.2

[Let me do it myself]

- Let us find the value of x for the following proportions.
(i) $10:35::x:42$ (ii) $x:50::3:2$
- Let us find the fourth proportional of the followings :
(i) $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ (ii) 9.6 kg, 7.6 kg, 28.8 kg. (iii) x^2y, y^2z, z^2x
(iv) $(p-q), (p^2-q^2), p^2-pq+q^2$
- Let us find the 3rd proportional of the following positive numbers :
(i) 5, 10 (ii) 0.24, 0.6 (iii) p^3q^2, q^2r (iv) $(x-y)^2, (x^2-y^2)^2$

4. Let us find the mean proportional of the following positive numbers :
(i) 5 and 80 (ii) 8.1 and 2.5 (iii) x^3y and xy^3 (iv) $(x-y)^2, (x+y)^2$
5. If the two ratios $a : b$ and $c : d$ express mutually opposite relations, let us write what relation will be expressed by their inverse relation.
6. Let us write how many continued proportions can be constructed by three numbers in continued proportions.
7. If the first and second of the five numbers in continued proportion are 2 and 6 respectively, let us find the fifth number.
8. Let us write by calculating what should be added to each of 6, 15, 20 and 43 to make the sums proportional.
9. Let us find what should be subtracted from each of 23, 30, 57 and 78 to make the results proportional.
10. Let us find what should be subtracted from each of p, q, r, s to have four numbers in proportion.

Sabba wrote four numbers on the board like Sharmisthadidi, which are in proportion.

Sabba wrote 3, 5, 6 and 10

We see, $3 : 5 :: 6 : 10$ i.e. $3 : 5 = 6 : 10$

Again, $3 : 6 = 1 : 2 = 5 : 10$

i.e. $3 : 6 :: 5 : 10$

We get, if $3 : 5 :: 6 : 10$ then, $3 : 6 :: 5 : 10$



- 12** I write any four proportional non zero numbers a, b, c and d on the board and let us prove some properties of proportional. If a, b, c and d are in proportion let us prove that a, b, c and d will be proportional.

Proof: a, b, c, d are in proportion i.e. $a : b :: c : d$

$$\therefore \frac{a}{b} = \frac{c}{d}$$

Multiplying both sides by $\frac{b}{c}$ we get

$$\frac{a}{b} \times \frac{b}{c} = \frac{c}{d} \times \frac{b}{c}$$

$$\text{or, } \frac{a}{c} = \frac{b}{d}$$

$$\therefore a : c = b : d$$

we get, if $a : b :: c : d$ then $a : c :: b : d$



- 13** But what is called this property of proportion ?

"If the second and third terms interchange their places, then also the four terms are in proportion", this property of their proportion is called alternendo.

We understand, if $2 : 3 :: 10 : 15$ then $2 : 10 :: 3 : \square$ [Let me do it myself]

Again we see, if $3 : 5 :: 6 : 10$ then $3 : 6 :: 5 : 10$

14 For four numbers a, b, c, d if $a:b::c:d$, let us prove the possibility of $b:a::d:c$

Proof, $a:b::c:d$

$$\text{i.e. } \frac{a}{b} = \frac{c}{d} \text{ i.e. } ad = bc$$

Dividing both sides by ac , we get

$$ad \div ac = bc \div ac$$

$$\text{or, } \frac{d}{c} = \frac{b}{a}$$

$$\text{or, } \frac{b}{a} = \frac{d}{c}$$

$$\therefore b:a::d:c$$

$\therefore$ we get, if $a:b::c:d$ then $b:a::d:c$



15 But what is called this property of proportion ?

"If two ratios are equal then their inverse ratios are also equal". This property of proportion is called inverse or invertendo.

We understand, if $6:10::9:15$ then $10:6::\square:\square$ [Let me do it myself]

My friend Bivas has done a funny work. He made different types of proportion with the help of four numbers 3, 5, 6 and 10, written by sabba.

He did, $(3+5):5 = (6+10):10$

16 If $a:b::c:d$, then let us prove the possibility of $(a+b):b::(c+d):d$.

Proof: $a:b::c:d$ i.e. $\frac{a}{b} = \frac{c}{d}$

$$\text{or, } \frac{a}{b} + 1 = \frac{c}{d} + 1 \text{ [adding 1 both sides, we get]}$$

$$\text{or, } \frac{a+b}{b} = \frac{c+d}{d}$$

$$\therefore (a+b):b::(c+d):d$$



We get, for four numbers a, b, c and d , if $a:b::c:d$ then $(a+b):b::(c+d):d$

17 What is called this property of proportion ?

This property is called Componendo.

We understand, if $4:5::8:10$, then with the help of componendo of proportion, we get $(4+5):5 = \square:10$ [Let me do it myself]

18 But if $a:b::c:d$, let us prove the possibility of $(a-b):b::(c-d):d$

Proof: $a:b::c:d$ i.e. $\frac{a}{b} = \frac{c}{d}$

$$\text{or, } \frac{a}{b} - 1 = \frac{c}{d} - 1 \text{ [Subtracting 1 from both sides we get]}$$

$$\text{or, } \frac{a-b}{b} = \frac{c-d}{d}$$

$$\therefore (a-b):b::(c-d):d$$

$\therefore$ we get, if $a:b::c:d$, then, $(a-b):b::(c-d):d$



19 What is called this property of proportion ?

For four numbers a, b, c, d if $a:b::c:d$, then $(a-b):b::(c-d):d$. This property of proportion is called **Dividendo**.

We understand, if $5:4=10:8$, then with the help of dividendo of proportion we get $(5-4):4=$: 8 [let us write it by verifying]

Application 47 : If $a:b::c:d$, then let us prove with the help of componendo and dividendo of proportion $(a+b):(a-b)::(c+d):(c-d)$

Proof, $a:b::c:d$

i.e. $\frac{a}{b} = \frac{c}{d}$ or, $\frac{a+b}{b} = \frac{c+d}{d}$ [we get by componendo]

Again, $\frac{a}{b} = \frac{c}{d}$ or, $\frac{a-b}{b} = \frac{c-d}{d}$ [we get by dividendo]

so, $\frac{a+b}{b} \div \frac{a-b}{b} = \frac{c+d}{d} \div \frac{c-d}{d}$

or, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

$\therefore (a+b):(a-b)::(c+d):(c-d)$



Alternative proof :

Let, $\frac{a}{b} = \frac{c}{d} = k$, (where k is a constant)

$\therefore a = bk$ and $c = dk$

$\therefore \frac{a+b}{a-b} = \frac{bk+b}{bk-b} = \frac{b(k+1)}{b(k-1)} = \frac{k+1}{k-1}$

Again, $\frac{c+d}{c-d} = \frac{dk+d}{dk-d} = \frac{d(k+1)}{d(k-1)} = \frac{k+1}{k-1}$

$\therefore (a+b):(a-b)::(c+d):(c-d)$

$\therefore$ We get if $a:b::c:d$ then $(a+b):(a-b)::(c+d):(c-d)$

20 But what is called this property of proportion ?

If ' $a:b::c:d$, then $(a+b):(a-b)::(c+d):(c-d)$ '. This property of proportion is called **componendo and dividendo**.

Application 48 : Taking the numbers of proportion $7:3::14:6$, let us apply componendo and dividendo of proportion.

$7:3=14:6 \therefore (7+3):(7-3)=10:4=5:2$

Again, $(14+6):(14-6)=20:8=5:2$

$\therefore (7+3):(7-3)::(14+6):(14-6)$



Application 49 : If $5:4::10:8$, then with the help of componendo and dividendo of proportion, we get,

$(5+4):(5-4)::(10+8):$ [Let me do it myself]

Application 50 : If $a : b :: c : d$, then let us prove that $(a^2+b^2) : (a^2-b^2) :: (c^2+d^2) : (c^2-d^2)$

$$a : b :: c : d \text{ i.e. } \frac{a}{b} = \frac{c}{d} \text{ or, } \frac{a^2}{b^2} = \frac{c^2}{d^2} \text{ [squaring we get]}$$

$$\text{or, } \frac{a^2+b^2}{a^2-b^2} = \frac{c^2+d^2}{c^2-d^2} \text{ [Applying componendo and dividendo we get]}$$

$$\therefore (a^2+b^2) : (a^2-b^2) :: (c^2+d^2) : (c^2-d^2)$$

Alternative proof, let $\frac{a}{b} = \frac{c}{d} = k$ (where k is a constant) $\therefore a = bk$ and $c = dk$

$$\text{L.H.S} = \frac{a^2+b^2}{a^2-b^2} = \frac{(bk)^2+b^2}{(bk)^2-b^2} = \frac{b^2(k^2+1)}{b^2(k^2-1)} = \frac{k^2+1}{k^2-1}$$

$$\text{R.H.S} = \frac{c^2+d^2}{c^2-d^2} = \frac{(dk)^2+d^2}{(dk)^2-d^2} = \frac{d^2(k^2+1)}{d^2(k^2-1)} = \frac{k^2+1}{k^2-1} \therefore \text{L.H.S} = \text{R.H.S} \text{ [proved]}$$



Application 51 : If $a:b::c:d$, let us prove that $(4a+7b):(4a-7b)::(4c+7d):(4c-7d)$ [Let me do it myself]

Tusar wrote, $2 : 5, 6 : 15, 16 : 40, 24 : 60$

$$\text{We see, } \frac{2}{5} = \frac{6}{15} = \frac{16}{40} = \frac{24}{60}$$



- 21** I make a ratio of which the antecedent term is the sum of antecedent terms and the consequent term is the sum of consequent terms of ratio which had been written by Tuser.

$$\frac{2+6+16+24}{5+15+40+60} = \frac{48}{120} = \frac{2}{5}$$

$$\therefore \text{We get } \frac{2}{5} = \frac{6}{15} = \frac{16}{40} = \frac{24}{60} = \frac{2+6+16+24}{5+15+40+60}$$

- 22** If $a : b = c : d = e : f$, let us prove that each ratio is equal to $(a+c+e) : (b+d+f)$

Proof, $a : b = c : d = e : f$

$$\text{Let } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k \text{ (where } k \text{ is a constant)}$$

$$\therefore a = bk, c = dk \text{ and } e = fk$$

$$\therefore \frac{a+c+e}{b+d+f} = \frac{bk+dk+fk}{b+d+f} = \frac{k(b+d+f)}{b+d+f} = k$$

$$\therefore \text{We get, } \frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{a+c+e}{b+d+f}$$

$$\therefore \text{In general we can write, } \frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \dots = \frac{a_n}{b_n} = \frac{a_1+a_2+a_3+\dots+a_n}{b_1+b_2+b_3+\dots+b_n}$$



- 23** But what is called this property of proportion?

$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \frac{a_3}{b_3} = \dots = \frac{a_n}{b_n} = \frac{a_1+a_2+a_3+\dots+a_n}{b_1+b_2+b_3+\dots+b_n}$ — this property of proportion is called Addendo.

- 24** If $a : b = c : d = e : f$, let us see whether each ratio is equal to (i) $(a+c+e) : (b+d+f)$ (ii) $(a-c+e) : (b-d+f)$ (iii) $(a-c-e) : (b-d-f)$

(i) $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$ (where k is a constant) $\therefore a = bk, c = dk, e = fk$

$$\frac{a+c+e}{b+d+f} = \frac{bk+dk+fk}{b+d+f} = \frac{k(b+d+f)}{(b+d+f)} = k \quad \therefore a : b = c : d = e : f = (a+c+e) : (b+d+f)$$

(ii) $\frac{a-c+e}{b-d+f} = \frac{bk-dk+fk}{b-d+f} = \frac{k(b-d+f)}{(b-d+f)} = k \quad \therefore a : b = c : d = e : f = (a-c+e) : (b-d+f)$

(iii) $\frac{a-c-e}{b-d-f} = \frac{bk-dk-fk}{b-d-f} = \frac{k(b-d-f)}{(b-d-f)} = k \quad \therefore a : b = c : d = e : f = (a-c-e) : (b-d-f)$

- 25** If $a : b = c : d = e : f$ let us prove that each ratio is equal to $\frac{am+cn+ep}{bm+dn+fp}$ [m, n, p are non zero numbers] [Let me do it myself]

Application 52 : We can write by applying the addendo property of proportion, $\frac{2}{3} = \frac{6}{9} = \frac{8}{12} =$

$$\frac{2+6+8}{\square+\square+\square} \quad \text{[Let me do it myself]}$$

Application 53 : If $a : b = c : d$, let us prove that $(a^2+c^2)(b^2+d^2) = (ab+cd)^2$

$$a : b = c : d \quad \text{or, } \frac{a}{b} = \frac{c}{d} \quad \text{or, } \frac{a^2}{ab} = \frac{c^2}{cd} = \frac{a^2+c^2}{ab+cd} \quad \text{[Applying addendo property, we get]}$$

$$\text{Again, } \frac{a}{b} = \frac{c}{d} \quad \text{or, } \frac{ab}{b^2} = \frac{cd}{d^2} = \frac{ab+cd}{b^2+d^2} \quad \text{[Applying addendo property, we get]}$$

$$\text{so, } \frac{a^2+c^2}{ab+cd} = \frac{ab+cd}{b^2+d^2}$$

$$\therefore (a^2+c^2)(b^2+d^2) = (ab+cd)^2 \quad \text{[proved]}$$

Alternative proof

$$\text{Let } \frac{a}{b} = \frac{c}{d} = k \quad (\text{where } k \text{ is a constant})$$

$$\therefore a = bk, c = dk$$

$$\therefore \text{L.H.S} = (a^2+c^2)(b^2+d^2) = (b^2k^2+d^2k^2)(b^2+d^2) = k^2(b^2+d^2)^2$$

$$\text{R.H.S} = (ab+cd)^2 = (bk \times b + dk \times d)^2 = (b^2k + d^2k)^2 = k^2(b^2+d^2)^2$$

$$\therefore \text{L.H.S} = \text{R.H.S} \quad (\text{proved})$$



Application 54 : If $x : a = y : b = z : c$, let us show that $\frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{3xyz}{abc}$

$$\text{Let, } \frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k \quad (\text{where } k \text{ is a constant})$$

$$\therefore x = ak, y = bk \text{ and } z = ck$$

$$\text{L.H.S} = \frac{x^3}{a^3} + \frac{y^3}{b^3} + \frac{z^3}{c^3} = \frac{a^3k^3}{a^3} + \frac{b^3k^3}{b^3} + \frac{c^3k^3}{c^3} = k^3 + k^3 + k^3 = 3k^3$$

$$\text{R.H.S} = \frac{3xyz}{abc} = \frac{3 \times ak \times bk \times ck}{abc} = 3k^3 \quad \therefore \text{L.H.S} = \text{R.H.S} \quad \text{[proved]}$$



Application 55 : If $a : b = b : c$, let us show that $(a+b+c)(a-b+c) = a^2 + b^2 + c^2$

Let, $\frac{a}{b} = \frac{b}{c} = k$ (where k is a constant)

$\therefore a = bk, b = ck$

So, $a = ck \times k = ck^2$

$$\begin{aligned} \text{L.H.S} &= (a+b+c)(a-b+c) \\ &= (ck^2+ck+c)(ck^2-ck+c) \\ &= c(k^2+k+1) \times c(k^2-k+1) \\ &= c^2(k^2+k+1)(k^2-k+1) \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= a^2+b^2+c^2 \\ &= (ck^2)^2+(ck)^2+c^2 \\ &= c^2k^4+c^2k^2+c^2 \\ &= c^2(k^4+k^2+1) \\ &= c^2[k^4+2k^2+1-k^2] \\ &= c^2[(k^2+1)^2-(k)^2] \\ &= c^2(k^2+1+k)(k^2+1-k) \\ &= c^2(k^2+k+1)(k^2-k+1) \end{aligned}$$

$\therefore \text{L.H.S} = \text{R.H.S}$ [Proved]



Application 56 : If a, b, c, d are in continued proportion, let us show that $(a^2-b^2)(c^2-d^2) = (b^2-c^2)^2$

Let, $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$ (where k is a constant),

so, $a = bk, b = ck, c = dk$

$\therefore b = dk \times k = dk^2$ and $a = dk^2 \times k = dk^3$

$$\begin{aligned} \text{L.H.S} &= (a^2-b^2)(c^2-d^2) \\ &= \{(dk^3)^2-(dk^2)^2\} \{(dk)^2-d^2\} \\ &= \{d^2k^6-d^2k^4\} \{d^2k^2-d^2\} \\ &= d^2k^4(k^2-1) \times d^2(k^2-1) \\ &= d^4k^4(k^2-1)^2 \end{aligned}$$

$$\begin{aligned} \text{R.H.S} &= (b^2-c^2)^2 \\ &= \{(dk^2)^2-(dk)^2\}^2 \\ &= \{d^2k^4-d^2k^2\}^2 \\ &= \{d^2k^2(k^2-1)\}^2 \\ &= d^4k^4(k^2-1)^2 \end{aligned}$$

$\therefore \text{L.H.S} = \text{R.H.S}$ [Proved]



Application 57 : If $\frac{ay-bx}{c} = \frac{cx-az}{b} = \frac{bz-cy}{a}$, let us prove that $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

$$\frac{ay-bx}{c} = \frac{cx-az}{b} = \frac{bz-cy}{a}$$

$$\therefore \frac{c(ay-bx)}{c^2} = \frac{b(cx-az)}{b^2} = \frac{a(bz-cy)}{a^2}$$

$$= \frac{c(ay-bx)+b(cx-az)+a(bz-cy)}{c^2+b^2+a^2} = \frac{cay-cbx+bcx-baz+abz-acy}{c^2+b^2+a^2} = 0 \text{ [using addendo we get]}$$

$$\text{So, } \frac{ay-bx}{c} = 0 \quad \text{or, } ay-bx = 0$$

$$\text{or, } ay = bx \quad \therefore \frac{y}{b} = \frac{x}{a}$$

$$\text{Again, } \frac{cx-az}{b} = 0 \quad \text{or, } cx = az \quad \therefore \frac{x}{a} = \frac{z}{c}$$

$$\therefore \frac{x}{a} = \frac{y}{b} = \frac{z}{c} \text{ [Proved]}$$



Alternative proof :

$$\text{Let, } \frac{ay-bx}{c} = \frac{cx-az}{b} = \frac{bz-cy}{a} = k \quad (\text{where } k \text{ is a constant})$$

$$\therefore ay-bx = ck, cx-az = bk \text{ and } bz-cy = ak$$

$$ay-bx = ck \quad \text{--- (i)} \qquad \qquad \qquad cx-az = bk \quad \text{--- (ii)}$$

Multiplying equation (i) by c and (ii) by b and adding them, we get

$$acy - bcx = c^2k$$

$$bcx - abz = b^2k$$

$$\hline a(cy-bz) = k(b^2+c^2)$$

$$\text{or, } cy-bz = \frac{k}{a}(b^2+c^2)$$

$$\text{or, } -ak = \frac{k}{a}(b^2+c^2) \quad [\because bz-cy = ak]$$

$$\text{or, } -a^2k = kb^2+kc^2$$

$$\text{or, } k(a^2+b^2+c^2) = 0 \quad \therefore k = 0 \quad [\because a^2+b^2+c^2 \neq 0]$$

$$\text{So, } \frac{ay-bx}{c} = 0 \quad \text{or, } ay-bx = 0 \quad \text{or, } ay = bx \quad \therefore \frac{x}{a} = \frac{y}{b}$$

$$\text{Again, } \frac{cx-az}{b} = 0 \quad \text{or, } cx-az = 0 \quad \text{or, } cx = az \quad \therefore \frac{x}{a} = \frac{z}{c} \quad \therefore \frac{x}{a} = \frac{y}{b} = \frac{z}{c} \quad \text{[Proved]}$$



Application 58 : If $\frac{x}{a+b-c} = \frac{y}{b+c-a} = \frac{z}{c+a-b}$, let us prove that each ratio $= \frac{x+y+z}{a+b+c}$ [Let me do it myself]

Application 59 : If $\frac{bz+cy}{a} = \frac{cx+az}{b} = \frac{ay+bx}{c}$ let us prove that

$$\frac{x}{a(b^2+c^2-a^2)} = \frac{y}{b(c^2+a^2-b^2)} = \frac{z}{c(a^2+b^2-c^2)}$$

$$\frac{bz+cy}{a} = \frac{cx+az}{b} = \frac{ay+bx}{c}$$

$$\text{or, } \frac{abz+acy}{a^2} = \frac{bcx+baz}{b^2} = \frac{cay+cbx}{c^2}$$

$$= \frac{(bcx+baz)+(cay+cbx)-(abz+acy)}{b^2+c^2-a^2} = \frac{2bcx}{b^2+c^2-a^2}$$

[Applying componendo and dividendo, we get]

Again, similarly we get

$$\text{Each ratio} = \frac{(abz+acy)+(acy+bcx)-(bcx+baz)}{a^2+c^2-b^2} = \frac{2acy}{c^2+a^2-b^2}$$

$$\text{and each ratio} = \frac{(abz+acy)+(bcx+abz)-(acy+bcx)}{a^2+b^2-c^2} = \frac{2abz}{a^2+b^2-c^2}$$

$$\therefore \frac{2bcx}{b^2+c^2-a^2} = \frac{2acy}{c^2+a^2-b^2} = \frac{2abz}{a^2+b^2-c^2} \quad \text{or, } \frac{2abcx}{a(b^2+c^2-a^2)} = \frac{2abcy}{b(c^2+a^2-b^2)} = \frac{2abcz}{c(a^2+b^2-c^2)}$$

$$\therefore \frac{x}{a(b^2+c^2-a^2)} = \frac{y}{b(c^2+a^2-b^2)} = \frac{z}{c(a^2+b^2-c^2)} \quad \text{[Dividing each by } 2abc, \text{ we get]}$$



Application 60 : If $x = \frac{4ab}{a+b}$, let us show that $\frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = 2$ [given $a \neq 0, b \neq 0$ and $a \neq b$]

$$x = \frac{4ab}{a+b} \text{ or, } \frac{x}{2a} = \frac{2b}{a+b} \text{ [dividing both sides by } 2a, \text{ we get]}$$

$$\text{so, } \frac{x+2a}{x-2a} = \frac{2b+a+b}{2b-a-b} \text{ [Applying componendo and dividendo, we get]}$$

$$= \frac{a+3b}{b-a}$$

$$\text{Again, } x = \frac{4ab}{a+b} \text{ or, } \frac{x}{2b} = \frac{2a}{a+b} \text{ [Dividing both sides by } 2b, \text{ we get]}$$

$$\text{so, } \frac{x+2b}{x-2b} = \frac{2a+a+b}{2a-a-b} \text{ [Applying componendo and dividendo, we get]}$$

$$= \frac{3a+b}{a-b}$$

$$\therefore \frac{x+2a}{x-2a} + \frac{x+2b}{x-2b} = \frac{3b+a}{b-a} + \frac{3a+b}{a-b} = \frac{3b+a}{b-a} - \frac{3a+b}{b-a} = \frac{3b+a-3a-b}{b-a} = \frac{2b-2a}{b-a} = \frac{2(b-a)}{b-a} = 2$$

Application 61 : If $\frac{a+b-c}{a+b} = \frac{b+c-a}{b+c} = \frac{c+a-b}{c+a}$ and $a+b+c \neq 0$, let us prove that $a=b=c$

$$\frac{a+b-c}{a+b} = \frac{b+c-a}{b+c} = \frac{c+a-b}{c+a}$$

$$\text{or, } 1 - \frac{c}{a+b} = 1 - \frac{a}{b+c} = 1 - \frac{b}{c+a} \text{ [subtracting each ratio from 1, we get]}$$

$$\text{or, } \frac{c}{a+b} = \frac{a}{b+c} = \frac{b}{c+a}$$

$$\text{or, } \frac{a+b}{c} = \frac{b+c}{a} = \frac{a+c}{b} \text{ [Applying converse process, we get]}$$

$$\text{or, } \frac{a+b+c}{c} = \frac{b+c+a}{a} = \frac{a+c+b}{b} \text{ [adding 1 to each ratio, we get]}$$

$$\text{or, } \frac{1}{c} = \frac{1}{a} = \frac{1}{b} \text{ [}\therefore a+b+c \neq 0, \text{ dividing by } a+b+c]$$

$\therefore a=b=c$ [Applying converse process, we get] **[Proved]**

Application 62 : If $\frac{bz-cy}{b-c} = \frac{cx-az}{c-a}$, let us show that each ratio is equal to $\frac{ay-bx}{a-b}$

$$\frac{bz-cy}{b-c} = \frac{cx-az}{c-a}$$

In ratio $\frac{ay-bx}{a-b}$, z is absent. Each term of first ratio is multiplied by a and each term of 2nd ratio is multiplied by b for eliminating z , we get,

$$\frac{abz-acy}{ab-ac} = \frac{bcx-abz}{bc-ba} = \frac{abz-acy+bcx-abz}{ab-ac+bc-ba} \text{ [applying addendo process, we get]}$$

$$= \frac{-acy+bcx}{-ac+bc} = \frac{-c(ay-bx)}{-c(a-b)} = \frac{ay-bx}{a-b}$$

$$\therefore \frac{bz-cy}{b-c} = \frac{cx-az}{c-a} = \frac{ay-bx}{a-b} \text{ **[Proved]**}$$



Application 63 : If $\frac{x}{y+z} = \frac{y}{z+x} = \frac{z}{x+y}$, let us prove that each ratio is equal to $\frac{1}{2}$ or (-1)

Let $\frac{x}{y+z} = \frac{y}{z+x} = \frac{z}{x+y} = k$ (where k is a non zero constant)

So, $x = k(y+z)$, $y = k(z+x)$ and $z = k(x+y)$

Now, $x+y+z = k(y+z) + k(z+x) + k(x+y)$

or, $x+y+z = k(y+z+z+x+x+y)$

or, $x+y+z = 2k(x+y+z)$

or, $(x+y+z) - 2k(x+y+z) = 0$

or, $(x+y+z)(1-2k) = 0$

Either, $x+y+z = 0$

or, $1-2k = 0$

or, $-2k = -1 \quad \therefore k = \frac{1}{2}$

$\therefore$ Each and every ratio is $= \frac{1}{2}$

Again, if $x+y+z = 0$, then $y+z = -x$

$\therefore \frac{x}{y+z} = \frac{x}{-x} = -1$

Again, $z+x = -y$; so, $\frac{y}{z+x} = \frac{y}{-y} = -1$ and $x+y = -z$; so, $\frac{z}{x+y} = \frac{z}{-z} = -1$

$\therefore$ if, $\frac{x}{y+z} = \frac{y}{z+x} = \frac{z}{x+y}$, the value of each ratio is equal to $\frac{1}{2}$ or (-1)



Application 64: If $\frac{b}{a+b} = \frac{a+c-b}{b+c-a} = \frac{a+b+c}{2a+b+2c}$ (where $a+b+c \neq 0$), let us show that $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$

$$\frac{b}{a+b} = \frac{a+c-b}{b+c-a} = \frac{a+b+c}{2a+b+2c}$$

$$\begin{aligned} \text{or, } \frac{2b}{2(a+b)} &= \frac{a+c-b}{b+c-a} = \frac{2(a+b+c)}{2(2a+b+2c)} = \frac{2b+a+c-b+2a+2b+2c}{2a+2b+b+c-a+4a+2b+4c} \quad [\text{using addendo process we get}] \\ &= \frac{3(a+b+c)}{5(a+b+c)} = \frac{3}{5} \quad (\because a+b+c \neq 0) \end{aligned}$$

$$\text{so, } \frac{b}{a+b} = \frac{3}{5} \quad \text{or, } 5b = 3a+3b$$

$$\text{or, } 2b = 3a \quad \therefore \frac{a}{2} = \frac{b}{3} \quad \text{--- (i)}$$

$$\text{Again, } \frac{a+c-b}{b+c-a} = \frac{3}{5}$$

$$\text{or, } 5(a+c-b) = 3(b+c-a)$$

$$\text{or, } 5a+5c-5b = 3b+3c-3a$$

$$\text{or, } 8a-8b = -2c$$

$$\text{or, } 8a-12a = -2c \quad [\because 2b = 3a]$$

$$\text{or, } -4a = -2c \quad \therefore \frac{a}{2} = \frac{c}{4} \quad \text{--- (ii)}$$

From (i) and (ii), we get $\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$ [Proved]



Application 65 : If $\frac{b+c-a}{y+z-x} = \frac{c+a-b}{z+x-y} = \frac{a+b-c}{x+y-z}$, let us prove that $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$

$$\frac{b+c-a}{y+z-x} = \frac{c+a-b}{z+x-y} = \frac{a+b-c}{x+y-z} = \frac{b+c-a+c+a-b+a+b-c}{y+z-x+z+x-y+x+y-z} \quad [\text{using addendo process, we get}]$$

$$\therefore \text{Each ratio} = \frac{a+b+c}{x+y+z}$$

$$\therefore \frac{b+c-a}{y+z-x} = \frac{a+b+c}{x+y+z} = \frac{(a+b+c) - (b+c-a)}{(x+y+z) - (y+z-x)} \therefore \text{Each ratio} = \frac{2a}{2x} = \frac{a}{x}$$

$$\text{similarly, } \frac{c+a-b}{z+x-y} = \frac{a+b+c}{x+y+z} = \frac{(a+b+c) - (c+a-b)}{(x+y+z) - (z+x-y)} \therefore \text{Each ratio} = \frac{2b}{2y} = \frac{b}{y}$$

$$\text{and } \frac{a+b-c}{x+y-z} = \frac{a+b+c}{x+y+z} = \frac{(a+b+c) - (a+b-c)}{(x+y+z) - (x+y-z)} \therefore \text{Each ratio} = \frac{2c}{2z} = \frac{c}{z}$$

$$\therefore \frac{a}{x} = \frac{b}{y} = \frac{c}{z} \quad [\text{Proved}]$$



Application 66 : If $(4a+5b)(4c-5d) = (4a-5b)(4c+5d)$, let us prove that a, b, c and d are in proportion. **[Let me do it myself]**

Let us work-out

5.3

1. If $a:b = c:d$, let us show that,

$$(i) (a^2+b^2):(a^2-b^2) = (ac+bd):(ac-bd)$$

$$(ii) (a^2+ab+b^2):(a^2-ab+b^2) = (c^2+cd+d^2):(c^2-cd+d^2)$$

$$(iii) \sqrt{a^2+c^2}:\sqrt{b^2+d^2} = (pa+qc):(pb+qd)$$

2. If $x:a = y:b = z:c$, let us prove that,

$$(i) \frac{x^3}{a^2} + \frac{y^3}{b^2} + \frac{z^3}{c^2} = \frac{(x+y+z)^3}{(a+b+c)^2} \quad (ii) \frac{x^3+y^3+z^3}{a^3+b^3+c^3} = \frac{xyz}{abc}$$

$$(iii) (a^2+b^2+c^2)(x^2+y^2+z^2) = (ax+by+cz)^2$$

3. If $a:b = c:d = e:f$, let us prove that,

$$(i) \text{ Each ratio} = \frac{5a-7c-13e}{5b-7d-13f} \quad (ii) (a^2+c^2+e^2)(b^2+d^2+f^2) = (ab+cd+ef)^2$$

4. If $a:b = b:c$, let us prove that,

$$(i) \left(\frac{a+b}{b+c}\right)^2 = \frac{a^2+b^2}{b^2+c^2} \quad (ii) a^2b^2c^2 \left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3}\right) = a^3+b^3+c^3 \quad (iii) \frac{abc(a+b+c)^3}{(ab+bc+ca)^3} = 1$$

5. If a, b, c, d are continued proportion, let us prove that

$$(i) (a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2 \quad (ii) (b-c)^2 + (c-a)^2 + (b-d)^2 = (a-d)^2$$

6. (i) If $\frac{m}{a} = \frac{n}{b}$, let us show that $(m^2+n^2)(a^2+b^2) = (am+bn)^2$
(ii) If $\frac{a}{b} = \frac{x}{y}$, let us show that, $(a+b)(a^2+b^2)x^3 = (x+y)(x^2+y^2)a^3$
(iii) If, $\frac{x}{m-n^2} = \frac{y}{mn-l^2} = \frac{z}{nl-m^2}$, let us show that $lx+my+nz = 0$
(iv) If $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$, let us show that $(b-c)x+(c-a)y+(a-b)z = 0$
(v) If $\frac{x}{y} = \frac{a+2}{a-2}$, let us show that $\frac{x^2-y^2}{x^2+y^2} = \frac{4a}{a^2+4}$
(vi) If $x = \frac{8ab}{a+b}$, let us write by calculating the value of $\left(\frac{x+4a}{x-4a} + \frac{x+4b}{x-4b}\right)$
7. (i) If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$, let us show that $\frac{a+b+c}{c} = 2$
(ii) If $\frac{a}{q-r} = \frac{b}{r-p} = \frac{c}{p-q}$, let us show that $a+b+c = 0 = pa+qb+rc$
(iii) If $\frac{ax+by}{a} = \frac{bx-ay}{b}$, let us show that each ratio is equal to x .
8. (i) If $\frac{a+b}{b+c} = \frac{c+d}{d+a}$, let us prove that $c=a$ or $a+b+c+d = 0$
(ii) If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{a+b}$, let us show that $\frac{a}{y+z-x} = \frac{b}{z+x-y} = \frac{c}{x+y-z}$
(iii) If $\frac{x+y}{3a-b} = \frac{y+z}{3b-c} = \frac{z+x}{3c-a}$, let us show that $\frac{x+y+z}{a+b+c} = \frac{ax+by+cz}{a^2+b^2+c^2}$
(iv) If $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$, let us show that $\frac{x^2-yz}{a^2-bc} = \frac{y^2-zx}{b^2-ca} = \frac{z^2-xy}{c^2-ab}$
9. (i) If $\frac{3x+4y}{3u+4v} = \frac{3x-4y}{3u-4v}$, let us show that $\frac{x}{y} = \frac{u}{v}$
(ii) If $(a+b+c+d) : (a+b-c-d) = (a-b+c-d) : (a-b-c+d)$, let us prove that, $a : b = c : d$
10. (i) If $\frac{a^2}{b+c} = \frac{b^2}{c+a} = \frac{c^2}{a+b} = 1$, let us show that $\frac{1}{1+a} + \frac{1}{1+b} + \frac{1}{1+c} = 1$
(ii) If $x^2 : (by+cz) = y^2 : (cz+ax) = z^2 : (ax+by) = 1$, let us show that, $\frac{a}{a+x} + \frac{b}{b+y} + \frac{c}{c+z} = 1$
11. (i) If $\frac{x}{xa+yb+zc} = \frac{y}{ya+zb+xc} = \frac{z}{za+xb+yc}$ and $x+y+z \neq 0$, let us show that each ratio is equal to $\frac{1}{a+b+c}$
(ii) If $\frac{x^2-yz}{a} = \frac{y^2-zx}{b} = \frac{z^2-xy}{c}$, let us prove that $(a+b+c)(x+y+z) = ax+by+cz$

(iii) If $\frac{a}{y+z} = \frac{b}{z+x} = \frac{c}{x+y}$, let us prove that $\frac{a(b-c)}{y^2-z^2} = \frac{b(c-a)}{z^2-x^2} = \frac{c(a-b)}{x^2-y^2}$

12. **Very short answer type questions (V.S.A) :**

(A) (M.C.Q) :

- (i) The fourth proportional of 3, 4 and 6 is (a) 8 (b) 10 (c) 12 (d) 24
- (ii) The 3rd proportional of 8 and 12 is (a) 12 (b) 16 (c) 18 (d) 20
- (iii) The mean proportional of 16 and 25 is (a) 400 (b) 100 (c) 20 (d) 40
- (iv) a is a positive number and if $a : \frac{27}{64} = \frac{3}{4} : a$, then the value of a is
(a) $\frac{81}{256}$ (b) 9 (c) $\frac{9}{16}$ (d) $\frac{16}{9}$
- (v) If $2a = 3b = 4c$, then $a : b : c$ is (a) 3:4:6 (b) 4:3:6 (c) 3:6:4 (d) 6:4:3

(B) Let us write true or false of the following statements :

- (i) Compound ratio of $ab : c^2$, $bc : a^2$ and $ca : b^2$ is 1:1
- (ii) x^3y , x^2y^2 and xy^3 are continued proportional.

(C) Let us fill in the blanks :

- (i) If the product of three positive consecutive numbers is 64, then the mean proportional of first and third term is _____.
- (ii) If $a:2 = b:5 = c:8$, then 50% of a = 20% of b = _____ % of c
- (iii) If mean proportional of $(x-2)$ and $(x-3)$ is x, then the value of x is _____.

13. **Short Answers (S.A.) :**

- (i) If $\frac{a}{2} = \frac{b}{3} = \frac{c}{4} = \frac{2a-3b+4c}{p}$, let us find the value of p.
- (ii) If $\frac{3x-5y}{3x+5y} = \frac{1}{2}$, let us find the value of $\frac{3x^2-5y^2}{3x^2+5y^2}$
- (iii) If $a:b = 3:4$ and $x:y = 5:7$, let us find the value of $(3ax-by):(4by-7ax)$
- (iv) If x, 12, y, 27 are in continued proportion, let us find the positive value of x and y.
- (v) If $a:b = 3:2$ and $b:c = 3:2$, let us find the value of $(a+b):(b+c)$

6

COMPOUND INTEREST AND UNIFORM RATE OF INCREASE OR DECREASE



Bappadada of our locality borrowed a sum of ₹ 1200 from a cooperative bank at 8% interest per annum for two years for buying a bicycle.

1 Let us calculate and see that after two years how much money Bappadada will repay as interest for 2 years

$$= ₹ \frac{1200 \times 8 \times 2}{100} = ₹ \boxed{}$$

∴ After 2 years Bappadada will repay the total money of ₹ (1200 + 192) = ₹ 1392. But after 2 years total ₹ 1399.68 has to be repaid by Bappadada.

2 How did it happen? i.e. let us calculate and see how much is the total amount more than ₹ 1392 after 2 years.

That cooperative bank lend ₹ 1200 for 2 years at Compound Interest at the rate of 8% per annum.

3 What is compound interest?

Really interest is of two types (i) Simple interest (ii) Compound interest.

Simple Interest : When interest is calculated on principal or capital only, that interest is said to be simple interest. [Simple Interest]

Compound Interest : After a definite period of time, the interest accrued is added to the principal to get the new principal. For the definite period of time, if the interest is calculated on the new principal, then the interest is called Compound Interest.

4 How do we know that the period of time after which the compound interest is due?

The period of time after which the compound interest is due, is called the period of compound interest, the period of compound interest are generally 3 months, 6 months and one year. In case of compound interest if the period of time is not mentioned, it is understood that the period of time is of 1 year.

5 What is amount subjected to compound interest?

The sum of the principal and compound interest for a fixed period of time is termed as amount.

First year principal of Bappadada = ₹ 1200

$$\text{Interest for the first year} = ₹ 96 \left[₹ 1200 \times \frac{8}{100} \right]$$

Principal for the 2nd year = ₹ (1200 + 96) = ₹ 1296

$$\therefore \text{Interest for the 2nd year} = \left[₹ 1296 \times \frac{8}{100} \right] = ₹ 103.68$$

∴ Amount for the 2nd year = ₹ (1296 + 103.68) = ₹ 1399.68

∴ The principal in case of compound interest does not remain same always at the end of every period of time, the principal changes.



Application 1. If I take a loan of ₹ 1400 at 5% compound interest per annum for 2 years. Let us write by calculating how much compound interest and total amount I shall pay.

Let, The principal for the first year = ₹ 100.00

At 5 % interest for the first year = ₹ 5.00

Principal for the 2nd year = ₹ 105.00

At 5 %, interest for the 2nd year = ₹ 5.25 [₹ 105 × $\frac{5}{100}$]

Amount for the 2nd year = ₹ 110.25

∴ Compound interest for 2nd year = ₹ (110.25 – 100.00) or, ₹ (5.00 + 5.25) = ₹ 10.25

In the language of Mathematics, the problem is Principal (in ₹) Compound interest (in ₹)

100

10.25

1400

?

If the principal is ₹ 100 then compound interest is ₹ 10.25

„ „ „ 1 „ „ „ „ „ $\frac{10.25}{100}$

„ „ „ 1400 „ „ „ „ „ $\frac{10.25 \times 1400}{100} = ₹ \boxed{}$

(∵ Principal and compound interest are ₹ $\boxed{}$ (directly/inversely proportional))

∴ Required compound interest is ₹ 143.50 and amount is ₹ (1400 + 143.50) = ₹ $\boxed{}$

Alternative Method

Principal for the first year = ₹ 1400.00

Interest for the first year = ₹ 70.00 [∵ ₹ 1400 × $\frac{5}{100}$]

Amount for the 2nd year = ₹ 1470.00

Interest for the 2nd year = ₹ 73.50 [₹ 1470 × $\frac{5}{100}$]

Amount for the 2nd year = ₹ 1543.50

∴ Compound interest for 2 years = ₹ (1543.50 – 1400.00)

or ₹ (70.00 + 73.50) = ₹ 143.50

Let us see by calculating alternatively : what I shall get.

Let the principal be ₹ p and the annual rate of compound interest be r%

∴ Interest for the first year = ₹ $\frac{p \times r \times 1}{100}$

Amount for the first year = ₹ $(p + \frac{pr}{100}) = ₹ p (1 + \frac{r}{100})$

Interest for the 2nd year = ₹ $p (1 + \frac{r}{100})$



$$\therefore \text{Interest for the 2nd year} = \left\{ \frac{p \left(1 + \frac{r}{100} \right) \times r \times 1}{100} \right\}$$



$$\begin{aligned} \text{Amount for the 2nd year} &= \left\{ p \left(1 + \frac{r}{100} \right) + \frac{p \left(1 + \frac{r}{100} \right) \times r \times 1}{100} \right\} \\ &= p \left(1 + \frac{r}{100} \right) \left\{ 1 + \frac{r}{100} \right\} \\ &= p \left(1 + \frac{r}{100} \right)^2 \end{aligned}$$

$$\therefore \text{We get amount for two years} = p \left(1 + \frac{r}{100} \right)^2 \text{ ----- (I)}$$

$$\begin{aligned} \therefore \text{Compound interest for 2nd year} &= \left\{ p \left(1 + \frac{r}{100} \right)^2 - p \right\} \\ &= p \left\{ 1^2 + (2 \times 1 \times \frac{r}{100}) + \left(\frac{r}{100} \right)^2 - 1^2 \right\} \\ &= p \frac{r}{100} (2 + \frac{r}{100}) \end{aligned}$$

$\therefore$ If $p = ₹1400$, $r = 5$

$$\begin{aligned} \text{From (I), we get amount for two years} &= ₹1400 \left(1 + \frac{5}{100} \right)^2 \\ &= ₹1400 \times \frac{105}{100} \times \frac{105}{100} = ₹ \boxed{} \end{aligned}$$

$$\text{Compound interest for two years} = ₹(1543.50 - 1400.00) = ₹143.50$$

Application :2. If principal amount is p and annual rate of compound interest is $r\%$, let us calculate how much amount will be for 3 years.

$$\text{From (I), we get principal for 3rd year} = p \left(1 + \frac{r}{100} \right)^2$$

$$\begin{aligned} \text{Amount for 3 years} &= p \left(1 + \frac{r}{100} \right)^2 + p \left(1 + \frac{r}{100} \right)^2 \times \frac{r}{100} \\ &= p \left(1 + \frac{r}{100} \right)^2 \left(1 + \frac{r}{100} \right) \\ &= p \left(1 + \frac{r}{100} \right)^3 \end{aligned}$$



$$\therefore \text{Amount for 3 years} = p \left(1 + \frac{r}{100} \right)^3 \text{ ----- (II)}$$

(p = Principal, r = rate of compound interest per annum and n = time (year)).

Application :3. Let us write by calculating what is the amount on ₹1000 for 2 years at the rate of 5% compound interest per annum. [Let me do it myself]

Application : 4. If ₹ 4000 are invested in a bank at the rate of 5% compound interest per annum, then let us write by calculating the amount after 3 years.

Let,	Principal for first year	=	₹ 4000.00
	At 5% interest for first year	=	₹ 200.00 ($\because ₹ 4000 \times \frac{5}{100}$)
	Principal for 2nd year	=	₹ 4200.00
	At 5% interest for 2nd year	=	₹ 210.00 ($\because ₹ 4200 \times \frac{5}{100}$)
	Principal for 3rd year	=	₹ 4410.00
	At 5% interest for 3rd year	=	₹ 220.50 ($\because ₹ 4410 \times \frac{5}{100}$)
	Amount after 3 years	=	₹ 4630.50
$\therefore$	Compound interest for 3 years	=	₹ (4630.50 – 4000)
		=	₹ 630.50

Let

Let us calculate alternatively $p = ₹ 4000$, $r = 5$ and $n = 3$ years

$$\begin{aligned}
 \text{From (II) we get amount after 3 years} &= ₹ p \left(1 + \frac{r}{100}\right)^3 \\
 &= ₹ 4000 \left(1 + \frac{5}{100}\right)^3 \\
 &= ₹ 4000 \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100} \\
 &= ₹ \boxed{}
 \end{aligned}$$



$$\text{Compound interest for 3 years} = ₹ (4630.50 - 4000) = ₹ \boxed{}$$

Application : 5. At 5% compound interest per annum, let us find the compound interest on ₹ 10,000 for 3 years [Let me do it myself]

Application : 6. If interest compounded half yearly, let us write by calculating compound interest and amount on ₹ 8,000 at the rate of 10% compound interest per annum for $1\frac{1}{2}$ years.

Principal for first 6 months	=	₹ 8000
At 10% interest for first 6 months or $\frac{1}{2}$ year	=	₹ 400 ($\because ₹ 8000 \times \frac{10}{100} \times \frac{1}{2}$)
Principal for 2nd 6 months	=	₹ 8400
At 10% interest for 2nd 6 months	=	₹ 420 ($\because ₹ 8400 \times \frac{10}{100} \times \frac{1}{2}$)
Principal for 3rd 6 months	=	₹ 8820
At 10% interest for 3rd 6 months	=	₹ 441 ($\because ₹ 8820 \times \frac{10}{100} \times \frac{1}{2}$)
$\therefore$ Amount for $1\frac{1}{2}$ years	=	₹ 9261
$\therefore$ Compound interest for $1\frac{1}{2}$ years	=	₹ (9261 – 8000) = ₹ 1261

Let us calculate alternatively and let us see what I shall get.



Let, Principal = ` p, rate of interest per annum = r%

$$\therefore \text{Interest for first 6 months or } \frac{1}{2} \text{ year} = \frac{p \times r \times \frac{1}{2}}{100}$$

$$\therefore \text{Amount for first 6 months} = \left(p + \frac{p \times r \times \frac{1}{2}}{100} \right) = p \left(1 + \frac{r}{200} \right)$$

$$\therefore \text{Principal for 2nd 6 months} = p \left(1 + \frac{r}{200} \right)$$

$$\text{Interest for 2nd 6 months} = \frac{p \left(1 + \frac{r}{200} \right) \times r \times \frac{1}{2}}{100}$$

$$\therefore \text{Amount for 2nd 6 months or } 2 \times \frac{1}{2} \text{ year} = p \left(1 + \frac{r}{200} \right)^2 = p \left(1 + \frac{r}{200} \right)^{2 \times 1}$$

$$\text{Similarly amount for 3rd 6 months or } 3 \times \frac{1}{2} \text{ years} = p \left(1 + \frac{r}{200} \right)^3 = p \left(1 + \frac{r}{200} \right)^{2 \times \frac{3}{2}}$$

We understand that if the rate of compound interest is r% per annum and the interest is compounded half yearly or if the number of phase of compound interest in a year is 2 then the amount in n years = $p \left(1 + \frac{r}{200} \right)^{2n}$ ----- (III)

Let, p = ` 8000, r = 10, n = $1\frac{1}{2}$ years = $\frac{3}{2}$ years

$$\begin{aligned} \therefore \text{From (III), we get amount for } 1\frac{1}{2} \text{ years} &= \text{` } 8000 \left( 1 + \frac{10}{200} \right)^{\frac{3}{2} \times 2} = \text{` } 8000 \left(1 + \frac{5}{100} \right)^3 \\ &= \text{` } 8000 \times \frac{105}{100} \times \frac{105}{100} \times \frac{105}{100} = \text{` } \square \end{aligned}$$

$\therefore$ Required compound interest for $1\frac{1}{2}$ years = ` $\square$ [Let me do it myself]



Application : 7. Let us find compound interest on ` 1000 at the rate of 10% compound interest per annum and compounded the interest at 6 months interval. [Let me do it myself]

Application : 8. Let us write by calculating compound interest on ₹ 10,000 at the rate of 8% compound interest per annum for 9 months compounded at the interval of 3 months.

Principal for first 3 months	= ₹ 10000.00
At 8% interest for first 3 months or $\frac{1}{4}$ years	= ₹ 200.00 [$\because ₹ 10000 \times \frac{8}{100} \times \frac{1}{4}$]
Principal for 2nd 3 months	= ₹ 10200.00
At 8% interest for 2nd 3 months	= ₹ 204.00 [$\because ₹ 10200 \times \frac{8}{100} \times \frac{1}{4}$]
Principal for 3rd 3 months	= ₹ 10404.00
At 8% interest for 3rd 3 months	= ₹ 208.08 [$\because ₹ 10404 \times \frac{8}{100} \times \frac{1}{4}$]
$\therefore$ Amount for 9 months	= ₹ 10612.08
$\therefore$ Compound interest for 9 months	= ₹ (10612.08 – 10000.00) = ₹ 612.08

Let us calculate alternatively

Let principal = ₹ p, Compound interest per annum = r%



$$\text{Interest for 3 months or } \frac{1}{4} \text{ year} = \frac{p \times r \times \frac{1}{4}}{100}$$

$$\text{Amount for 3 months or } \frac{1}{4} \text{ year} = \left(p + \frac{p \times r \times \frac{1}{4}}{100} \right) = p \left(1 + \frac{r}{400} \right) = p \left(1 + \frac{r}{100} \times \frac{1}{4} \right)^{4 \times \frac{1}{4}}$$

$$\therefore \text{Principal for 2nd 3 months} = p \left(1 + \frac{r}{400} \right) \quad [\because 3 \text{ months} = \frac{1}{4} \text{ year}]$$

$$\begin{aligned} \therefore \text{Amount for 2nd 3 months} &= p \left(1 + \frac{r}{400} \right) + \frac{p \left(1 + \frac{r}{400} \right) \times r \times \frac{1}{4}}{100} \\ &= p \left(1 + \frac{r}{400} \right)^2 = p \left(1 + \frac{r}{400} \right)^{4 \times \frac{2}{4}} \quad (\because 6 \text{ months} = \frac{2}{4} \text{ year}) \end{aligned}$$

$$\text{Similarly amount for 3 months} = p \left(1 + \frac{r}{400} \right)^3 = p \left(1 + \frac{r}{400} \right)^{4 \times \frac{3}{4}} \quad (\because 9 \text{ months} = \frac{3}{4} \text{ years})$$

We understand that if the rate of compound interest is r% per annum and the interest is compounded quarterly, the number of phase of compound interest in a year is 4, then the amount

$$\text{for } n \text{ years} = p \left(1 + \frac{r}{400} \right)^{4n} \quad \text{———— (IV)}$$

Let, $p = ₹ 10000$, $r = 8$ and $n = \frac{9}{12}$ years $= \frac{3}{4}$ years

$$\begin{aligned}\therefore (\text{From IV we get}), \text{ amount for 9 months or } \frac{3}{4} \text{ years} &= ₹ 10000 \left(1 + \frac{\frac{8}{100}}{1}\right)^{4 \times \frac{3}{4}} \\ &= ₹ 10000 \left(1 + \frac{2}{100}\right)^3 \\ &= ₹ 10000 \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \\ &= ₹ \boxed{}\end{aligned}$$

$\therefore$ Compound interest for 9 months $= ₹ (\boxed{} - \boxed{}) = ₹ \boxed{}$ [Let me do it myself]

Note: Here it is very important that in our present syllabus we shall restrict our discussion regarding compound interest up to three phase of compounding only and deposited value up to three years.

Application : 9. Rokeyabibi borrowed ₹ 30000 in such a way that the rate of compound interest per annum for first, 2nd and 3rd year are 4%, 5% and 6% respectively. Let us write by calculating how much total money she will repay after 3 years.

Let, Principal for first year	= ₹ 100.00
4% interest for first year	= ₹ 4.00 $[\because ₹ 100 \times \frac{4}{100}]$
Principal for 2nd year	= ₹ 104.00
5% interest for first year	= ₹ 5.20 $[\because ₹ 104 \times \frac{5}{100}]$
Principal for 3rd year	= ₹ 109.20
6% interest for first year	= ₹ 6.552 $[\because ₹ 109.20 \times \frac{6}{100}]$
$\therefore$ Amount for 3rd year	= ₹ 115.752



$\therefore$ Compound interest $= ₹ (115.752 - 100.00)$ or $₹ (4.00 + 5.20 + 6.552) = ₹ 15.752$

In the language of Mathematics, the problem is

Principal (in ₹)	Compound interest (in ₹)
100	15.752
30000	?

If Principal amount is ₹ 100 then compound interest is ₹ 15.752

$$\begin{array}{rcll} 1 & \text{,,} & \text{,,} & \text{,,} \text{,,} \frac{15.752}{100} \\ 30000 & \text{,,} & \text{,,} & \text{,,} \text{,,} \frac{15.752 \times 30000}{100} = ₹ 4725.60 \end{array}$$



$\therefore$ After 3 years Rokeyabibi will repay total amount $= ₹ (30000 + 4725.60)$
 $= ₹ 34725.60$

Let us see by calculating what I shall get alternatively.

Let principal = ` p and rate of compound interest per annum for first, 2nd and 3rd year are $r_1\%$, $r_2\%$ and $r_3\%$

$$\therefore \text{Interest for first year} = \frac{p \times r_1 \times 1}{100} = \frac{pr_1}{100}$$

$$\therefore \text{Amount for first year} = \text{` } p + \frac{pr_1}{100} = \text{` } p \left(1 + \frac{r_1}{100}\right)$$

$$\therefore \text{Principal for 2nd year} = \text{` } p \left(1 + \frac{r_1}{100}\right)$$

$$\therefore \text{Compound interest for 2nd year} = \frac{p \left(1 + \frac{r_1}{100}\right) \times r_2 \times 1}{100}$$

$$\begin{aligned} \therefore \text{Amount for 2nd year} &= \left[p \left(1 + \frac{r_1}{100}\right) + \frac{p \left(1 + \frac{r_1}{100}\right) \times r_2}{100} \right] \\ &= \text{` } p \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \end{aligned}$$

$$\therefore \text{Similarly we get total amount for 3 years} = \text{` } p \left(1 + \frac{r_1}{100}\right) \left(1 + \frac{r_2}{100}\right) \left(1 + \frac{r_3}{100}\right)$$

Let, $p = \text{` } 30000$, $r_1 = 4\%$, $r_2 = 5\%$ and $r_3 = 6\%$

$$\begin{aligned} \therefore \text{Amount for 3 years} &= \text{` } 30000 \times \left(1 + \frac{4}{100}\right) \left(1 + \frac{5}{100}\right) \left(1 + \frac{6}{100}\right) \\ &= \text{` } 30000 \times \frac{104}{100} \times \frac{105}{100} \times \frac{106}{100} = \text{Rs } \boxed{} \end{aligned}$$

At the end of 3 years Rokeyabibi will repay total amount ` 34725.60

Application : 10. If the rate of compound interest for the first year is 4% and 2nd year is 5%, Let us find the compound interest on ` 25000 for 2 years. [Let me do it myself]

Application : 11. I lend ` 10000 at the rate of 4% compound interest per annum for $2\frac{1}{2}$ years, let us write by calculating how much total money I shall pay.

Amount for first 2 years = `  $p \left(1 + \frac{r}{100}\right)^2$  [ $p = \text{` } 10000$, $r = 4$]

$$\begin{aligned} &= \text{` } 10000 \left(1 + \frac{4}{100}\right)^2 &= \text{` } 10000 \times \frac{104}{100} \times \frac{104}{100} \\ &= \text{` } 10816 \end{aligned}$$

The total amount at the end of 2 years = ` 10816

$$\text{After } \frac{1}{2} \text{ year interest} = \frac{10816 \times 4 \times \frac{1}{2}}{100} = \text{` } 216.32$$

$$\therefore \text{Total amount after } 2\frac{1}{2} \text{ year is } \text{` } (10816 + 216.32) = \text{` } \boxed{}$$



Let us calculate alternatively.

Let us calculate alternatively.

$$\begin{aligned}\text{Amount for } 2\frac{1}{2} \text{ years} &= ₹ 10000 \left(1 + \frac{4}{100}\right)^2 + 10000 \times \left(1 + \frac{4}{100}\right)^2 \times \frac{6}{12} \times \frac{4}{100} \\ &= ₹ 10000 \left(1 + \frac{4}{100}\right)^2 \left(1 + \frac{6}{12} \times \frac{4}{100}\right) \\ &= ₹ 10000 \times \frac{104 \times 104}{100 \times 100} \times \frac{102}{100} = ₹ \frac{1103232}{100} = ₹ 11032.32\end{aligned}$$

Application : 12. Let us find the amount on ₹ 30000, at the rate of 6% compound interest per annum for $2\frac{1}{2}$ years. [Let me do it myself]

Application : 13. Let us write by calculating on what sum of money the compound interest is ₹ 246 at the rate of 5% compound interest per annum for 2 years

Let, Principal for first year	= ` 100.00
At 5% interest for first year	= ` 5.00
Principal for 2nd year	= ` 105.00
At 5% , interest for 2nd year	= ` 5.25 [$\therefore 105 \times \frac{5}{100}$]
$\therefore$ Amount for 2 years	= ` 110.25



$$\therefore \text{Compound interest for 2 years} = ₹ (110.25 - 100.00) = ₹ 10.25$$

∴ In the language of Mathematics, the problem is : compound interest (in `) Principal (in `)

10.25	100
246	?

If compound interest is ₹ 10.25 then principal is ₹ 100

$$\begin{aligned} \text{,,} \quad \text{,,} \quad 1 \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \frac{100}{10.25} \\ \text{,,} \quad \text{,,} \quad 246 \quad \text{,,} \quad \text{,,} \quad \text{,,} \quad \frac{100 \times 246}{10.25} = \text{、} 2400 \end{aligned}$$

Let us calculate alternatively.

Let principal = x

At the rate of 5% compound interest per annum the amount for 2 years = ` $x \left(1 + \frac{5}{100}\right)^2$

$$\therefore \text{Compound interest for 2 years} = \left(\frac{441x}{100} - x \right)$$

By the condition, $\frac{441x}{100} - x = 246$

$$\text{or, } \frac{441x - 400x}{400} = 246$$

$$\text{or, } \frac{41x}{400} = 246$$

$$\therefore x = \frac{246 \times 400}{41} = 2400$$

$\therefore$ Principal is ` 2400 .



Application :14. Let us write by calculating what sum of money will amount ` 3528 after 2 years at the rate of 5% compound interest per annum. [Let me do it myself]

Application : 15. Let us write by calculating on what sum of money, the difference between simple interest and compound interest for 2 years becomes ` 80 at 4% interest per annum.

Let principal = ` 100

∴ At the rate of simple interest 4% per annum, the interest on ` 100 for 2 years = ` $\frac{100 \times 2 \times 4}{100}$ = ` 8.

At the rate of 4% compound interest per annum, the compound interest on ` 100 for 2 years

$$= \text{`} \left\{ 100 \left(1 + \frac{4}{100} \right)^2 - 100 \right\}$$

$$= \text{`} 100 \left\{ \left( \frac{26}{25} \right)^2 - 1 \right\} = \text{`} 100 \left(\frac{676}{625} - 1 \right) = \text{`} \frac{100 \times 51}{625} = \frac{204}{25} = \text{`} 8.16$$

∴ Difference between compound interest and simple interest = ` (8.16 – 8.00) = ` 0.16.

∴ In the language of Mathematics, the problem is,

Difference between interest (in `)	Principal (in `)
0.16	100
80	?



If difference between compound interest and simple interest is ` 16 then principal is ` 100

” ” ” ” ” ” ” ” ” 1 ” ” ” ” 100
” ” ” ” ” ” ” ” ” 80 $\frac{100 \times 80}{0.16}$ = ` 50000

∴ Principal is = ` 50000.

Let us calculate alternatively and let us see what I shall get.

Let Principal = ` x

∴ Simple interest at 4% for 2 years = `  $\frac{x \times 4 \times 2}{100}$  = ` $\frac{2x}{5}$

Compound interest at 4% for 2 years = ` $x \left(1 + \frac{4}{100} \right)^2$
= `  $x \times \frac{104}{100} \times \frac{104}{100}$  = ` $\frac{676x}{625}$

∴ Compound interest for 2 years = `  $\left( \frac{676x}{625} - x \right)$  = ` $\frac{51x}{625}$

By the condition, $\frac{51x}{625} - \frac{2x}{5} = 80$

$$\text{or, } \frac{51x - 50x}{625} = 80$$

$$\text{or, } x = 80 \times 625$$

$$\therefore x = 50000$$

∴ Principal is ` 50000



Application : 16. Minatididi deposited a sum of money in a bank at 8% compound interest per annum for 3 years and she gets ₹ 37791.36 on maturity. Let us write by calculating how much money. Minatididi deposited in the bank.

Let Minatididi deposited ₹ x in the bank.

The amount at the rate of 8% compound interest per annum for 3 years = ₹ $x \left(1 + \frac{8}{100}\right)^3$ = ₹ $x \times \frac{108}{100} \times \frac{108}{100} \times \frac{108}{100}$

$$\therefore \text{By the condition, } x \times \frac{108}{100} \times \frac{108}{100} \times \frac{108}{100} = 37791.36$$

$$\text{or, } \frac{x \times 27 \times 27 \times 27}{25 \times 25 \times 25} = 37791.36$$

$$\text{or, } x = \frac{37791.36 \times 25 \times 25 \times 25}{27 \times 27 \times 27} \therefore x = 30000$$



∴ Minatididi deposited ₹ 30000 in the bank.

Application : 17. Let us see by calculating alternatively that, Minatididi deposits ₹ 300000 in the bank.

Application : 18. The simple interest and compound interest of a certain sum of money for two years are ₹ 400 and ₹ 410 respectively. Let us calculate that sum of money and the rate of interest per annum.

Simple interest for 2 years = ₹ 400

Simple interest for one year $(400 \div 2)$ = ₹ 200

Difference between the compound interest and simple interest for 2 years = ₹ $(410 - 400)$ = ₹ 10.

∴ Interest on ₹ 200 for 1 year = ₹ 10

Interest on ₹ 100 for 1 year = ₹ $(10 \div 2)$ = ₹ 5

∴ Rate of interest = 5%

In the language of Mathematics, the problem is

Principal (in ₹)	time (in year)	Interest (in ₹)
100	1	5
?	2	400

If interest is ₹ 5 for 1 year then, principal is ₹ 100

$$\begin{array}{ccccccc} \text{,,} & \text{,,} & 5 & \text{,,} & 2 & \text{,,} & \text{,,} & \text{,,} & \text{,,} & \frac{100}{2} \\ \text{,,} & \text{,,} & 1 & \text{,,} & 2 & \text{,,} & \text{,,} & \text{,,} & \text{,,} & \frac{100}{2 \times 5} \\ \text{,,} & \text{,,} & 1 & \text{,,} & 2 & \text{,,} & \text{,,} & \text{,,} & \text{,,} & \frac{100 \times 400}{2 \times 5} = ₹ 4000 \end{array}$$

∴ Required principal is ₹ 4000

and rate of interest per annum is 5%

Let us calculate alternatively and let us see what I shall get.

Let, the principal = ₹ p and the rate of interest per annum = r%

∴ The interest for 2 years at the rate of simple interest r% per annum = ₹ $\frac{pr \times 2}{100}$ = ₹ $\frac{2pr}{100}$

Again the amount at the rate of compound interest r% per annum for 2 years = ₹ $p \left(1 + \frac{r}{100}\right)^2$

$$\begin{aligned} \therefore \text{Compound interest for 2 years} &= ₹ \left[p \left(1 + \frac{r}{100}\right)^2 - p \right] = ₹ \left[p \left[1 + \frac{2r}{100} + \left(\frac{r}{100}\right)^2 - 1 \right] \right] \\ &= ₹ \frac{pr}{100} \left[2 + \frac{r}{100} \right] \end{aligned}$$



By the condition, $\frac{2pr}{100} = 400$ (i)

$$\frac{pr}{100} \left[2 + \frac{r}{100} \right] = 410 \text{ (ii)}$$

Dividing equation (ii) by equation (i), we get

$$\frac{\frac{pr}{100} \left[2 + \frac{r}{100} \right]}{\frac{2pr}{100}} = \frac{410}{400}$$

$$\text{or, } \frac{2 + \frac{r}{100}}{2} = \frac{41}{40}$$

$$\text{or, } 80 + \frac{40r}{100} = 82$$

$$\text{or, } \frac{2r}{5} = 82 - 80 = 2$$

$$\text{or, } 2r = 10 \quad \therefore r = 5$$

$$\therefore \text{ We get from (i) } \frac{2 \times p \times 5}{100} = 400 \quad \therefore p = 4000$$

$\therefore$ Principal is ` 4000 and rate of interest is 5%

Application : 19. The simple interest and compound interest of a certain sum of money for 2 years are ` 840 and ` 869.40 respectively. Let us calculate that sum of money and the rate of interest. [Let me do it myself]

Application : 20. Let us calculate at what rate of compound interest, ` 10000 amount to ` 11664 in 2 years.

Let the rate of compound interest be $r\%$ per annum The amount at rate of $r\%$ compound interest per annum for 2 years = ` 10000 $\left(1 + \frac{r}{100}\right)^2$

By the condition, $10000 \left(1 + \frac{r}{100}\right)^2 = 11664$

$$\text{or, } \left(1 + \frac{r}{100}\right)^2 = \frac{11664}{10000}$$

$$\text{or, } \left(1 + \frac{r}{100}\right)^2 = \frac{729}{625}$$

$$\text{or, } \left(1 + \frac{r}{100}\right)^2 = \left(\frac{27}{25}\right)^2$$

$$\text{or, } 1 + \frac{r}{100} = \frac{27}{25}$$

$$\text{or, } \frac{r}{100} = \frac{27}{25} - 1$$

$$\text{or, } \frac{r}{100} = \frac{2}{25}$$

$$\text{or, } r = \frac{200}{25} \quad \therefore r = 8$$

$\therefore$ Rate of interest per annum is 8%.



Application : 21. Let us calculate at what rate of compound interest, ₹ 5000 amount to ₹ 5832 in 2 years. [Let me do it myself]

Application : 22. Let us write by calculating in how many years will ₹ 4000 in compound interest at the rate of 10% per annum amount to ₹ 5324.

Let it be in 'n' years ₹ 4000 in compound interest at the rate of 10% per annum will amount to ₹ 5324.

$$\therefore \text{Amount for } n \text{ years} = ₹ 4000 \left(1 + \frac{10}{100}\right)^n$$

$$\text{By the condition, } 4000 \left(1 + \frac{10}{100}\right)^n = 5324$$

$$\text{or, } \left(\frac{11}{10}\right)^n = \frac{5324}{4000}$$

$$\text{or, } \left(\frac{11}{10}\right)^n = \frac{1331}{1000}$$

$$\text{or, } \left(\frac{11}{10}\right)^n = \left(\frac{11}{10}\right)^3$$

$$\therefore n = 3$$



$\therefore$ In the rate of compound interest of 10% per annum ₹ 4000 will amount to ₹ 5324 in 3 years.

Application : 23. Let us write by calculating in how many years will ₹ 5000 in compound interest at the rate of 10% per annum amount to ₹ 6050.

Let us see by calculating 6.1

1. I have ₹ 5000 in my hand. I deposited that money in a bank at the rate of 8.5% compound interest per annum for two years. Let us write by calculating how much money I shall get at the end of 2 years.
2. Let us calculate the amount on ₹ 5000 at the rate of 8% compound interest per annum for 3 years.
3. Goutam babu borrowed ₹ 2000 at the rate of 6% compound interest per annum for 2 years. Let us write by calculating how much compound interest at the end of 2 years he will pay.
4. Let us write by calculating compound interest on ₹ 30000 at the rate of 9% compound interest per annum for 3 years.
5. Let us write by calculating the amount on ₹ 80000 for $2\frac{1}{2}$ years at the rate of 5% compound interest per annum.
6. Chandadavi borrowed some money for 2 years in the compound interest at the rate of 8% per annum. Let us calculate if the compound interest is ₹ 2496, then how much money she had lended.
7. Let us write by calculating the principal whose compound interest becomes ₹ 2648 after getting 10% compound interest per annum for 3 years.
8. Rahaman chacha deposited some money in a cooperative bank at the rates of 9% compound interest per annum and he received amount ₹ 29702.50 after 2 years. Let us calculate how much money Rahaman chacha had deposited in cooperative bank.

9. Let us write by calculating what sum of money at the rate of 8% compound interest per annum for 3 years will amount to ` 31492.80.
10. Let us calculate the difference between the compound interest and simple interest on ` 12000 for 2 years, at 7.5% interest per annum.
11. Let us write by calculating the difference between compound interest and simple interest on ` 10,000 for 3 years at 5% per annum.
12. Let us write by calculating the sum of money, if the difference between compound interest and simple interest for 2 years at the rate of 9% interest per annum is ` 129.60.
13. Let us write by calculating the sum of money if the difference between compound interest and simple interest for 3 years becomes ` 930 at the rate of 10% interest per annum.
14. If the rates of compound interest for the first and the second year are 7% and 8% respectively, let us write by calculating compound interest on ` 6000 for 2 years.
15. If the rate of compound interest for the first and second year are 5% and 6% respectively, let us calculate the compound interest on ` 5000 for 2 years
16. If simple interest of a certain sum of money for 1 year is ` 50 and compound interest for 2 years is ` 102, let us write by calculating the sum of money and the rate of interest.
17. If simple interest and compound interest of a certain sum of money for two years are ` 8400 and ` 8652, then let us write by calculating the sum of money and the rate of interest.
18. Let us calculate compound interest on ` 6000 for 1 year at the rate of 8% compound interest per annum compounded at the interval of 6 months.
19. Let us write by calculating compound interest on ` 6250 for 9 months at the rate of 10% compound interest per annum compounded at the interval of 3 months.
20. Let us write by calculating at what rate of interest per annum will ` 6000 amount to ` 69984 in 2 years .
21. Let us calculate in how many years will ` 40000 amount to ` 46656 at the rate of 8% compound interest per annum.
22. Let us write by calculating at what rate of compound interest per annum, the amount on ` 10,000 for 2 years is ` 12100.
23. Let us calculate in how many years will ` 50000 amount to ` 60500 at the rate of 10% compound interest per annum.
24. Let us write by calculating in how many years will ` 300000 amount to ` 399300 at the rate of 10% compound interest per annum.
25. Let us calculate the compound interest and amount on ` 1600 for $1\frac{1}{2}$ years at the rate of 10% compound interest per annum compounded at the interval of 6 months.

A Computer training centre was inaugurated last year in our village. Willing students from class V to X and other willing persons can acquire preliminary knowledge on different topics of computer with their own efficiency at minimum cost in this institution.



Application : 24. At present 4000 students have been taking training from this training centre. In last 2 years it has been decided that the facility to get chance for training programme in this centre will be increased by 5% in comparison to its previous year. Let us see by calculating how many students will get chance to join this training programme at the end of 2 year?

$$\begin{aligned}
 \text{At present number of students} &= 4000 \\
 \text{5\% increase for first year i.e. increased} &= 200 \left[\because 4000 \times \frac{5}{100} = 200 \right] \\
 \text{Number of students at the beginning of 2nd year} &= 4200 \\
 \text{5\% increase for 2nd year i.e. increased} &= 210 \left[\because 4200 \times \frac{5}{100} = 210 \right] \\
 \text{number of students after two years} &= 4410
 \end{aligned}$$

6 What is called this equal/same rate of growth?

It is called **Uniform rate of growth** and if the rate decreases equally, then it is called **Uniform rate of decreases or depreciation**.

Increase in population, increase the number of learners, increase production in the field of agriculture and industry are examples of uniform rate of growth. On the other hand fall of efficiency of a Machine due to constant use, decrease valuations of old building and furniture, decrease in valuation of the movable properties or the transports, decrease in the number of decreases as a result of alertness come under uniform rate of decrease or depreciation.

Alternative method :

$$\begin{aligned}
 \text{Present number of students} &= 4000 \\
 \text{At the end of first year number of students} &= 4000 \times \frac{105}{100} \\
 \text{At the end of 2nd year number of students} &= 4000 \times \frac{105}{100} \times \frac{105}{100} = 4410 \\
 \text{Number of students after 2nd year} &= 4410
 \end{aligned}$$

The formula of rate of uniform growth is similar to that of compound interest.

Let us calculate by the formulas $p(1 + \frac{r}{100})^n$ and let us see what I shall get, to where p = Present population, $r\%$ = rate of increase of population in one year and n = number of years.



Let the present population in that city be p and the rate of increase of population be $r\%$ in 1 year

$$\therefore \text{Population after } n \text{ year} = p \left(1 + \frac{r}{100} \right)^n$$

I understand, here $p = 4000$, $r = 5$ and $n = 2$ years

$$\begin{aligned}
 \therefore \text{At the end of 2nd year, the number of students will be} &= 4000 \times \left(1 + \frac{5}{100} \right)^2 = 4000 \times \frac{105}{100} \times \frac{105}{100} \\
 &= \boxed{}
 \end{aligned}$$

Application : 25. The rate of increase of population of a town at the end of a year is 2% of the population that had been at the beginning of the year. If the present population of the town be 2000000, then let us write by calculating what will be the population after 3 years.

$$\text{Present population} = 2000000$$

$$\text{Increase in population in the first year at the rate of 2\%} = 40000 \left[\because 2000000 \times \frac{2}{100} = 40000 \right]$$

$$\text{Population at the beginning of the 2nd year} = 2040000$$

$$\text{Increase in population in the 2nd year at the rate of 2\%} = 40800 \left[\because 2040000 \times \frac{2}{100} = 40800 \right]$$

$$\text{Population at the beginning of the 3rd year} = 2080800$$

$$\text{Increase in population in the 3rd year at the rate of 2\%} = 41616 \left[\because 2080800 \times \frac{2}{100} = 41616 \right]$$

$$\text{Population at the end of 3rd years} = 2122416$$

$\therefore$ Population after 3 years = 2122416.

Let us calculate alternatively

Let, $p = 2000000$, $r = 2$ and $n = 3$

$$\begin{aligned} \therefore \text{Population after 3 years} &= 2000000 \left(1 + \frac{2}{100} \right)^3 \\ &= 2000000 \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \\ &= 2122416 \end{aligned}$$



Application : 26. The price of a machine in a factory of my uncle depreciates at the rate of 10% of the price at the beginning of the year. If its present price is ₹ 60000, then let us calculate what will be its estimated price after 3 years.

$$\text{Let the present price of the machine be} = ₹ 100.00$$

$$10\% \text{ decrease in price after the first year} = ₹ 10.00$$

$$\text{Decrease price at the beginning of the 2nd year} = ₹ 90.00$$

$$10\% \text{ decrease in price after the 2nd year} = ₹ 9.00 \left[\because 90 \times \frac{10}{100} = 9 \right]$$

$$\text{Decrease price at the beginning of the 3rd year} = ₹ 81.00$$

$$10\% \text{ decrease in price after the 3rd year} = ₹ 8.10 \left[\because 81 \times \frac{10}{100} = 8.10 \right]$$

$$\text{Price of Machine after 3 years} = ₹ 72.90$$

$\therefore$ In the language of Mathematics, the problem is

Present price (in ₹)	Decreased price (in ₹)
100	72.90
60000	?

A direct relation exists between present price and decreased price.



∴ Estimated price of the machine after 3 years = ₹ $72.90 \times \frac{60000}{100}$ = Rs 43740

Alternative method : Present price of the machine = ₹ 60000

10% decrease in the first year = ₹ 6000 [$\because 60000 \times \frac{10}{100} = 6000$]

Price at the beginning of the 2nd year = ₹ 54000

10% decrease in the 2nd year = ₹ 5400 [$\because 54000 \times \frac{10}{100} = 5400$]

Price at the beginning of the 3rd year = ₹ 48600

10% decrease in the 3rd year = ₹ 4860 [$\because 48600 \times \frac{10}{100} = 4860$]

Price of the machine after 3 years = ₹ 43740

∴ Price of the machine after 3 years = ₹ 43740

Let us calculate alternatively and let us see what I shall get.

Let the present price of machine be ₹ p , the rate of decrease the in a year = $r\%$

∴ Decrease for 1 year = ₹ $\frac{p \times r}{100}$

∴ Decrease in the first year = ₹ $(p - \frac{pr}{100}) = ₹ p(1 - \frac{r}{100})$

Decrease for 2nd year is ₹ $\frac{p(1 - \frac{r}{100}) \times r}{100}$

∴ Price at the beginning of 3rd year = ₹ $[p(1 - \frac{r}{100}) - \frac{pr}{100}(1 - \frac{r}{100})] = ₹ p(1 - \frac{r}{100})^2$

Similarly price of the machine after 3 years = $p(1 - \frac{r}{100})^3$

I understand, that the price of Machine after n years = $p(1 - \frac{r}{100})^n$

∴ Price of machine after 3 years = $p(1 - \frac{r}{100})^n$ [where $p = ₹ 60000$, $r = 10\%$ and $n = 3$]
= ₹ $60000(1 - \frac{10}{100})^3 = ₹ 60000 \times \frac{90}{100} \times \frac{90}{100} \times \frac{90}{100} = ₹ \square$

∴ Price of the machine after 3 years = ₹ 43740.

Application : 27. The price of a motor car is ₹ 3 lakh. If the price of the car depreciates at the rate of 30% every years, let us write by calculating, the price of the car after 3 years. [Let me do it myself]

Application : 28. Through the publicity of road safety programmes the street accident in a state has been decreased by 10% in comparison to its previous year. In the present year if the number of street accidents be 2916, let us write by calculating the number of street accidents that had been in the state 3 years before.

Let the number of street accidents 3 years before the state be x

∴ At present the number of street accidents in that state = $x(1 - \frac{10}{100})^3$

By the condition, $x(1 - \frac{10}{100})^3 = 2916$

or, $x \times \frac{90}{100} \times \frac{90}{100} \times \frac{90}{100} = 2916$ or, $x = \frac{2916 \times 100 \times 100 \times 100}{90 \times 90 \times 90}$ ∴ $x = 4000$

∴ The number of street accidents 3 years before in the state was 4000.

Application : 29. At present the population of a city is 576000; if it is being increased at the rate of $6\frac{2}{3}\%$ every year, let us calculate its population 2 years ago. **[Let me do it myself]**

Let us work-out 6.2

1. At present the population of village of Pahanpur is 10000; if population is being increased at the rate of 3% every year, let us write by calculating its population after 2 years.
2. Rate of increase in population of a state is 2% in a year. The present population is 80000000; let us calculate the population of the state after 3 years.
3. The price of a machine in a leather factory depreciates at the rate of 10% every year. If the present price of the machine be ₹ 100000, let us calculate what will be the price of that machine after 3 years.
4. As a result of Sarva Shiksha Abhiyan, the students leaving the school before completion, are readmitted, so the students in a year is increased by 5% in comparison to its previous year. If the number of such readmitted students in a district be 3528 in the present year. Let us write by calculating, the number of students readmitted 2 years before in this manner.
5. Through the publicity of road-safety programme, the street accidents in Purulia district are decreased by 10% in comparison to its previous year. If the number of street accidents in this year be 8748, let us write by calculating, the number of street accidents 3 years before in the district.
6. A cooperative society of fisherman implemented such an improved plan for the production of fishes that the production in a year will be increased 10% in comparison to its previous year. In the present year if the cooperative society can produce 406 quintals of fishes, let us write by calculating, what will be the production of fishes after 3 years.
7. The height of tree increases at the rate of 20% every year. If the present height of tree is 28.8 metre, let us calculate the height of tree 2 years before.
8. Three years before from today a family had planned to reduce the expenditure of electric bill by 5% in comparison to its previous year. 3 years ago, that family had to spend ₹ 4000 in a year for electric bill. Let us write by calculating how much amount will the family have to spend to pay the electric bill in the present year.
9. The weight of Savan babu is 80kg . In order to reduce his weight, he started regular morning walk. He decided to reduce his weight every year by 10% . Let us write by calculating, his weight after 3 years.
10. At present the sum of the number of students in all M.S.K in a district is 3993. If the

number of students increased in a year was 10% of its previous year, let us calculate the sum of the number of students 3 years before in all the M.S.K in the district.

11. As the farmers are becoming more alert of the harmful effects of using the chemical fertilisers and insecticides in agricultural lands, the number of farmers using fertilisers and insecticides in the Rasulpur village decreases by 20% in a year in comparison to its previous year. Three years ago, the number of such farmers was 3000, let us calculate the number of such farmers in that village now.
12. The price of a machine of a factory is ₹ 180000. The price of that machine decreases by 10% in each year. Let us calculate its price after 3 years.
13. For the families having no electricity in their house, a Panchayat samity of Bakultala village accepted a plan to offer electric connections. 1200 families in this village have no electric connection in their house. In comparison to its previous year, it is possible to arrange electricity every year for 75% of the families having no electricity, let us write by calculating, the number of families without electricity after 2 years.
14. As a result of continuous publicity about harmful reactions in the use of cold drinks, the number of users of cold drinks has decreased by 25% every year in comparison to its previous year. 3 years ago the number of users of cold drink in a town was 80000. Let us write by calculating, the number of users of cold drink in the present year.
15. As a result of the non smoking campaign, the number of smoker has decreased by $6\frac{1}{4}\%$ every year in comparison to its previous year. If the number of smokers at present in a city is 33750, let us write by calculating, the number of smokers in that city 3 years before.

16. **Very short answer type questions (V.S.A.)**

(A) (M.C.Q.) :

- (i) In case of compound interest, the rate of compound interest per annum is —
(a) equal (b) unequal (c) both equal or unequal (d) none of these
- (ii) In case of compound interest
(a) The Principal remains unchanged each year
(b) Principal changes in each year
(c) Principal may be equal or unequal in each year (d) none of these
- (iii) At present the population of a village is p and if increase rate of population per year be $2r\%$, the population will be after n years
(a) $p \left(1 + \frac{r}{100}\right)^n$ (b) $p \left(1 + \frac{r}{50}\right)^n$ (c) $p \left(1 + \frac{r}{100}\right)^{2n}$ (d) $p \left(1 - \frac{r}{100}\right)^n$
- (iv) Present price of a machine is ₹ $2p$ and if price of the machine decreases by $2r\%$ in each year, the price of machine after $2n$ years will be.

(a) $\text{₹ } p \left(1 - \frac{r}{100}\right)^n$ (b) $\text{₹ } 2p \left(1 - \frac{r}{50}\right)^n$ (c) $\text{₹ } p \left(1 - \frac{r}{50}\right)^{2n}$ (d) $\text{₹ } 2p \left(1 - \frac{r}{50}\right)^{2n}$

- (v) A person deposited ₹ 100 in a bank and got the amount ₹ 121 for two years, the rate of compound interest per annum is

- (a) 10% (b) 20% (c) 5% (d) $10\frac{1}{2}\%$

(B) Let us write true or false for the following statements

- (i) The compound interest will be always less than simple interest for some money at fixed rate of interest for fixed time.
- (ii) In case of compound interest, interest is to be added to principal at the fixed time interval. i.e. the amount of principal increases continuously.

(C) Let us fill in the blanks :

- (i) The compound interest and simple interest for one year at the fixed rate of interest on fixed sum of money are _____.
- (ii) If some things are increased by fixed rate with respect to time, that is _____ of growth.
- (iii) If some things are decreased by fixed rate with respect to time this is uniform rate of _____.

17. Short answer (S.A.):

- (i) Let us write the rate of compound interest per annum, so that the amount on ₹ 400 for 2 years becomes ₹ 441.
- (ii) If a sum of money doubles itself at the fixed rate of compound interest per annum in n years, let us write in how many years will it become four times.
- (iii) Let us calculate the principal that at the rate of 5% compound interest per annum becomes ₹ 615 after two years.
- (iv) The price of a machine depreciates at the rate of $r\%$ per annum. If the price of the machine after n years be ₹ v , let us find the price of the machine that was n years before.
- (v) If the rate of increase in population is $r\%$ per year, the population after n years is p ; let us find the population that was n years before.

7

THEOREMS RELATED TO ANGLES IN A CIRCLE

Like every year, this year too an exhibition will be arranged in the month of February in the field of our school. We, the students of class X will make some models on Mathematics. We shall still get-time about two weeks.



We have collected some small and big circular rings of different size and small and big sticks of different lengths.

We have decided to put sticks inside the circular ring and find what we get, we know from different information and we shall try to make model as per information.

At first Sathi puts a stick inside a circular ring like the picture mentioned beside.

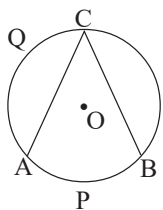
We see that the stick AB is a chord of circular ring.

Minor arc of circular ring is $\widehat{APB}$ and Major arc of circular ring is $\widehat{AQB}$.

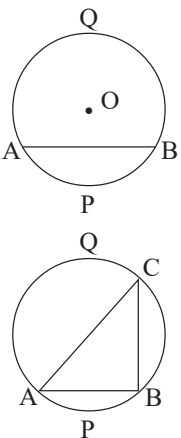
I Put two sticks AC and BC like the picture mentioned beside.

AC and BC are another two chords of circle.

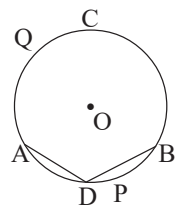
1 But the chord AB of circle made a front angle at the point C. What this angle is called ?



Point C lies on the circle except the front circular arc $\widehat{APB}$, so, $\angle ACB$ subtended front angle by the circular arc $\widehat{APB}$ is called front angle of the circle.



Again if the point D lies on the circle except the circular arc $\widehat{AQB}$, $\angle ADB$ subtended front angle by the circular arc $\widehat{AQB}$.



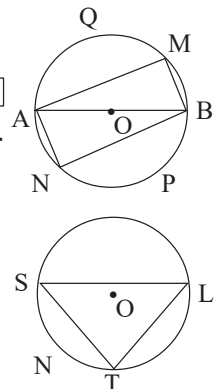
Let us Do 7.1

1) In the adjoining figure, $\angle AMB$, formed by the circular arc $\widehat{APB}$ is angle and $\angle ANB$, formed by circular arc is front angle of the circle.

2) $\angle SLT$ in the adjoining figure formed at the point L by the chord is front angle.

Again since the point L lies on the circle, so, angle formed by the chord ST is a front angle in the segment.

Again, $\angle SLT$, formed by circular arc is a front angle of the circle.

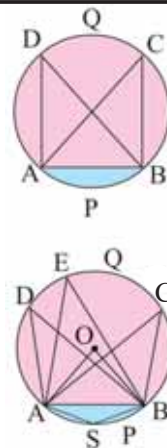


Soham puts two more sticks in the circular ring like the adjoining figure. We see that another front angle $\angle ADB$ in the segment made by arc $\widehat{APB}$ is formed. But $\angle ACB$ and $\angle ADB$ in the segment stand on same arc.

I drew a circle with centre O on my copy and drew angle in the segment with same arc.

The four angles in the segment of a circle with centre O in the adjoining figure are , , and .

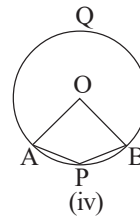
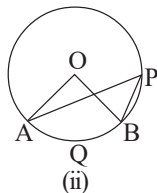
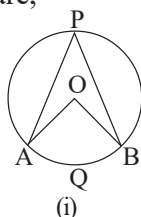
Let us write the angles in the segment at major arc $ADEQCB$ and the angle in the segment at minor arc $ASPB$.



2 The chord AB of a circle with centre O has made an angle $\angle AOB$. What is called this $\angle AOB$?

$\angle AOB$ is called the angle at the centre which is made by the arc $\widehat{ASB}$.

Mariya and Shakil have drawn many angles at the centre and the angles in the segment. These are,



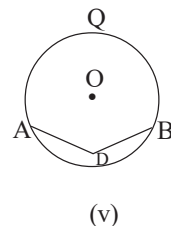
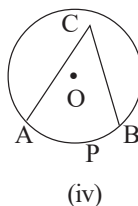
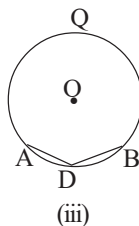
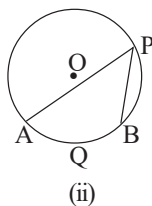
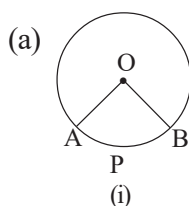
We see in no. (i), no. (ii) circles the angle at the center formed by the arc $\widehat{AQB}$ is and an angle in the segment is . But in no. (iii) circle, the angle at the centre formed by the arc $\widehat{ASB}$ is and the angle in the segment formed by the arc $\widehat{ASQ}$ is .



$\therefore$ In no (iii) circle, $\angle AOB$ is the angle at the centre and $\angle APQ$ in the segment are not formed with the same arc. Again in no. (iv) circle, $\angle AOB$ at the centre and $\angle APB$ in the segment is formed with the arc $\widehat{AQB}$.

Let us do 7.2

- I drew a circle and I drew two angles in the segment in that circle which (a) are formed with same arc (b) are not formed with same arc.
- I drew angles at the centre and the angle in the segment which (a) are formed with same arc (b) are not formed with the same arc.
- Let us look at the picture and let us give answer (O is the centre of the circle)



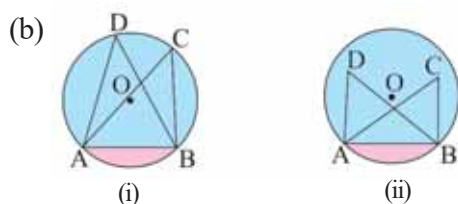
In picture no. (i) $\angle AOB$ is formed with arc $\widehat{APB}$ at the .

In picture no. (ii) angle in the segment is formed with arc $\widehat{AQB}$.

In picture no. (iii) $\angle ADB$ is formed with arc .

In picture no. (iv) $\angle ACB$ is formed with arc in the segment.

In picture no. (v) angle is not an angle in the segment formed with the arc .



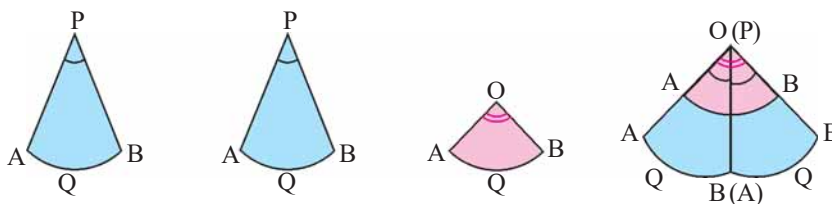
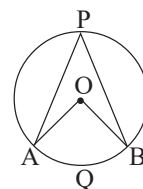
In picture no. (i), two angles and are in the same arc.

In picture no. (ii), and are two angles lie in the segment .

Beena drew a circle, I drew an angle at the centre and an angle in the segment with the same arc and find the relation between them.

Hands on trial

- In the circle with centre O, I draw $\angle AOB$ at the centre and $\angle APB$ in the segment with the same arc $\widehat{AQB}$.
- Two angles $\angle APB$ and one angle $\angle AOB$, are cut off with tracing paper.



- The two angles $\angle APB$ are placed side by side on $\angle AOB$ of a circle.

We see by hands on trial, $2\angle APB = \angle AOB$.

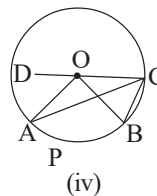
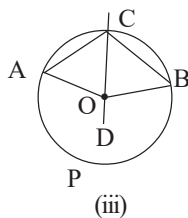
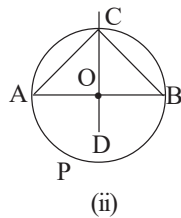
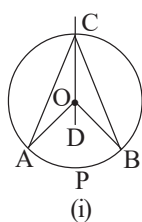
i.e. we get by hands on trial that, **the angle at the centre is the double of the angle in the segment which is formed with the same arc.**

Drawing another circle and drawing an angle at the centre and an angle in the segment with same arc, I see by hands on trial that the angle at the centre is the double of the angle in the segment. [Let me do it myself]



Let us prove with reason

Theorem : 34. The front angle formed at the centre of a circle by an arc, is the double of the angle formed by the same arc at any point on the circle.



Given : $\angle AOB$ is the angle at the centre of the circle with centre O and $\angle ACB$ is the angle at any point on the circle formed by circular arc APB.

To Prove : $\angle AOB = 2\angle ACB$

According to the length of the circular arc APB, these may be of three types. In the fig. no. (i) and (iv) APB are Minor arcs. In the fig. no. (ii) APB is semi circular arc, in fig. no. (iii) APB is Major arc.

Construction : C, O are joined and C O is extended up to the point D.

Proof : In $\triangle AOC$, $OA = OC$ in each case [radii of same circle]

$$\therefore \angle OCA = \angle OAC$$

Again, in each case, side CO of $\triangle AOC$ is extended up to the point D.

Exterior $\angle AOD = \angle OAC + \angle OCA$

$$= 2 \angle OCA \dots\dots (I) \quad [\because \angle OAC = \angle OCA]$$

In $\triangle BOC$, $OB = OC$ in each case [$\because$ radii of same circle]

Again, since side CO of $\triangle BOC$ is extended up to the point D.

$\therefore$ Exterior $\angle BOD = \angle OCB + \angle OBC$

$$= 2 \angle OCB \dots\dots (II) \quad [\because \angle OBC = \angle OCB]$$

In figure (i) and (ii) $\angle AOD + \angle BOD = 2\angle OCA + 2\angle OCB$ [We get from I & II]

$$\therefore \angle AOB = 2(\angle OCA + \angle OCB) = 2\angle ACB \dots\dots (III)$$

But in fig. no.(iii) i.e. when APB is major arc, we shall write (III)

$$\therefore \angle AOB = 2\angle ACB$$

In fig. (iv) $\angle BOD - \angle AOD = 2\angle OCB - 2\angle OCA$

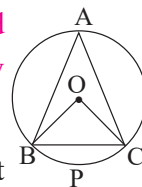
$$\text{Or, } \angle AOB = 2(\angle OCB - \angle OCA)$$

$$\therefore \angle AOB = 2\angle ACB \text{ (Proved)}$$



Application : 1. I drew a circle with centre P and by drawing an angle $\angle APB$ at the centre and an angle $\angle AQB$ at the point on the circle by the arc $\widehat{ACB}$, we shall prove that, $\angle APB = 2\angle AQB$ [Let me do it myself]

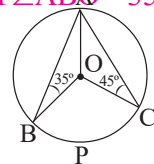
Application : 2. O is the centre of the circumcircle of $\triangle ABC$ and the point A and the chord BC lie on opposite sides of the centre. If $\angle BOC = 120^\circ$, let us write by calculating the value of $\angle BAC$.



By drawing figure we see that, $\angle BOC$ and $\angle BAC$ are calculating two angles at the centre and on the circle respectively lying on the same arc $\widehat{BPC}$ of a circle with centre O.

$$\therefore \angle BAC = \frac{1}{2} \angle BOC = \frac{1}{2} \times 120^\circ = 60^\circ$$

Application : 3. The circle with centre O passes through three points A, B, C, if $\angle ABO = 35^\circ$ and $\angle ACO = 45^\circ$, let us write by calculating the value of $\angle BOC$.



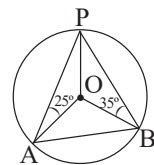
In $\triangle OAB$, $OA = OB$ (radii of same circle) $\therefore \angle OAB = \angle OBA = 35^\circ$

In $\triangle OAC$, $OA = OC$ (radii of same circle) $\therefore \angle OAC = \angle OCA = 45^\circ$

$$\therefore \angle BAC = \boxed{} + \boxed{} = \boxed{} \text{ [Let me write if myself]}$$

Since the angle $\angle BOC$ is at the centre of the circle and $\angle BAC$ is on the circle formed by circular arc $\widehat{BPC}$ of a circle with centre O, $\therefore \angle BOC = 2\angle BAC = 160^\circ$

Application : 4. We see the circle in the adjoining figure with centre O and let us write by calculating the value of $\angle OAB$, when $\angle OAP = 25^\circ$ and $\angle OBP = 35^\circ$ |



Construction : O, P are joined

Proof : $OA = OP$ (radii of same circle) $\therefore \angle OAP = \angle OPA$

$$\therefore \angle OAP = 25^\circ, \therefore \angle OPA = 25^\circ$$

$$OB = OP \text{ (radii of same circle) } \therefore \angle OBP = \angle OPB$$

$$\therefore \angle OBP = 35^\circ, \text{ so, } \angle OPB = 35^\circ$$

$$\angle APB = \angle OPA + \angle OPB = 25^\circ + 35^\circ = 60^\circ$$

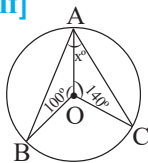
$$\angle AOB = 2\angle APB = 2 \times 60^\circ = 120^\circ$$

$$\text{In } \triangle AOB, \angle OAB + \angle OBA = 180^\circ - 120^\circ = 60^\circ \text{ [sum of three angles of a triangle is } 180^\circ]$$

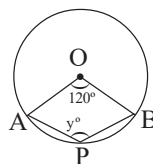
$$\therefore AO = BO \text{ [radii of same circle]}$$

$$\therefore \angle OAB = \angle OBA = \frac{1}{2} \times 60^\circ = 30^\circ$$

Application : 5. Let us write by calculating the value of x and y from the two figures given below.
[Let me do it myself]



(i)



(ii)



Application : 6. Two circles with radii of same lengths intersect each other at the points A and B. A straight line passing through the point A, in opposite sides of the chord AB, intersects one of the circles at the point P and the other circle at the point Q, let us prove that $BP = PQ$

Given : Two equal circles intersect each other at the points A and B. A straight line passing through the point A intersects one of the circles at P and the other circle at the point Q.

To prove : $BP = BQ$

Construction : Let X, Y be the centres of the first circle and the second circle respectively. A, B; A, X; B, X; A, Y; and B, Y are joined.

Proof : In $\triangle AXB$ and $\triangle AYB$, $AX = AY$ [radii of same circle]

$$BX = BY \quad [\because \square]$$

AB is common side

$\therefore \triangle AXB \cong \triangle AYB$ (By S-S-S axiom of congruency)

$\therefore \angle AXB = \angle AYB$ [corresponding angles]

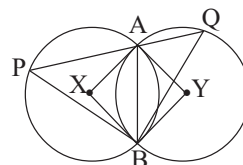
$$\therefore \frac{1}{2} \angle AXB = \frac{1}{2} \angle AYB$$

Again, $\angle APB = \frac{1}{2} \angle AXB$ [$\because \angle AXB$ is an angle at the centre and $\angle APB$ is an angle on the circle formed by minor arc AB of first circle]

Similarly, $\angle AQB = \frac{1}{2} \angle AYB$ [$\because \angle AYB$ is an angle at the centre and $\angle AQB$ is angle on the circle formed by minor arc AB of 2nd circle]

$\therefore \angle APB = \angle AQB$ i.e. $\angle QPB = \angle PQB$

$\therefore$ In $\triangle PQB$, $\angle QPB = \angle PQB \therefore BP = BQ$ [Proved]



[If the straight-line intersects the circle at the points P and Q on the same side of the chord AB, then by drawing figure, let us prove $BP = BQ$]

Application : 7. ABC is an isosceles triangle with sides $AB = AC$; we draw $\triangle DBC$ in such a way that $\triangle DBC$ and $\triangle ABC$ lie on the same side of BC and $\angle BAC = 2\angle BDC$. Let us prove that, the circle drawn with centre A and with radius AB is passing through the point D i.e. the point D lies on the circle. **[Let me do it myself]**

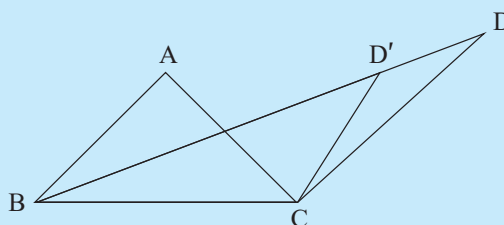
[Answer hints : If point D does not lie on that circle, let the circle intersects BD or extended BD at the point D'. D', C are joined.

$$\therefore \angle BAC = 2\angle BD'C;$$

$$\text{But, } \angle BAC = 2\angle BDC$$

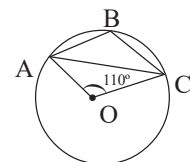
$$\therefore \angle BD'C = \angle BDC, \text{ But this is impossible if the}$$

points D and D' do not coincide. Because exterior angle of triangle can not be equal to one interior opposite angle.

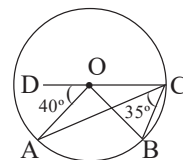


Let us work-out 7.1

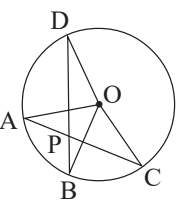
1. O is the circumcentre of the isosceles triangle ABC, whose $AB = AC$, the points A and O are opposite in side of BC. If $\angle BOC = 100^\circ$, let us write by calculating, the values of $\angle ABC$ and $\angle ABO$.



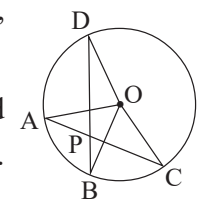
2. In the adjoining figure, if O is the centre of circumcircle of $\triangle ABC$ and $\angle AOC = 110^\circ$; let us write by calculating, the value of $\angle ABC$.



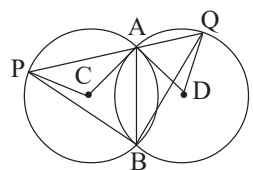
3. ABCD is a cyclic quadrilateral of a circle with centre O; DC is extended to the point P. If $\angle BCP = 108^\circ$, let us write by calculating, the value of $\angle BOD$.



4. In the adjacent figure O is the centre of the circle; $\angle AOD = 40^\circ$ and $\angle ACB = 35^\circ$; let us write by calculating the value of $\angle BCO$ and $\angle BOD$, and answer with reason.



5. O is the centre of circle in the picture beside, if $\angle APB = 80^\circ$, let us find the sum of the measures of $\angle AOB$ and $\angle COD$ and answer with reason.



6. Like the adjoining figure, we draw two circles with centres C and D which intersect each other at the points A and B. We draw a straight line through the point A which intersects the circle with centre C at the point P and the circle with centre D at the point Q. Let us prove that (i) $\angle PBQ = \angle CAD$ (ii) $\angle BPC = \angle BQD$

7. If the circumcentre of triangle ABC is O; let us prove that $\angle OBC + \angle BAC = 90^\circ$

8. Each of two equal circles passes through the centre of the other and the two circles intersect each other at the points A and B. If a straight line through the point A intersects the two circles at points C and D, let us prove that $\triangle BCD$ is an equilateral triangle.

9. S is the centre of the circumcircle of $\triangle ABC$ and if $AD \perp BC$, let us prove that $\angle BAD = \angle SAC$

10. Two chords AB and CD of a circle with centre O intersect each other at the point P, let us prove that $\angle AOD + \angle BOC = 2\angle BPC$.

If $\angle AOD$ and $\angle BOC$ are supplementary to each other, let us prove that the two chords are perpendicular to each other.

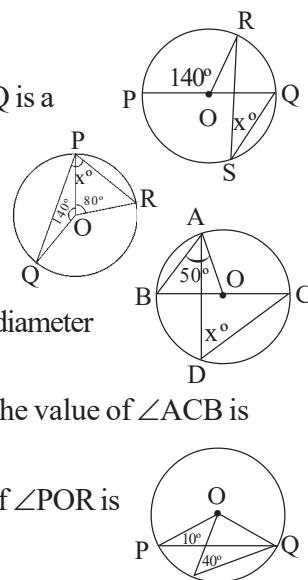
11. If two chords AB and CD of a circle with centre O, when produced intersect each other at the point P, let us prove that $\angle AOC - \angle BOD = 2\angle BPC$

12. We drew a circle with the point A of quadrilateral ABCD as centre which passes through the points B, C and D. Let us prove that $\angle CBD + \angle CDB = \frac{1}{2} \angle BAD$

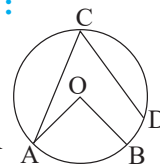
13. O is the circumcentre of $\triangle ABC$ and OD is perpendicular on the side BC; let us prove that $\angle BOD = \angle BAC$

14. **Very short answer type questions (V.S.A.) :****(A) (M.C.Q.) :**

- (i) In the adjoining figure, if 'O' is the centre of circle and PQ is a diameter then the value of x is
(a) 140 (b) 40 (c) 80 (d) 20
- (ii) In the adjoining figure, if O is the centre of circle, then the value of x is (a) 70 (b) 60 (c) 40 (d) 200
- (iii) In the adjoining figure, if O is centre of circle and BC is the diameter then the value of x is (a) 60 (b) 50 (c) 100 (d) 80
- (iv) O is the circumcentre of $\triangle ABC$ and $\angle OAB = 50^\circ$, then the value of $\angle ACB$ is
(a) 50° (b) 100° (c) 40° (d) 80°
- (v) In the adjoining figure, if O is centre of circle, the value of $\angle POR$ is
(a) 20° (b) 40° (c) 60° (d) 80°

**(B) Let us write whether the following statements are true or false :**

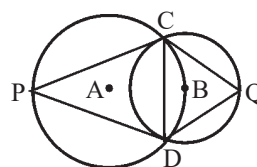
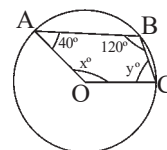
- (i) In the adjoining figure, if O is centre of circle, then $\angle AOB = 2\angle ACD$
- (ii) The point O lies within the triangular region ABC in such a way that $OA = OB$ and $\angle AOB = 2\angle ACB$. If we draw a circle with centre O and length of radius OA, then the point C lies on the circle.

**(C) Let us fill in the blanks**

- (i) The angle subtended at a point on the circle is _____ of the angle subtended at the centre by the same arc.
- (ii) The length of two chords AB and CD are equal of a circle with centre O. If $\angle APB$ and $\angle DQC$ are angles on the circle, then the values of the two angles are _____.
- (iii) If O is the circumcentre of equilateral triangle, then the value of the front angle formed by any side of the triangle is _____

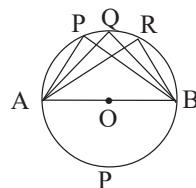
15. **Short answer type questions (S.A.) :**

- (i) In the adjoining figure, O is the centre of the circle, if $\angle OAB = 40^\circ$, $\angle ABC = 120^\circ$, $\angle BCO = y^\circ$ and $\angle COA = x^\circ$, let us find x and y.
- (ii) O is the circumcentre of the triangle ABC and D is the mid point of the side BC. If $\angle BAC = 40^\circ$, let us find the value of $\angle BOD$.
- (iii) Three points A, B and C lie on the circle with centre O in such a way that AOCB is a parallelogram, let us calculate the value of $\angle AOC$.
- (iv) O is the circumcentre of isosceles triangle ABC and $\angle ABC = 120^\circ$; if the length of the radius of the circle is 5cm, let us find the value of the side AB.
- (v) Two circles with centres A and B intersect each other at the points C and D. The centre B of the other circle lies on the circle with centre A. If $\angle CQD = 70^\circ$, let us find the value of $\angle CPD$





Like Rabeya, Shakil also drew a circle with centre O on the blackboard of our school and drew a diameter AB of that circle. Trisha drew three semicircular angles $\angle APB$, $\angle AQB$ and $\angle ARB$.



Each semicircular angle is right angle [let us write the angle by measuring]
i.e. All semicircular angles are [equal/unequal]

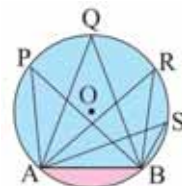
I draw many angles in the same segment and measure those angles with protractor and try to find the relation among them.

$\angle APB$, $\angle AQB$, $\angle ARB$ and $\angle ASB$ are the angles in the segment ABSRQP

Measuring with protractor and see.

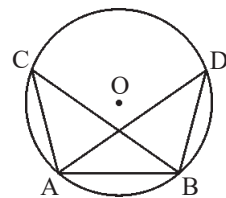
$\angle APB$ $\angle AQB$ $\angle ARB = \angle ASB$ [Let us write = / $\neq$]

I draw one or more angles in the same segment on my copy and by measuring with protractor I see that angles in the same segment of a circle are [equal/unequal] [let us write by measuring or drawing]



Hands on trial

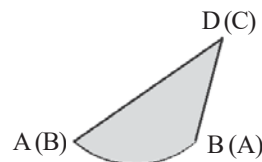
- (1) We draw a circle with centre O on the white art paper.
- (2) We get chord AB joining two points A & B of circle with centre O like the adjoining figure.
- (3) We draw two angles $\angle ACB$ and $\angle ADB$ at the points C and D of a circle in the same side of chord AB. i.e. $\angle ACB$ and $\angle ADB$ are two angles in the segment ABDC of the circle with centre O.
- (4) I cut off $\angle ACB$ and $\angle ADB$ by drawing it with the help of tracing paper.
- (5) The two angles we draw on tracing paper are placed one upon another in such a way that the vertices C and D are coincided.



We see, $\angle ACB$ and $\angle ADB$ are coincided.

$\therefore$ We get by hands on trial, $\angle ACB = \angle ADB$

$\therefore$ We get all the angles in the same segment of circle are equal.



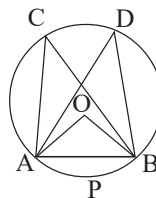
Let us prove with reason

Theorem 35. Angles in the same segment of a circle are equal.

Given : Let $\angle ACB$ and $\angle ADB$ are any two angles in the segment ABDC of a circle with centre O.

To prove : All angles in the same segment ABDC of the circle are equal.

Since $\angle ACB$ and $\angle ADB$ are any two angles on that segment on the circle, the theorem will be proved if it can be shown that $\angle ACB$ and $\angle ADB$ are equal to each other.



Construction : O, A and O, B are joined.

Proof : The angle $\angle AOB$ at the centre; the angles $\angle ACB$ and $\angle ADB$ on the circle, are on the same arc APB.

$$\therefore \angle AOB = 2\angle ACB$$

$$\text{and } \angle AOB = 2\angle ADB$$

$$\text{i.e. } 2\angle ACB = 2\angle ADB$$

$$\therefore \angle ACB = \angle ADB$$

**Let us do 7.3**

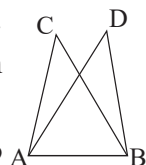
- (1) Let us prove that all angles subtended at the points on the circle formed by same arc are equal.
- (2) Let us prove that if all angles of a circle subtended by circular arcs be equal then the lengths of the arcs are equal.

By hands on trial and with reason we have proved that all angles of a circle subtended by the same arcs are equal.

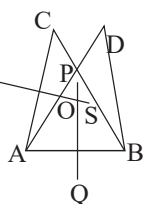
Is its converse theorem possible? i.e. if the line segment joining any two points subtends equal angles at other two points on the same side of line segment will four points be concyclic?

**Hands on trial**

- (i) On a white paper, we draw a line segment AB by joining two points A and B. Two equal angles $\angle ACB$ and $\angle ADB$ are formed at the points C and D on the same side of the line segment AB.



- (ii) By folding the paper we got perpendicular bisectors PQ and RS of two line segments AB and CD respectively, they intersect each other at the point O.



- (iii) By measuring with scale we see, AO OB OC OD
[By hands on trial we put = / ≠]

- (iv) We draw a circle with centre O and taking length of radius OA, it will pass through the points A, B, C, D,

So, we get a fixed circle which passes through the non-collinear points A, B and C also it will pass through the point D.

$\therefore$ We get four points A, B, D and C which are concyclic.

Let us prove with reason

Theorem: 36. If a line segment joining two points subtends equal angles at two other points on the same side of it, then the four points are concyclic.

Given : Let the line segment joining two points A and B subtends equal angles at the two points C and D.

$$\therefore \angle ACB = \angle ADB$$

To prove : The points A, B, D and C are concyclic.

Proof : A fixed circle is drawn through three non-collinear points A, B and C. If this fixed circle does not pass through the point D, the circle intersects AD or extended AD at a point.

Let the circle intersects AD or extended AD at the point D'.

$\therefore$ Four points A, B, D' and C are concyclic D' and B are joined by a line segment.

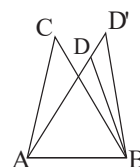
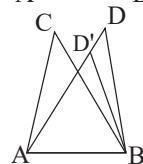
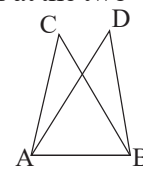
$$\therefore \angle ACB = \angle AD'B \text{ (angles in the same segment)}$$

$$\text{But } \angle ACB = \angle ADB$$

$$\therefore \angle AD'B = \angle ADB$$

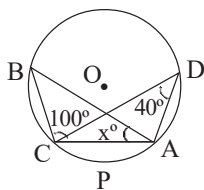
But it is absurd, unless D and D' coincides

$$\therefore \text{A, B, D and C are concyclic.}$$

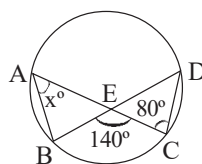


[As the exterior angle of a triangle can not be equal to interior opposite angle]

Application : 8. Let us see the figure of circle below and let us find the value of x. O is the centre of circle.



(i)



(ii)



(i) O is the centre of the circle.

$\angle ABC$ and $\angle ADC$ are the front angles on the circle formed by the minor arc CPA.

$$\therefore \angle ABC = \angle ADC = 40^\circ \text{ (} \because \text{ Given that } \angle ADC = 40^\circ \text{)}$$

and in $\triangle ABC$, $\angle ABC + \angle ACB + \angle BAC = 180^\circ$

$$\therefore 40^\circ + 100^\circ + x = 180^\circ$$

$$\text{or, } x^\circ = 180^\circ - 140^\circ = \boxed{}$$

$$\therefore x = 40$$

Similarly let us calculate the value of x in figure no. (ii)

Application : 9. In the adjoining figure, $\angle BDC = 50^\circ$, $\angle APB = 65^\circ$, $\angle CBD = 28^\circ$; Let us determine the values of $\angle ADB$, $\angle ABD$, $\angle BAC$, $\angle ACB$, $\angle CAD$ and $\angle ACD$.

$$\angle BAC = \angle BDC = \boxed{} [\because \text{angle in the same segment}]$$

$$\text{Again, } \angle CAD = \boxed{} = 28^\circ [\because \text{angle in the same segment}]$$

In $\triangle BPC$, Exterior $\angle APB = \angle PBC + \angle PCB$.

$$\therefore 65^\circ = 28^\circ + \angle ACB ; \therefore \angle ACB = \boxed{}$$

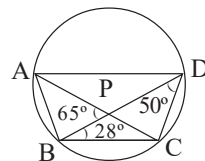
$$\therefore \angle ADB = \angle ACB = \boxed{} [\text{angles in the same segment}]$$

In $\triangle ABP$ - $\angle ABP + \angle BPA + \angle PAB = 180^\circ$

$$\therefore \angle ABP = \boxed{}$$

$$\therefore \angle ABD = \boxed{} \text{ [Let me do it myself]}$$

$$\angle ACD = \angle ABD = \boxed{} [\because \text{Front angles in the same segment}]$$



Application : 10. Let us prove that the bisectors of all the angles in the same segment of a circle pass through a fixed point.

Given : For a circle with centre O, $\angle APB$ is an angle in the segment APB; bisector of $\angle APB$ intersects the circle at the point Q.

To prove : The bisector passes through a fixed point.

Proof : $\angle APQ = \angle BPQ$ [$\because$ PQ, is a bisector of $\angle APB$]

$$\angle AOQ = 2\angle APQ ; \text{ Again } \angle BOQ = 2\angle BPQ$$

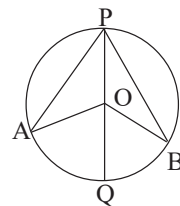
$$\text{i.e. } \angle APQ = \angle BPQ , \text{ so, } \angle AOQ = \angle BOQ$$

$$\therefore \text{Arc AQ} = \text{Arc BQ (i.e. Arc AQ and arc BQ make front angle at the centre of the circle)}$$

$$\therefore \text{Q is the mid point of arc AQB.}$$

If AQB is fixed, Q is also fixed point.

$\therefore$ For any position of P on the arc APB, bisectors of $\angle APB$ will pass through the mid point of its opposite arc i.e. passes through fixed point.



Application : 11. In the adjoining figure, AB and CD are two chords of a circle. If we extend BA and DC, they intersect each other at the point P, let us prove that $\angle PCB = \angle PAD$.

Given : AB and CD are two chords of a circle. BA and DC, when extended, meet each other at the point P.

To prove : $\angle PCB = \angle PAD$

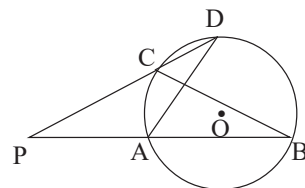
Proof : $\angle BCD = \angle BAD$ (angles in the same segment)

$$\text{Again, } \angle PCB = 180^\circ - \angle BCD$$

$$\text{and } \angle PAD = 180^\circ - \angle BAD$$

$$\therefore \angle BCD = \angle BAD, \text{ So, } 180^\circ - \angle BCD = 180^\circ - \angle BAD$$

$$\therefore \angle PCB = \angle PAD \text{ [Proved]}$$



Application : 12. The equilateral triangle ABC is inscribed in a circle. P is any point on minor arc BC, let us prove that $PA = PB + PC$

Given : The equilateral triangle ABC is inscribed in a circle. P is any point on minor arc BC.

To prove : $PA = PB + PC$

Construction : PX is cut off from PA making equal to PB. B, X are joined.

Proof: $PB = PX$ [by construction]

$$\therefore \angle PBX = \angle PXB \text{ _____ (I)}$$

Again, $\angle ACB = \angle APB$ [angles in the same segment]

$$= 60^\circ [\because \text{ABC is an equilateral triangle } \therefore \angle ACB = 60^\circ]$$

$$\therefore \angle PBX + \angle PXB = 180^\circ - 60^\circ = \boxed{}$$

$$\therefore \angle PBX = \angle PXB = 60^\circ \text{ [we get from (I)]}$$

$\therefore$ PBX is an equilateral triangle.

$$\therefore \angle PBC = \angle PBX - \angle CBX = \angle CBA - \angle CBX = \angle XBA \text{ _____ (II)}$$

In $\triangle BPC$ and $\triangle ABX$, $AB = BC$ [$\because \triangle ABC$ is an equilateral triangle]

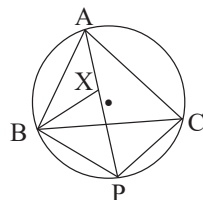
$$\angle ABX = \angle PBC \text{ [we get from (II)]}$$

$$BX = PB (\because \triangle BPX \text{ is an equilateral triangle})$$

$$\therefore \triangle BPC \cong \triangle ABX \text{ [S-A-S congruence]}$$

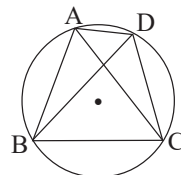
$$\therefore AX = PC$$

$$\therefore PA = PX + XA = PB + PC \text{ [Proved]}$$

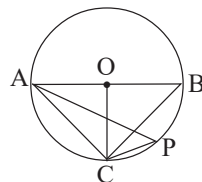


Let us work-out 7.2

- In the adjoining figure, $\angle DBA = 40^\circ$, $\angle BAC = 60^\circ$ and $\angle CAD = 20^\circ$; let us find the values of $\angle DCA$ and $\angle BCA$ also let us see by calculating, what the sum of $\angle BAD$ and $\angle DCB$ will be



- In the adjoining figure, AOB is the diameter of the circle and O is the centre of the circle. The radius OC is perpendicular on AB. If P is any point on minor arc CB, let us write by calculating the values of $\angle BAC$ and $\angle APC$



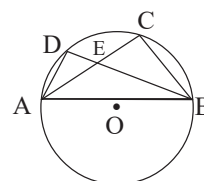
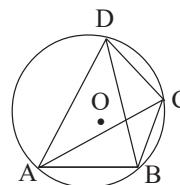
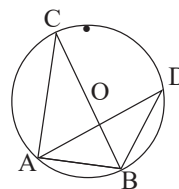
- O is the orthocentre, of the triangle ABC and the perpendicular AD drawn on BC, when extended, intersects the circumcircle of $\triangle ABC$ at the point G, let us prove that $OD = DG$

4. I is the centre of the incircle of $\triangle ABC$; AI produced intersects the circumcircle of that triangle at the point P, let us prove that $PB = PC = PI$
5. Timir drew two circles which intersect each other at the points P and Q. Through the point P two straight lines are drawn so that they intersect one of the circles at the points A and B, and the other circle at the points C and D respectively, let us prove that $\angle AQC = \angle BQD$.
6. Two chords AB and CD of a circle are perpendicular to each other. If a perpendicular drawn to AD from the point of intersection P of those two chords AB and CD is produced to meet BC at the point E, let us prove that the point E is the mid point of BC.
7. If in a cyclic quadrilateral ABCD, $AB = DC$, let us prove that $AC = BD$
8. OA is the radius of a circle with centre at O, AQ is its chord and C is any point on the circle. A circle passes through the points O, A, C intersects the chord AQ at the point P, let us prove that $CP = PQ$
9. The triangle ABC is inscribed in a circle, the bisectors AX, BY and CZ of the angles $\angle BAC$, $\angle ABC$ and $\angle ACB$ intersect at the points X, Y, Z, on the circle respectively, let us prove that AX is perpendicular to YZ.
10. The triangle ABC is inscribed in a circle. The bisectors of the angles $\angle BAC$, $\angle ABC$ and $\angle ACB$ intersect at the points X, Y and Z on the circle respectively, let us prove that in $\triangle XYZ$, $\angle YXZ = 90^\circ - \frac{\angle BAC}{2}$
11. A perpendicular drawn on BC from the point A of $\triangle ABC$ intersects the side BC at the point D and a perpendicular drawn on the side CA from the point B intersects the side CA from the point B at the point E, let us prove that four points A, B, D, E are concyclic.

12. **Very short answer type questions (V.S.A) :**

M.C.Q.

- (i) In the adjoining figure, O is the centre of the circle, if $\angle ACB = 30^\circ$, $\angle ABC = 60^\circ$, $\angle DAB = 35^\circ$ and $\angle DBC = x^\circ$, the value of x is
(a) 35 (b) 70 (c) 65 (d) 55
- (ii) In the adjoining figure, O is the centre of the circle, if $\angle BAD = 65^\circ$, $\angle BDC = 45^\circ$, then the value of $\angle CBD$ is (a) 65° (b) 45° (c) 40° (d) 20°
- (iii) In the adjoining figure, the O is the centre of circle, if $\angle AEB = 110^\circ$ and $\angle CBE = 30^\circ$, the value of $\angle ADB$ is (a) 70° (b) 60° (c) 80° (d) 90°



- (iv) In the adjoining figure, O is the centre of the circle, if $\angle BCD = 28^\circ$, $\angle AEC = 38^\circ$, then the value of $\angle AXB$ is

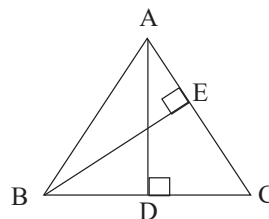
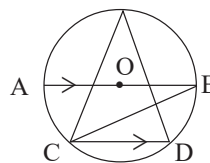
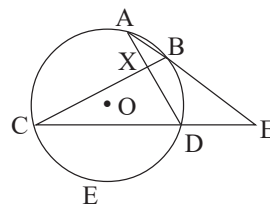
(a) 56° (b) 86° (c) 38° (d) 28°

- (v) In the adjoining figure, O is the centre of the circle and AB is diameter. If $AB \parallel CD$. $\angle ABC = 25^\circ$, the value of $\angle CED$ is

(a) 80° (b) 50° (c) 25° (d) 40°

(B) Write true or false :

- (i) In the adjoining figure AD and BE are the perpendiculars on side BC and CA of the triangle ABC. A, B, D, E are concyclic.



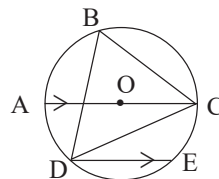
- (ii) In $\triangle ABC$, $AB = AC$, BE and CF are the bisectors of the angles $\angle ABC$ and $\angle ACB$ and they intersect AC and AB at the points E and F respectively. Four points B, C, E, F, are not concyclic.

(C) Fill in the blanks :

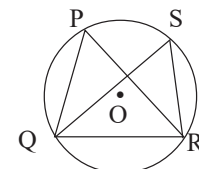
- (i) All angles in the same segment are _____ .
 (ii) If the line segment joining two points subtends equal angle at two other points on the same side, then the four points are _____ .
 (iii) If two angles on the circle formed by two arcs are equal then the lengths of arcs are _____ .

13. Short answer type (S.A) :

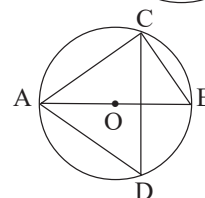
- (i) In the adjoining figure, O is centre of the circle, AC is diameter and chord DE is parallel to the diameter AC. If $\angle CBD = 60^\circ$; let us find the value of $\angle CDE$.



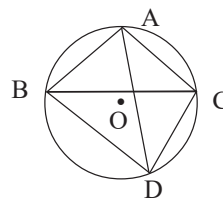
- (ii) In the adjoining figure, QS is the bisectors of an angle $\angle PQR$, if $\angle SQR = 35^\circ$ and $\angle PRQ = 32^\circ$, let us find the value of $\angle QSR$.



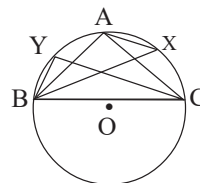
- (iii) In the adjoining figure, O is centre of the circle and AB is diameter. If AB and CD are mutually perpendicular to each other and $\angle ADC = 50^\circ$; let us find the value of $\angle CAD$.



- (iv) In the adjoining figure, O is centre of the circle and $AB = AC$; if $\angle ABC = 32^\circ$, let us find the value of $\angle BDC$.



- (v) In the adjoining figure, BX and CY are the bisectors of the angles $\angle ABC$ and $\angle ACB$ respectively. If $AB = AC$ and $BY = 4$ cm, let us find the length of AX .

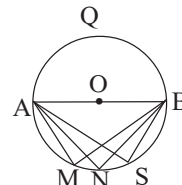
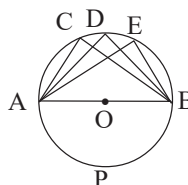


Rabeya has drawn a circle with centre O, Shakil drew a diameter AB in the circle drawn by Rabeya.

By measuring we see that, the diameter AB or circular arc APB subtended angles at the points C, D & E on the circle

[let us write by drawing and measuring]

Again diameter AB or circular arc AQB subtended angles at each of the points M, N, S on the



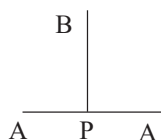
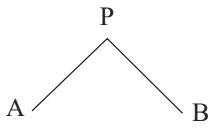
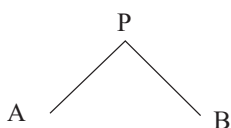
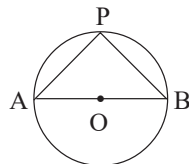
3 These $\angle ACB$, $\angle ADB$, $\angle AEB$, $\angle AMB$, $\angle ANB$ and $\angle ASB$ angles i.e. what the angles formed by a diameter of a circle at different points on a semicircle are called?

The angles formed by a diameter at different points of this semicircle are called semicircular angles. Here each of $\angle ACB$, $\angle ADB$, $\angle AEB$ and $\angle AMB$, $\angle ANB$ and $\angle ASB$, is **Semicircular** angle.

By hands on trial, let us see and verify that the angle in a semicircle is a right angle.

Hands on trial

- (1) We draw a semicircular angle $\angle APB$ of a circle with centre O.
- (2) We draw $\angle APB$ twice with the help of a tracing paper and cut-off and placing two angles at the point O on the diameter AB of the circle with centre O side by side like the figure given below, two angles $\angle APB$ are supplementary to each other and equal. [$\because$ same measure of angle]
 $\therefore$ Each angle is right angle.



$\therefore$ We get by hands on trial, $\angle APB = \frac{1}{2} \times 180^\circ = 90^\circ$

$\therefore$ We get by hands on trial, that the angle in a semicircle is a right angle.



Let us prove with reason

Theorem: 37. Angle in a semicircle is a right angle.

Given : In the circle with centre O, $\angle ACB$ is an angle in a semicircle.

To prove : $\angle ACB = 1$ right angle.

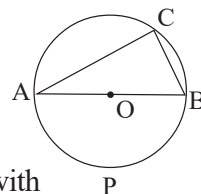
Proof : $\angle AOB$ is the angle at the centre lying on the arc $\widehat{APB}$ of a circle with centre O, and $\angle ACB$ is the angle subtended by the same arc $\widehat{APB}$.

$$\therefore \angle AOB = 2\angle ACB \dots\dots\dots (I)$$

Since AB is a line segment, so, $\angle AOB$ is a straight angle. $\therefore \angle AOB = 2$ right angles

$$\therefore 2\angle ACB = 2 \text{ right angles [We get from (I)]}$$

$$\therefore \angle ACB = 1 \text{ right angle [Proved]}$$

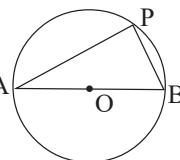


$\angle PQR$ is a semicircular angle of another circle with centre O, let us prove with reason that $\angle PQR = 90^\circ$. [Let me do it myself]

Application : 13. AB is a diameter of a circle and P is any point on the circle, if $\angle PAB = 30^\circ$, let us find the value of $\angle PBA$.

Answer : $\angle APB$ is a semicircular angle $\therefore \angle APB = 90^\circ$; $\angle PAB = 30^\circ$

$$\therefore \angle PBA = 90^\circ - 30^\circ = 60^\circ$$



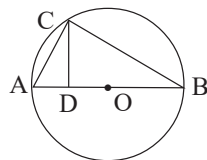
Application : 14. In adjoining figure, AB is a diameter of a circle with centre O. C is any point on the circle. If $\angle BAC = 50^\circ$ and CD is perpendicular on AB, let us find the value of $\angle BCD$.

Answer : $\angle ACB$ is a semicircular angle. $\therefore \angle ACB = 90^\circ$

In right angled triangle ACD, $\angle ADC = 90^\circ$,

$$\angle DAC = 50^\circ, \therefore \angle ACD = 90^\circ - 50^\circ = 40^\circ$$

$$\text{Again, } \angle ACB = 90^\circ; \text{ hence } \angle BCD = 90^\circ - 40^\circ = 50^\circ$$



Application : 15. In the adjoining figure, the line segments AB and CD intersect at the centre O of the circle. If $\angle AOC = 80^\circ$, $\angle CDE = 40^\circ$, let us find the value of (i) $\angle DCE$ and (ii) $\angle ABC$.

Solution : $\angle CED$ is a semicircular angle $\therefore \angle CED = 90^\circ$

In right angled triangle CED, $\angle CED = 90^\circ$, $\angle CDE = 40^\circ$,

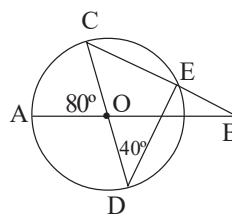
$$\therefore \angle DCE = 90^\circ - 40^\circ = 50^\circ \dots\dots\dots (i)$$

In $\triangle BOC$, Exterior $\angle AOC = \angle OBC + \angle OCB$

$$80^\circ = \angle OBC + 50^\circ (\because \angle DCE = 50^\circ)$$

$$\therefore \angle OBC = 30^\circ$$

So, $\angle ABC = 30^\circ \dots\dots\dots (ii)$



Application : 16. let us prove with reason that the angle in the segment of a circle which is greater than a semicircle is an acute angle.

Given : The segment $\widehat{ACB}$ of the circle with centre O is greater than the semicircle.

To prove: $\angle ACB$ is an acute angle.

Proof : Since the segment $\widehat{ACB}$ of the circle is greater than the semicircle.

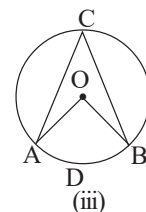
$\therefore \widehat{ADB}$ is a minor arc.

$\therefore \angle AOB$, the angle at the centre lying on that arc is less than 2 right angle.

Again, $\angle ACB$ is an angle on the circle also lying on the same arc

$\widehat{ADB} \therefore \angle AOB < 180^\circ$ i.e. $2\angle ACB < 180^\circ \therefore \angle ACB < 90^\circ$

$\therefore \angle ACB$ is less than 1 right angle. i.e. $\angle ACB$ is an acute angle. **[Proved]**



Application: 17. Let us prove that the angle in the segment of a circle which is less than a semicircle is an obtuse angle. **[Let me do it myself]**

Application : 18. Let us prove with reason that the middle point of hypotenuse of any right angled triangle is equidistant from three vertices.

Given : In right angled triangle ABC, $\angle ACB = 90^\circ$ and O is the middle point of the hypotenuse AB.



To prove : $OA = OB = OC$

Construction : We draw a circle taking length of radius OB with centre O.

Proof : The circle drawn with length of radius OB, taking O as centre passes through the points A and B, if the circle does not pass through the point C, the circle intersects AC or AC produced at the point C'. B', C' are joined.

$\therefore \angle AC'B = 90^\circ$ [$\because$ semicircular angle]

Again, $\angle ACB = 90^\circ$

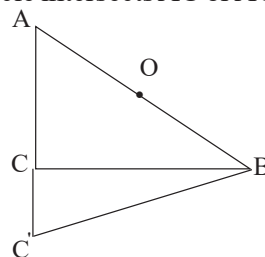
$\therefore \angle ACB = \angle AC'B$

It is impossible, as exterior angle of a triangle cannot be equal to interior opposite angle.

It is possible, if C and C' are same point.

$\therefore$ The circle passes through the point C.

$\therefore OA = OB = OC$ **[Proved]**



Application : 19. Let us prove with reason that the circle drawn with hypotenuse of a right angled triangle as diameter passes through the right angular vertex. **[Let me do it myself]**

Application : 20. Let us prove with reason that the point of intersection of two circles drawn with two smaller sides than the greatest side of a scalene triangle, as diameters, lies on third side.

Given : AC is the greatest side of $\triangle ABC$. A circle drawn with AB as diameter intersects AC at the point D.

To prove : The circle drawn with BC as diameter also passes through the point D.

Construction : B, D are joined by line segment.

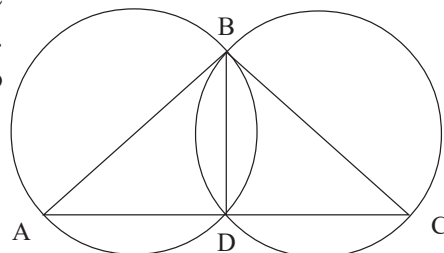
Proof : $\angle ADB$ is an angle in the semicircle.

$\therefore \angle ADB = 1$ right angle.

$\therefore \angle CDB = 1$ right angle.

Since in right angled triangle CDB, $\angle CDB = 1$ right angle,

So, BC is the diameter of the circle drawn with BC as diameter also passes through the point D.



Application :21. AC is the common hypotenuse of two right angled triangles ABC and ADC, let us prove that $\angle CAD = \angle CBD$;

Given : AC is the common hypotenuse of right angled triangles ABC and ADC

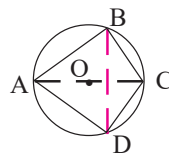
To prove : $\angle CAD = \angle CBD$

Proof : In $\triangle ABC$, $\angle ABC = 90^\circ$

$\therefore$ If the circle drawn with AC as diameter, it passes through the point B. Similarly, the circle drawn with diameter AC, it passes through the point D.

$\angle CAD$ and $\angle CBD$ are the angles on the circle lie on the same minor arc DC.

$\therefore \angle CAD = \angle CBD$ (**Proved**)

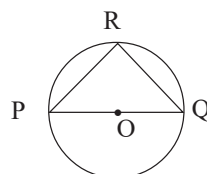


Let us work out 7.3

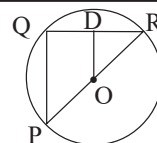
- In $\triangle ABC$, $\angle B$ is a right angle. A circle drawn taking AC as diameter intersects AB at the point D, let us write the correct information from the followings :
(i) $AB > AD$ (ii) $AB = AD$ (iii) $AB < AD$
- Let us prove that the circle drawn with any one of the equal sides of an isosceles triangle as diameter bisects the unequal side.
- Sahana drew two circles which intersect each other at the points P and Q. If the diameters of the two circles are PA and PB respectively, then let us prove that A, Q, B are collinear.
- Rajat drew a line segment PQ of which mid point is R and two circles are drawn with PR and PQ as diameter. I drew a straight line through the point P which intersects the first circle at the point S and the second circle at the point T. Let us prove with reason that $PS = ST$
- Three points P, Q, R lie on a circle. The two perpendiculars PQ and PR at the point P intersect the circle at the points S and T respectively. Let us prove that $RQ = ST$
- ABC is an acute angled triangle. AP is the diameter of the circumcircle of the triangle ABC; BE and CF are perpendiculars on AC and AB respectively and they intersect each other at the point Q. Let us prove that BPQC is a parallelogram.
- The internal and external bisectors of the vertical angle of a triangle intersect the circumcircle of the triangle at the points P and Q. Let us prove that PQ is a diameter of the circle.
- AB and CD are two diameters of a circle. Let us prove that ACBD is a rectangular figure.
- Let us prove that if the circles are drawn having sides of a rhombus as diameter then the circles pass through the fixed points.

- Very short answer (V.S.A.)**
(M.C.Q.) :

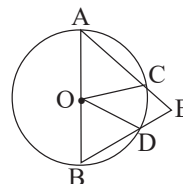
- PQ is a diameter of a circle with centre O, and $PR = RQ$; the value of $\angle RPQ$ is
(a) 30° (b) 90° (c) 60° (d) 45°



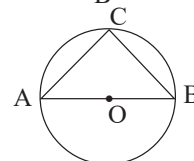
- (ii) QR is a chord of a circle and POR is a diameter of a circle. OD is perpendicular on QR. If $OD = 4$ cm, the length of PQ is
(a) 4cm. (b) 2cm. (c) 8cm. (d) none of these



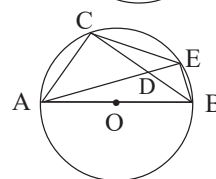
- (iii) AOB is a diameter of a circle. The two chords AC and BD when extended meet at the point E. If $\angle COD = 40^\circ$, the value of $\angle CED$ is
(a) 40° (b) 80° (c) 20° (d) 70°



- (iv) AOB is diameter of a circle. If $AC = 3$ cm, $BC = 4$ cm, then the length of AB is
(a) 3 cm. (b) 4 cm. (c) 5 cm. (d) 8 cm.



- (v) In the adjoining figure, O is centre of circle & AB is a diameter, if $\angle BCE = 20^\circ$, $\angle CAE = 25^\circ$, the value of $\angle AEC$ is
(a) 50° (b) 90° (c) 45° (d) 20°



(B) Let us write true or false.

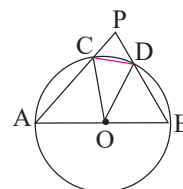
- (i) The angle in the segment of a circle which is greater than a semicircle is an obtuse angle.
(ii) O is the mid point of the side AB of the triangle ABC, and $OA = OB = OC$; if we draw a circle with side AB as diameter, then the circle passes through the point C.

(C) Let us fill in the blanks.

- (i) Semicircular angle is _____.
(ii) The angle in the segment of a circle which is less than a semicircle is an _____ angle.
(iii) The circle drawn with hypotenuse of a right angled triangle as diameter passes through the _____.

11. Short answer

- (i) In isosceles triangle ABC, $AB = AC$; a circle drawn taking AB as diameter meets the side BC at the point D. If $BD = 4$ cm, let us find the value of CD.
(ii) Two chords AB and AC of a circle are mutually perpendicular to each other. If $AB = 4$ cm, $AC = 3$, let us find the length of the radius of the circle.
(iii) Two chords PQ and PR of a circle are mutually perpendicular to each other. If the length of the radius of the circle is r cm., let us find the length of the chord QR.
(iv) AOB is a diameter of a circle. The point C lies on the circle. If $\angle OBC = 60^\circ$, let us find the value of $\angle OCA$.
(v) In the picture beside, O is centre of the circle and AB is the diameter. The length of chord CD is equal to the length of the radius of the circle. AC and BD produced meet at the point P, let us find the value of $\angle APB$.



8

RIGHT CIRCULAR CYLINDER



A fine wooden penstand has been placed on the table of the reading room of our house. It has become old for few years. Some part of it has broken, so one more penstand is required.

I and my elder sister both have decided that we shall make that type of penstand.

1 But what is the shape of this penstand?

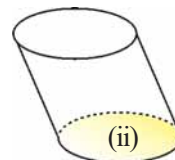
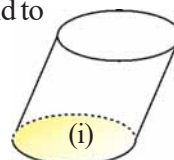
The shape of this penstand is like a cylinder.

In the **picture beside**, the penstand is like the right Circular Cylinder.

But in the pictures beside, no. (i) and (ii) cylinders are not right circular cylinder.

2 Which cylinder will we understand when only it is said to be a cylinder?

Here we shall understand right circular cylinder when only it is said to be a cylinder.



I have a coloured rectangular paper.

3 How shall we get the minimum coloured paper that will be required for wrapping out side of the penstand excluding top and bottom.

We shall know it by the **measuring lateral surface** of the circular cylinder.

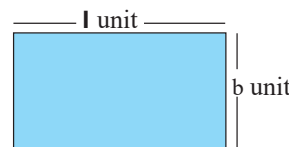
Let us calculate lateral surface area of a right circular cylinder by hands on trial.



- (1) We take a coloured rectangular paper.

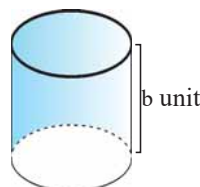
Let, length of the rectangular paper is l units and breadth is b units.

- (2) That penstand is covered by rectangular paper along breadth and fixed two end points by cello tape like the adjoining figure.



We see, length of the rectangular paper = l units = perimeter of the base of the right circular cylinder = $2\pi r$ units [where r unit = length of the radius of the base of the right circular cylinder]

Again, breadth of the rectangular paper = b units = height of the cylinder [let h units]



$\therefore$ Area of that rectangular paper = length $\times$ breadth = $l \times b$

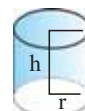
$$= 2\pi r \times h \text{ sq units}$$

$$= 2\pi rh \text{ sq units}$$

Radius of the right circular

cylinder means radius of the base.

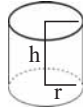
$\therefore$ We get by hands on trial, the lateral surface area of the right circular cylinder is $2\pi rh$ sq units. [Where r = length of radius of base of right circular cylinder, h = height of the right circular cylinder]



We understand, if the radius of the base of the penstand is r units and height of the penstand is h units, then minimum $2\pi rh$ sq units coloured paper will be required for covering the penstand.

4 But what will be the total surface area of a right circular cylinder?

In a right circular cylinder we have a curved surface or lateral surface and two circular plane surfaces with same radius. Lateral surface area of a right circular cylinder $= 2\pi rh$ [where r = length of the radius of the base of the right circular and h = height of the cylinder]



$$= 2\pi r \times h$$

= Perimeter of the circular plane surface of the cylinder $\times$ height.

$\therefore$ Total surface area of a right circular cylinder
 = Area of lateral surface + area of two circular plane surfaces.
 = $2\pi rh + 2\pi r^2$ [where r = length of radius of the base of the cylinder, h = height of cylinder]
 = $2\pi r (h+r)$

But we wrapped penstand by paper, which is in the shape of right circular cylinder and its top surface is open.

$\therefore$ Total surface area of that penstand is $= 2\pi rh + \pi r^2$
 [r = length of the radius of the circular base of the cylinder and h = height of the cylinder]

Application : 1. If the length of radius of the base of a right circular cylinder is 12 cm. and height is 21 cm. Let us write by calculating, its lateral surface area.

The length of radius of the cylinder is $\frac{12}{2}$ cm. = 6 cm.

$\therefore$ The lateral surface area of cylinder is $= 2 \times \pi \times 6 \times 21$ sq cm.
 $= 2 \times \frac{22}{7} \times 6 \times 21$ sq cm. = sq cm.



Application : 2. The perimeter of a right circular cylinder is 44 meter and height is 14 meter, let us write by calculating its lateral surface area. [Let me do it myself]

Application : 3. The base area of a closed cylindrical water tank is 616 sq meter and height is 21 meter. Let us write by calculating the total surface area of that tank.

Let the length of radius of circular base of water tank = r meter

$\therefore$ Base area $= \pi r^2$ sq meter

By the condition $\pi r^2 = 616$

$$\text{or, } \frac{22}{7} \times r^2 = 616$$

$$\text{or, } r^2 = 616 \times \frac{7}{22} \quad \therefore r = \text{ }$$

$\therefore$ Total surface area of water tank is $= (2\pi r^2 + 2\pi rh)$ sq meter [where height of cylinder = h meter]

$$= 2 \times \frac{22}{7} \times r (r + h) \text{ sq meter}$$

$$= 2 \times \frac{22}{7} \times 14 (14 + 21) \text{ sq meter}$$

$$= \text{ } \text{ sq meter}$$



Application : 4. If perimeter of the base of any closed cylindrical pot is 22 dm. and height is 5 dm. Let us write by calculating the area that will be coloured to paint the out side of that pot .

[Let me do it myself]

Application : 5. (i) The total surface area of a right circular cylinder with one end open is 1474 sq cm. If the length of diameter of the base is 14 cm. Let us write by calculating, its height.



(ii) Again if the pot would be closed at two ends, let us write by calculating, what would be its total surface area.

(i) The length of radius of base of right circular cylindrical pot = $\frac{14}{2}$ cm. = 7 cm.

Let the height of the pot is h cm.

$$\begin{aligned}\therefore \text{Total surface area of pot} &= \text{area of the base} + \text{area of the lateral surface} \\ &= \left(\frac{22}{7} \times 7^2 + 2 \times \frac{22}{7} \times 7 \times h \right) \text{ sq cm.} \\ &= (154 + 44h) \text{ sq cm.}\end{aligned}$$

By the condition, $154 + 44h = 1474$

$\therefore h = \boxed{}$ [Let us write]

$\therefore$ The height of the pot is 30 cm.

If the pot would be closed at two ends, surface area of that pot

$$\begin{aligned}&= \text{Area of the upper end surface} + 1474 \text{ sq cm.} \\ &= \left(\frac{22}{7} \times 7^2 + 1474 \right) \text{ sq cm.} = \boxed{} \text{ sq cm.}\end{aligned}$$



Application : 6. The length of the diameter of the base of a drum with lid made of steel sheet is 4.2 dcm. If 112.20 sq dcm. of steel sheet is required to make the drum, let us write by calculating the height of the drum.

Again if the price of 1 sq dcm. steel is ₹ 25, let us calculate the production cost of the drum.

[Let me do it myself]

My brother and sister gathered right circular cylindrical solids, used by them, in the corner of the room. There, they placed their cylindrical drinking water glasses also.

We see that the diameters of bases and heights of two glasses are different.



5 But how we shall understand that the glass which will contain maximum water?

we shall understand it by measuring the volume of the cylindrical glass.

But how we shall get the volume of a right circular cylinder?

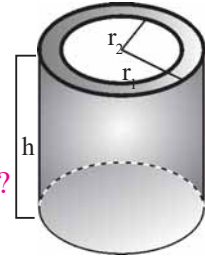
$$\begin{aligned}\text{Volume of a right circular cylinder} &= \text{Area of base} \times \text{height} \\ &= \pi r^2 \times h \\ &= \pi r^2 h\end{aligned}$$



Application : 7. If length of diameter of the base of glass is 11.2 cm. and height is 15 cm., let us calculate the volume of water that the glass will contain.

We understand, if the length of diameter of base of glass is 11.2 cm. and height is 15 cm., the glass will contain water

$$\begin{aligned}
 &= \pi \times \left(\frac{11.2}{2}\right)^2 \times 15 \text{ cubic cm.} \\
 &= \frac{22}{7} \times \frac{56}{10} \times \frac{56}{10} \times 15 \text{ cubic cm.} \\
 &= \boxed{} \text{ cubic cm.}
 \end{aligned}$$



6 But how we shall get, how much metal will be in a hollow cylindrical pipe?

The length of outer radius of a hollow pipe is r_1 units,

inner radius is r_2 units and height h units.

$$\begin{aligned}
 \text{Volume of hollow pipe} &= (\pi r_1^2 h - \pi r_2^2 h) \text{ cubic units} \\
 &= \pi (r_1^2 - r_2^2) h \text{ cubic units}
 \end{aligned}$$

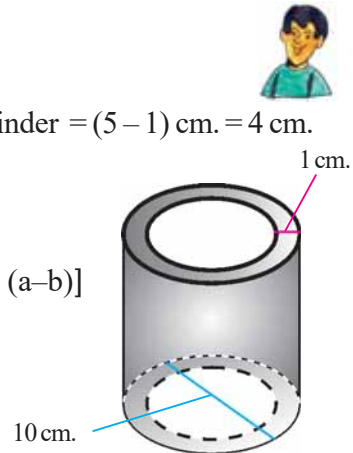
Application : 8. Height of a right circular cylinder made of iron open at two ends is 42 cm. If the thickness of the cylinder is 1 cm. and length of its external diameter is 10 cm., let us calculate the volume of iron in it ?

The length of external radius of cylinder = $\frac{10}{2}$ cm. = 5 cm.

It is of 1 cm. thickness. $\therefore$ length of internal radius of the cylinder = $(5 - 1)$ cm. = 4 cm.

volume of iron

$$\begin{aligned}
 &= \frac{22}{7} (5^2 - 4^2) \times 42 \text{ cubic cm.} \\
 &= \frac{22}{7} \times (5+4) (5-4) \times 42 \text{ cubic cm.} [\because a^2 - b^2 = (a+b)(a-b)] \\
 &= \frac{22}{7} \times 9 \times 1 \times 42 \text{ cubic cm.} \\
 &= 22 \times 9 \times 6 \text{ cubic cm.} \\
 &= \boxed{} \text{ cubic cm.}
 \end{aligned}$$



Application : 9. If we coloured inside and outside of the hollow right circular cylinder (Application 8) open at two ends, let us write by calculating how much area we shall colour?

The sum of inner and outer surface area of this hollow right circular cylinder open at two ends

$$\begin{aligned}
 &= (2 \times \pi \times 5 \times 21 + 2 \times \pi \times 4 \times 21) \text{ sq cm.} \\
 &= 2 \times \pi \times 21 (5+4) \text{ sq cm.} \\
 &= 2 \times \frac{22}{7} \times 21^3 \times 9 \text{ sq cm.} \\
 &= \boxed{} \text{ sq cm.}
 \end{aligned}$$



If length of outer radius and inner radius of a hollow right circular cylinder are r_1 units and r_2 units respectively and height is h units, sum of outer and inner curved surface area = $2\pi(r_1 + r_2)h$ sq units

7 But what will be the total surface area of this hollow right circular cylinder open at two ends?

The total surface area of hollow right circular cylinder open at the two ends = $2\pi(r_1 + r_2)h + 2\pi(r_1^2 - r_2^2)$ sq units. [where lengths of radius of outer and inner surface are r_1 and r_2 units, the height is h units]

Application : 10. The lengths of outer and inner radii of hollow right circular pipe are 5 cm. and 4 cm. respectively. If the total surface area of the pipe is 1188 sq cm., let us calculate its length.

The total surface area of cylinder = $[2\pi(r_1 + r_2)h + 2\pi(r_1^2 - r_2^2)]$ sq cm.

where, length of outer radius = r_1 cm. length of inner radius = r_2 cm. and height = h cm.

By the condition,

$$2\pi(r_1 + r_2)h + 2\pi(r_1^2 - r_2^2) = 1188$$

$$\text{or, } \pi[(5 + 4)h + (5^2 - 4^2)] = 594$$

$$\text{or, } \frac{22}{7}[9h + 9] = 594$$

$$\text{or, } 9h + 9 = 594 \times \frac{7}{22}$$

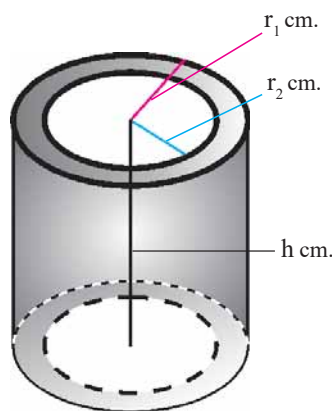
$$\text{or, } 9h + 9 = 189$$

$$\text{or, } 9h = 189 - 9$$

$$\text{or, } 9h = 180$$

$$\text{or, } h = \frac{180}{9}$$

$$\therefore h = 20$$



$\therefore$ Length of the cylinder is 20 cm.

Application : 11. If the inner and outer radii of a hollow right circular cylindrical pipe of 6 metre length, are 3.5 cm. and 4.2 cm. respectively let us write by calculating the volume of the iron that the pipe contains.

If 1 cubic decimetre of iron weighs 5 kilogram, let us write by calculating, the weight of the pipe.

[Let me do it myself]

Application : 12. If the area of base of cylinder is 13.86 sq metre and height is 8 metre, let us calculate the volume of the cylinder.

Let the length of radius of the base of the cylinder is r metre.

By the condition, $\pi r^2 = 13.86$

$$\text{or, } \frac{22}{7}r^2 = \frac{1386}{100}$$

$$\text{or, } r^2 = \frac{1386}{100} \times \frac{7}{22} = \boxed{}$$

$$\therefore r = \boxed{}$$

$\therefore$ The volume of the cylinder is $\frac{22}{7} \times \frac{21}{10} \times \frac{21}{10} \times 8$ cubic metre = $\boxed{}$ cubic metre.



Application : 13. If the perimeter of the base of a cylinder is 15.4 cm. and height is 10 cm., let us calculate its volume. **[Let me do it myself]**

Application : 14. If the glass of a tubelight is 105 cm. long and external circumference is 11 cm. and it is 0.2 cm. thick, let us write by calculating the volume (in cc) of the glass that will be required to make 5 such tube light.

Let the length of external radius of tubelight = r_1 cm. length of internal radius = r_2 cm. and height is h cm.

By the condition,

$$\text{External circumference is } = 2\pi r_1 = 11 \quad \text{or, } r_1 = \frac{11 \times 7}{2 \times 22} = \frac{7}{4}$$

$\therefore$ The length of external radius is = $\frac{7}{4}$ cm. = 1.75 cm.

The glass of tubelight is 0.2 cm. thick

$\therefore$ Length of internal radius of tubelight = $r_2 = (1.75 - 0.2)$ cm. = 1.55 cm.

External volume of the tube light - internal volume of tube light

$$\begin{aligned} &= \pi (r_1^2 - r_2^2) h \\ &= \frac{22}{7} \{ (1.75)^2 - (1.55)^2 \} \times 105 \text{ cubic cm.} \\ &= \frac{22}{7} \times (1.75 + 1.55) (1.75 - 1.55) \times 105 \text{ cubic cm.} \\ &= \frac{22}{7} \times 3.30 \times 0.2 \times 105 \text{ cubic cm.} \\ &= \boxed{} \text{ cubic cm.} \end{aligned}$$

$\therefore$ The glass will be required to make 5 tubelight is 5×217.8 cubic cm. = $\boxed{}$ cubic cm.

Application : 15. Through a hole, 110 kiloliter water enters into a ship. After closing the hole a pump is connected for the drainage of water. If the length of diameter of the pipe of the pump is 10 cm. and the speed of water flow is 350 metre/minute, let us write by calculating, the time that will be required by the pump to clear off all water in the ship.

volume of water can be cleared off by the pump in 1 minute

$$= \frac{22}{7} \times \frac{10}{2} \times \frac{10}{2} \times \frac{1}{100} \times 3500 \text{ cubic dcm.}$$

$$= 2750 \text{ cubic dcm.} = 2750 \text{ litres. [1 cubic dcm. = 1 litres.]}$$

$$\therefore \text{ Time required to clear off 110 kilolitres of water} = \frac{110000}{2750} \text{ minutes} = \boxed{} \text{ minutes}$$

So, time required by the pump to clear off all water in the ship = $\boxed{}$ minutes

Application : 16. A right circular cylindrical tank of 5 metre height is filled with water. Water comes out from there through a pipe having the length of diameter 8 cm. at a speed of 225 metre / minute and the tank becomes empty after 45 minutes. Let us write by calculating, the length of diameter of the tank.

[Let me do it myself]



Application : 17. The length, breadth and height of a parallelopiped made by copper are 11 cm. 9 cm. and 6 cm. respectively, let us write by calculating, the number of coins having the length of diameter 3 cm. and $\frac{1}{4}$ cm. thickness will be obtained by melting the parallelopiped.



Volume of parallelopiped = $11 \times 9 \times 6$ cubic cm.

The coins are right cylindrical,

So, length of radius of cylinder is $\frac{3}{2}$ cm. and height $\frac{1}{4}$ cm.

volume of each coin = $\frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{1}{4}$ cubic cm.

$$\text{Number of coins} = \frac{11 \times 9 \times 6}{\frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{1}{4}} = \frac{11 \times 9 \times 6 \times 7 \times 2 \times 4}{22 \times 3 \times 3} = 336$$

Let us work out

8



- Let us look at the picture of solid beside and answer the followings.
 - Solid has surfaces.
 - Number of curved surface is and number of plane surface is .
- Let us write the name of five solid objects of my house, the shapes of which are right circular cylinder?
- The length of diameter of a drum made of steel covered with lid is 28 cm. If 2816 sq cm. steel sheet is required to make the drum, let us write by calculating the height of the drum.
- Let us write by calculating how many cubic decimetre of concrete materials will be necessary to construct two cylindrical pillars each of whose diameter is 5.6 decimetre and height is 2.5 metre.
Let us write by calculating the cost of plastering the two pillars at ` 125 per square metre.
- If a gas cylinder for fuel purpose having the length of 7.5 dcm and the length of the inner diameter of 2.8 dcm carries 15.015 kg of gas, let us write by calculating the weight of the gas of per cubic dcm.
- Out of three jars of equal diameter and height, $\frac{2}{3}$ part of the first, $\frac{5}{6}$ part of the second and $\frac{7}{9}$ part of the third were filled with dilute Sulphuric acid. Whole of acid in the three jars were poured into a jar of 2.1 dcm. diameter, as a result the height of acid in the jar becomes 4.1 dcm. If the length of diameter of each of the three equal jars is 1.4 dcm, let us write by calculating the heights of the three jars.
- Total surface area of a right circular pot open at one end is 2002 sq cm. If the length of diameter of base of the pot is 14 cm. let us write by calculating. how much liter of water may the drum contain.
- If a pump set with a pipe of 14 cm. diameter can drain 2500 metre water per minute. Let us write by calculating, how much kiloliter water will that pump drain per hour. [1liter = 1 cubic dcm].

9. There are some water in a long gas jar of 7 cm. diameter. If a solid right circular cylindrical pipe of iron having 5 cm. length and 5.6 cm. diameter be immersed completely in that water, let us write by calculating how much the level of water will rise
10. If the surface area of a right circular cylindrical pillar is 264 sqmetre and volume is 924 cubic metre, let us write by calculating height and length of diameter of this pillar.
11. A right circular cylindrical tank of 9 metre height is filled with water. Water comes out from there through a pipe having length of 6 cm. diameter with a speed of 225 metre per minute and the tank becomes empty after 36 minutes, let us write by calculating the length of diameter of the tank.
12. Curved surface area of a right circular cylindrical log of wood of uniform density is 440 sq dcm. If 1 cubic dcm of wood weighs 1.5 kg and weight of the log is 9.24 quintals. Let us write by calculating the length of diameter of log and its height.
13. The length of inner and outer diameter of a right circular cylindrical pipe open at two ends are 30 cm. and 26 cm. respectively and length of pipe is 14.7 metre. Let us write by calculating the cost of painting its all surfaces with coaltar at ` 2.25 per square dcm.
14. Height of a hollow right circular cylinder, open at both ends, is 2.8 metre. If length of inner diameter of the cylinder is 4.6 dcm and the cylinder is made up of 84.48 cubic dcm of iron, let us calculate the length of outer diameter of the cylinder.
15. Height of a right circular cylinder is twice of its radius. If the height would be 6 times of its radius, then the volume of the cylinder would be greater by 539 cubic dcm, let us write by calculating the height of the cylinder.
16. A group of firebrigade personnel carried a right circular cylindrical tank filled with water and pumped out water at a speed of 420 metre per minute to put out the fire in 40 minutes by three pipes of 2cm. diameter each. If the diameter of tank is 2.8 metre and its length is 6 metre, then let us calculate (i) what volume of water has been spent in putting out the fire and (ii) the volume of water that still remains in the tank.
17. It is required to make a plastering of sand and cement with 3.5 cm. thick, surrounding four cylindrical pillars, each of whose diameter is 17.5 cm.
 - (i) If each pillar is of 3 metre height, let us write by calculating how many cubic dcm of plaster materials will be needed.
 - (ii) If the ratio of sand and cement in the plaster material be 4:1, let us write how many cubic dcm of cement will be needed.
18. The length of outer and inner diameter of hollow right circular cylinder are 16 cm. and 12 cm. respectively. Height of cylinder is 36 cm. Let us calculate how many solid cylinders of 2 cm. diameter and 6 cm. length may be obtained by melting this cylinder.
19. **Very short answer type question. (V.S.A.)**
 (A) (M.C.Q.) :
 - (i) If the length of radii of two solid right circular cylinder are in the ratio 2:3 and their heights are in the ratio 5:3, then ratio of the area of their lateral surfaces is
 (a) 2:5 (b) 8:7 (c) 10:9 (d) 16:9

- (ii) If the length of radii of two solid right circular cylinder are in the ratio 2:3 and their height are in the ratio 5:3, then the ratio of their volumes is
(a) 27:20 (b) 20:27 (c) 4:9 (d) 9:4
- (iii) If volume of two solid right circular cylinder are same and their height are in the ratio 1:2, then the ratio of lengths of radii is
(a) $1:\sqrt{2}$ (b) $\sqrt{2}:1$ (c) 1:2 (d) 2:1
- (iv) In a right circular cylinder, if the length of radius is halved and height is doubled, volume of cylinder will be
(a) equal (b) double (c) half (d) 4 times
- (v) If the length of radius of a right circular cylinder is doubled and height is halved, the lateral surface area will be.
(a) equal (b) double (c) half (d) 4 times

(B) Let us write true or false for the following statements:

- (i) The length of radius of right circular drum is r cm. and height is h cm. If half part of the drum is filled with water then the volume of water will be $\pi r^2 h$ cubic cm.
- (ii) If the length of radius of a right circular cylinder is 2 unit, the numerical value of volume and surface area of cylinder will be equal for any height.

(C) Let us fill in the blanks.

- (i) The length of a rectangular paper is l units and breadth is b units. The rectangular paper is rolled and a cylinder is formed of which perimeter is equal to the length of the paper. The lateral surface area of the cylinder is _____ sq unit.
- (ii) The longest rod that can be kept in a right circular cylinder having the diameter of 3 cm and height of 4 cm, then the length of rod is _____ cm.
- (iii) If the numerical values of volume and lateral surface area of a right circular cylinder are equal then the length of diameter of cylinder is _____ unit.

20. **Short answer type question (S.A.):**

- (i) If the lateral surface area of a right circular cylindrical pillar is 264 sq metre and volume is 924 cubic metre. Let us write the length of radius of the base of the pillar.
- (ii) If the lateral surface area of a right circular cylinder is c square unit, length of radius of base is r unit and volume is v cubic unit, let us write the value of $\frac{cr}{v}$.
- (iii) If the height of a right circular cylinder is 14 cm and lateral surface area is 264 sq cm, let us write the volume of the cylinder.
- (iv) If the heights of two right circular cylinder are in the ratio of 1:2 and perimeters are in the ratio of 3:4, let us write the ratio of their volumes.
- (v) The length of radius of a right circular cylinder is decreased by 50% and height is increased by 50%, let us write how much percent of the volume will be changed.

9

QUADRATIC SURD

Grandfather of Reba has presented a white board to Reba. We draw picture on that board and use this board for playing funny game. Today we took this board to the garden of Motiour, for playing a funny game.

Our friend Tapen draws a house on that board and writes some positive and negative integers in that house.



We have decided that each of us will write any two numbers from that house of integers and arrange them in different manners and we will know their nature.

Seema wrote 5 and 4

$5+4$, $5-4$, 5×4 each is integer.

again $\frac{5}{4}$ and $\frac{4}{5}$, each is [integer/rational number]



1 By squaring 5 and 4, let us see what I get.

$$5^2 = \text{input type="text"} \text{ and } 4^2 = \text{input type="text"}$$

∴ We see that squares of 5 and 4 are number.

We know that the square roots of any non negative real number a are $\pm\sqrt{a}$ or $\pm a^{1/2}$, because $(+\sqrt{a})^2 = (a^{1/2})^2 = a^1 = a$ and $(-\sqrt{a})^2 = a$

2 But what will be the value of $\sqrt{0}$?

According to definition $\sqrt{0} = 0$

3 By taking square root of 4 and 5, Let us see that I get.

Square roots of 4 are $\pm\sqrt{4}$ i.e. +2 and -2 [∵ $(+2)^2=4$ and $(-2)^2=4$]

Positive square root of 4 can be written as $\sqrt{4}$

i.e. In language of Mathematics $\sqrt{4} = 2$, ∴ square root of 4 is (rational/irrational)

4 But $\sqrt{4} = \sqrt{(-2)^2} = -2$, is it possible?

[Let me do it myself]

$$\sqrt{a^2} = |a| \text{ [according to definition]}$$

$$\sqrt{4} = \sqrt{2^2} = |2| = 2 \text{ and } \sqrt{4} = \sqrt{(-2)^2} = |-2| = 2$$

Square roots of 5 are $\pm\sqrt{5}$ [∵ We shall not get any integer whose square will be 5]

We understand that, 5 is not a square number but 4 is a square number.

5 What $\pm\sqrt{5}$ type of number is called?

If a is such a positive rational number which is not the square of rational number, then $\pm\sqrt{a}$ type of numbers is called a **pure quadratic surds**. Again a number of the form $a \pm \sqrt{b}$ is called a **mixed quadratic surds**, where a is rational number (non-zero) and $\sqrt{b}$ is a pure quadratic surd.



We know that $5^2 = 25$ so $\sqrt{25} = 25^{1/2} = (5^2)^{1/2} = 5^{2 \times 1/2} = 5^1 = 5$ [5 is square root of 25]

$5^3 = 125$ so $\sqrt[3]{125} = 125^{1/3} = (5^3)^{1/3} = 5^{3 \times 1/3} = 5^1 = 5$ [5 is cube root of 125]

$5^4 = 625$ so $\sqrt[4]{625} = 625^{1/4} = (5^4)^{1/4} = 5^{4 \times 1/4} = 5^1 = 5$ [5 is fourth root of 625]

Usually, to denote square root, we use $\sqrt{\quad}$ instead of $2\sqrt{\quad}$. The roots we mentioned above are all rational numbers. But it may not happen always.

Such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt[3]{7}$, $\sqrt[4]{20}$, $\sqrt[5]{25}$, i.e. they are all irrational number.

$\therefore \pm\sqrt{5}$ is purely quadratic surd.

6 I write by understanding 4 pure quadratic surds and 4 mixed quadratic surds.



4 pure quadratic surds are $\sqrt{3}$, $-\sqrt{5}$, $\sqrt{\frac{2}{3}}$, $\square$ [Let me write it myself]

4 mixed quadratic surds are $2-\sqrt{3}$, $2+\sqrt{6}$, $\frac{3}{5}-\sqrt{10}$, $\square$ [Let me write it myself]

But are all quadratic surds irrational?

The value of quadratic surds can not be evaluated completely in decimal. So these are irrational numbers. But all irrational numbers are not surds. i.e. $\sqrt{\pi}$ is irrational number but it is not quadratic surds.

7 Are $\sqrt{4}$, $\sqrt{25}$ quadratic surds?

Apparently $\sqrt{4}$, $\sqrt{25}$ are in the form of surds but they are not surds.

Rational number, $\sqrt{4} = 2$ and $\sqrt{25} = 5$

I apply Sridhar Acharyya's formula for solving the equation $x^2 - 2ax + (a^2 - b^2) = 0$ sec that the roots are $a + \sqrt{b}$ and $\square$ both of which are mixed surds, where b is a positive rational number which is not a square number of any rational number [Let me do it myself]

Our discussion will be restricted within the quadratic surds, where by the word 'surd' we shall generally mean to quadratic surd.

Monidipa wrote 8 and 12 on the board.

8 What type of number we get by the addition, subtraction, multiplication, division and square of these two numbers 8 and 12. [Let me do it myself]



Since $\sqrt{a} = a^{1/2}$, hence we get from the laws of indices.

$\sqrt{ab} = (ab)^{1/2} = a^{1/2} \times b^{1/2} = \sqrt{a} \times \sqrt{b}$, where a and b are non negative real number.

$\sqrt{\frac{a}{b}} = \left(\frac{a}{b}\right)^{1/2} = \frac{a^{1/2}}{b^{1/2}} = \frac{\sqrt{a}}{\sqrt{b}}$, where a is non negative real number and b is positive real number.

9 I take square root of 8, 32 and 9 and write what I shall get,

$\sqrt{8}$ is a pure quadratic surd as 8 is not a square rational number. Again $\sqrt{32}$ is a pure quadratic surd as 32 is not a square rational number.

We can write $\sqrt{8}$ as $\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \cdot \sqrt{2} = 2\sqrt{2}$

And we can write $\sqrt{32}$ as $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

10 We see that two quadratic surds $\sqrt{8}$ and $\sqrt{32}$ are rational multiple of the same surd $\sqrt{2}$. What is called by this type of surds? Two or more quadratic surds are said to be **Similar Surds** if they can be expressed as rational multiple of the same surd.

We understand $\sqrt{8}$ and $\sqrt{32}$ are similar surds.



Application : 1. Are $\sqrt{8}$ and $\sqrt{\frac{25}{2}}$ similar quadratic surds? Let us see by calculating.

$$\sqrt{8} = 2\sqrt{2} \text{ and } \sqrt{\frac{25}{2}} = \sqrt{\frac{25 \times 2}{4}} = \frac{\sqrt{25 \times 2}}{\sqrt{4}} = \frac{\sqrt{25} \times \sqrt{2}}{\sqrt{4}} = \frac{5}{2}\sqrt{2} \therefore \sqrt{8} \text{ and } \sqrt{\frac{25}{2}} \text{ are similar surds.}$$

Application : 2. Let us calculate whether $\sqrt{12}$ and $\sqrt{28}$ are similar surds.

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3} \text{ and } \sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}$$

Two pure quadratic surds $\sqrt{12}$ and $\sqrt{28}$ are not rational multiples of same surds.

What is called this type of pure quadratic surd?

The pure quadratic surd which are not similar surd, are **dissimilar surds**.

We understand that $\sqrt{12}$ and $\sqrt{28}$ are **dissimilar surds**.

i.e., If m and n are such numbers prime to each other [i.e. H.C.F of m and n is 1] and neither of which is perfect square, then $\sqrt{m}$ and $\sqrt{n}$ are dissimilar surds.

Example ; 15 and 22 are prime to each other as H.C.F of 15 and 22 is 1 and 15, 22 are not square numbers. So $\sqrt{15}$ and $\sqrt{22}$ are dissimilar surds.

11 Let us see how we shall get which number is greater between dissimilar surds $\sqrt{5}$ and $\sqrt{7}$?

Since $7 > 5 \therefore \sqrt{7} > \sqrt{5}$ ($\because$ a, b are positive number and if $a^2 > b^2$ then $a > b$)

Application : 3. We write similar surds in a specific place from the following quadratic surds.

$$\sqrt{45}, \sqrt{80}, \sqrt{147}, \sqrt{180} \text{ and } \sqrt{500}$$

$$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

$$\sqrt{147} = \sqrt{7 \times 7 \times 3} = 7\sqrt{3}$$

$$\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$$

$$\sqrt{180} = \sqrt{6 \times 6 \times 5} = 6\sqrt{5}$$

$$\sqrt{500} = \boxed{} \text{ [Let me do it myself]}$$

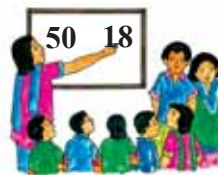
$\therefore$ The similar surds are $\sqrt{45}, \sqrt{80}, \sqrt{180}$ and $\sqrt{500}$



Application : 4. Let us write the similar surds among the quadratic surds $\sqrt{48}, \sqrt{27}, \sqrt{20}$ and $\sqrt{75}$ [Let me do it myself]

Reba has written 50 and 18 on the board.

$\sqrt{50}$ and $\sqrt{18}$ are pure quadratic surds.



Application : 5. Let us see whether $(\sqrt{50} + \sqrt{18})$ and $(\sqrt{50} - \sqrt{18})$ can be expressed in pure quadratic surds.

$$\sqrt{50} = 5\sqrt{2} \text{ and } \sqrt{18} = 3\sqrt{2}$$

$\therefore$ We see that $\sqrt{50}$ and $\sqrt{18}$ are similar surds

Since $5x + 3x = 8x$ and $5x - 3x = \square$

$$\therefore \sqrt{50} + \sqrt{18} = 5\sqrt{2} + 3\sqrt{2} = 8\sqrt{2}$$

$$\sqrt{50} - \sqrt{18} = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$$

$\therefore$ We see that $(\sqrt{50} + \sqrt{18})$ and $(\sqrt{50} - \sqrt{18})$ can be expressed in pure quadratic surds.



Application : 6. Let us write by calculating the value of $(\sqrt{2} + \sqrt{8})$ and $(\sqrt{2} - \sqrt{8})$ and see whether they can be expressed in pure quadratic surds. [Let me do it myself]

Mrinal wrote 12 and 45 on the board.

Application : 7. Let us write by calculating the value of $(\sqrt{12} + \sqrt{45})$ and $(\sqrt{12} - \sqrt{45})$

$$\sqrt{12} = 2\sqrt{3} \text{ and } \sqrt{45} = 3\sqrt{5}$$

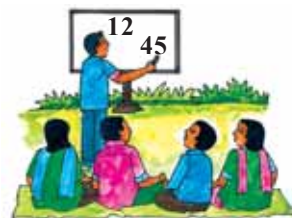
$\therefore \sqrt{12}$ and $\sqrt{45}$ are dissimilar surds

Since, sum of $2x$ and $3y = 2x + 3y$

$$\therefore \sqrt{12} + \sqrt{45} = 2\sqrt{3} + 3\sqrt{5}$$

$$\text{similarly } \sqrt{12} - \sqrt{45} = 2\sqrt{3} - 3\sqrt{5}$$

$\therefore$ Again we see, $(\sqrt{12} + \sqrt{45})$ and $(\sqrt{12} - \sqrt{45})$ are irrational numbers but they can not be written in the form of pure surds.



We understand, as sum of a and $b = a + b$, $\therefore \sqrt{5} + \sqrt{7} = \square$ [Let me do it myself]

Application : 8. Let us calculate the sum of the numbers $2\sqrt{3}$, $3\sqrt{2}$ and $4\sqrt{3}$

$$2\sqrt{3} + 3\sqrt{2} + 4\sqrt{3} = 6\sqrt{3} + 3\sqrt{2}$$

Application : 9. Let us write by calculating the sum of $\sqrt{12}$, $-4\sqrt{3}$ and $6\sqrt{3}$

[Let me do it myself]

Application : 10. Let us calculate, the sum of two mixed quadratic surds $(2 + \sqrt{3})$ and $(2 - 2\sqrt{3})$

Sum of 30 and $2x = 30 + 2x$ and $(30 + 2x) + (30 - 4x) = \square$

$$\therefore (2 + \sqrt{3}) + (2 - 2\sqrt{3}) = 2 + 2 + \sqrt{3} - 2\sqrt{3} = 4 - \sqrt{3}$$

$\therefore$ We get the sum of $(2 + \sqrt{3})$ and $(2 - 2\sqrt{3})$ is a mixed quadratic surd.



Application : 11. $(9 - 2\sqrt{5}) + (12 + 7\sqrt{5}) = \square$ [Let me do it myself]

Application : 12. Let us calculate the sum of $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$

$$(2 + \sqrt{3}) + (2 - \sqrt{3}) = 2 + 2 + \sqrt{3} - \sqrt{3} = 4$$

∴ We see that, we get the sum of two quadratic surds is a rational number.



Application : 13. I write any other two quadratic surds whose sum is a rational number.

[Let me do it myself]

Let us work out 9.1

- Let us write the following numbers in the form of product of rational and irrational numbers.
(i) $\sqrt{175}$ (ii) $2\sqrt{112}$ (iii) $\sqrt{108}$ (iv) $\sqrt{125}$ (v) $5\sqrt{119}$
- Let us prove that, $\sqrt{108} - \sqrt{75} = \sqrt{3}$
- Let us show that, $\sqrt{98} + \sqrt{8} - 2\sqrt{32} = \sqrt{2}$
- Let us show that, $3\sqrt{48} - 4\sqrt{75} + \sqrt{192} = 0$
- Let us simplify :
 $\sqrt{12} + \sqrt{18} + \sqrt{27} - \sqrt{32}$
- Let us write what should be added with $\sqrt{5} + \sqrt{3}$ to get the sum $2\sqrt{5}$.
 - Let us write what should be subtracted from $7 - \sqrt{3}$ to get $3 + \sqrt{3}$
 - Let us write the sum of $2 + \sqrt{3}$, $\sqrt{3} + \sqrt{5}$ and $2 + \sqrt{7}$.
 - Let us subtract $(-5 + 3\sqrt{11})$ from $(10 - \sqrt{11})$ and let us write the value of subtraction.
 - Let us subtract $(5 + \sqrt{2} + \sqrt{7})$ from the sum of $(-5 + \sqrt{7})$ and $(\sqrt{7} + \sqrt{2})$ and find value of subtraction.
 - I write two quadratic surds whose sum is a rational number.

12 Now our friend Amiya wrote 7 and 11 on the board.

We see that the numbers are prime numbers which is written on the board.

Let us write the sum and difference of two pure quadratic surds $\sqrt{7}$ and $\sqrt{11}$.

13 Let us write by calculating the product of two numbers $\sqrt{7}$ and $\sqrt{11}$.

Since $a^m \times b^m = (ab)^m$ [$a \neq 0$, $b \neq 0$, m is a rational number]

$$\begin{aligned} \therefore \sqrt{7} \times \sqrt{11} &= 7^{\frac{1}{2}} \times 11^{\frac{1}{2}} \\ &= (7 \times 11)^{\frac{1}{2}} \\ &= 77^{\frac{1}{2}} \\ &= \sqrt{77} \end{aligned}$$

Here $\sqrt{77}$ is a pure quadratic surd.



Application : 14. Let us determine the product of the following quadratic surds.

(i) $2\sqrt{5} \times 3\sqrt{2}$ (ii) $7\sqrt{3} \times 2\sqrt{3}$ (iii) $(2+\sqrt{3})(4+\sqrt{3})$ (iv) $(5-\sqrt{3})(2-\sqrt{3})$

(i) $2\sqrt{5} \times 3\sqrt{2} = 2 \times 3 \times \sqrt{5 \times 2} = 6\sqrt{10}$

(ii) $7\sqrt{3} \times 2\sqrt{3} = 7 \times 2 \times \sqrt{3} \times \sqrt{3}$

$$= 14\sqrt{3^2} = 14(3^2)^{\frac{1}{2}} = 14 \times 3^{(2 \times \frac{1}{2})}$$

$$= 14 \times 3 = 42 \left[\because (a^m)^n = a^{mn}, a \neq 0 \text{ and } m, n \text{ are rational numbers} \right]$$

(iii) $(2+\sqrt{3})(4+\sqrt{3}) = 8 + 4\sqrt{3} + 2\sqrt{3} + (\sqrt{3})^2$

$$= 8 + 6\sqrt{3} + 3 = 11 + 6\sqrt{3} \left[\because (x+y)(a+b) = ax+ay+bx+by \right]$$

(iv) $(5-\sqrt{3})(2-\sqrt{3}) = 5 \times 2 - 2\sqrt{3} - 5\sqrt{3} + (\sqrt{3})^2$

$$= 10 - 2\sqrt{3} - 5\sqrt{3} + 3 = 13 - 7\sqrt{3}$$

Application : 15. Let us calculate the product of $(2+\sqrt{3}+\sqrt{5})(3-\sqrt{5})$

$$(2+\sqrt{3}+\sqrt{5})(3-\sqrt{5}) = 2 \times (3-\sqrt{5}) + \sqrt{3} \times (3-\sqrt{5}) + \sqrt{5} \times (3-\sqrt{5})$$

$$= 6 - 2\sqrt{5} + 3\sqrt{3} - \sqrt{15} + 3\sqrt{5} - 5$$

$$= 6 - 5 + \sqrt{5} + 3\sqrt{3} - \sqrt{15}$$

$$= 1 + \sqrt{5} + 3\sqrt{3} - \sqrt{15}$$

Application : 16. Let us write by calculating the product of $(3+\sqrt{7}-\sqrt{5})(2\sqrt{2}-1)$

[Let me do it myself]

Nathura wrote 13 and 5 on the board.

$\sqrt{13}$ and $\sqrt{5}$ are pure quadratic surds.

Application : 17. Let us write the value of $\sqrt{13} \div \sqrt{5}$

$$\sqrt{13} \div \sqrt{5} = \frac{\sqrt{13}}{\sqrt{5}}$$

We see that surd is in denominator. But how shall we make the

denominator of $\frac{\sqrt{13}}{\sqrt{5}}$ free from surd?

Let us see what we get if we multiply numerator and denominator by $\sqrt{5}$

$$\frac{\sqrt{13}}{\sqrt{5}} = \frac{\sqrt{13} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{65}}{5} \text{ i.e.}$$

Rationalising denominator of $\frac{\sqrt{13}}{\sqrt{5}}$, we get $\frac{\sqrt{65}}{5}$



If $\frac{a}{b}$ is said to be algebraic fraction, then a is said to be numerator and b is said to be denominator i.e. for $\frac{\sqrt{13}}{\sqrt{5}}$ numerator is $\sqrt{13}$ and denominator is $\sqrt{5}$

But what is called this method when a surd is multiplied by a factor to make the product free from surd.

Any factor multiplying with any surd or forming irrational number by adding and subtracting more than one surds, if the product is free from surds i.e. the process of getting rational number is said to be **Rationalisation** and this factor is said to be **Rationalising Factor** of that surd or that irrational number.

Rationalising factor of $\sqrt{5}$ is $\sqrt{5}$; further $k\sqrt{5}$ where k is a non-zero irrational number.

∴ A rationalise factor of $\sqrt{a}$ is $\sqrt{a}$; further $k\sqrt{a}$, k is non zero rational number.

Application : 18. Let us write two rationalise factors of $\sqrt{7}$ [Let me do it myself]

Application : 19. Let us see what will be the rationalising factor of $(5+\sqrt{7})$

$$(5+\sqrt{7}) \times (5-\sqrt{7}) = (5)^2 - (\sqrt{7})^2 = 25-7 = \boxed{} \quad [(a+b)(a-b) = a^2-b^2]$$

$$\text{Again, } (5+\sqrt{7}) \times (-5+\sqrt{7}) = (\sqrt{7}+5) \times (\sqrt{7}-5) = (\sqrt{7})^2 - (5)^2 = \boxed{}$$

∴ We see that the rationalising factors of $5+\sqrt{7}$ are $(5-\sqrt{7})$ and $(-5+\sqrt{7})$

We understand that, the rationalising factor of $a+\sqrt{b}$ is $a-\sqrt{b}$ or $-a+\sqrt{b}$ or their rational multiple.

Application : 20. Let us write two rationalising factors of $7-\sqrt{3}$ [Let me do it myself]

Application : 21. Let us see the rationalising factors of $(\sqrt{11}-\sqrt{6})$

$$(\sqrt{11}-\sqrt{6})(\sqrt{11}+\sqrt{6}) = (\sqrt{11})^2 - (\sqrt{6})^2 = 11-6 = 5$$

$$\text{Again, } (\sqrt{11}-\sqrt{6})(-\sqrt{11}-\sqrt{6}) = -[(\sqrt{11}-\sqrt{6})(\sqrt{11}+\sqrt{6})] \\ = \boxed{} \quad [\text{Let me do it myself}]$$



The rationalising factor of irrational number $(\sqrt{a}-\sqrt{b})$ is $(\sqrt{a}+\sqrt{b})$ or $(-\sqrt{a}-\sqrt{b})$ or their any rational multiple.

Application : 22. Let us write two rationalising factors of $(\sqrt{15}+\sqrt{3})$ [Let me do it myself]

Application : 23. I write a rationalising factor of mixed surd $(7+\sqrt{2})$ with which adding $(7+\sqrt{2})$ we get rational number.

$$(7+\sqrt{2}) \times (7-\sqrt{2}) = 7^2 - (\sqrt{2})^2 = \boxed{}$$

$$\text{again, } (7+\sqrt{2}) + (7-\sqrt{2}) = 7+7 = 14$$

∴ We see, that the sum and product of mixed quadratic surd $7-\sqrt{2}$ with the factor $(7+\sqrt{2})$ is rational number.

But what is said this type of rationalising factor of any mixed quadratic surd?

If the sum and product of any mixed quadratic surd with rationalising factor are both rational number, the mixed quadratic surd is said to be **conjugate or complementary surd**.

We understand that conjugate surd of mixed quadratic surd $(7+\sqrt{2})$ is $7-\sqrt{2}$, but $(-7+\sqrt{2})$ is not conjugate surd, though, it is a rationalising factor of given surd.

Because $7+\sqrt{2}+7-\sqrt{2} = 14$ (rational number)

Again $(7+\sqrt{2})(7-\sqrt{2}) = (7)^2 - (\sqrt{2})^2 = 49-2=47$ (rational number) |

But $(7+\sqrt{2})+(-7+\sqrt{2}) = 7+\sqrt{2}-7+\sqrt{2} = 2\sqrt{2}$ (irrational number)



Application : 24. Let us write the conjugate surds of $(\sqrt{5}-1)$, $\sqrt{3}$, $(\sqrt{3}-2)$

The conjugate surd of $(\sqrt{5}-1)$ is $-\sqrt{5}-1$

Conjugate surd of $\sqrt{3}$ is $-\sqrt{3}$

Conjugate surd of $(\sqrt{3}-2)$ is $(-\sqrt{3}-2)$

∴ we get the conjugate surd of this type of surd $a+\sqrt{b}$ is $a-\sqrt{b}$

Conjugate surd of this type of surd $a-\sqrt{b}$ is $a+\sqrt{b}$



Application : 25. Let us write the conjugate surds of the following mixed and pure surds.

(i) $2+\sqrt{3}$ (ii) $5-\sqrt{2}$ (iii) $\sqrt{5}-7$ (iv) $\sqrt{11}+6$ (v) $\sqrt{5}$ [Let me do it myself]

Application : 26. Let us rationalise the denominator of $(2\sqrt{2} \div \sqrt{5})$

$$2\sqrt{2} \div \sqrt{5} = \frac{2\sqrt{2}}{\sqrt{5}} \text{ ————— (i)}$$

$$\frac{2\sqrt{2}}{\sqrt{5}} = \frac{2\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{2\sqrt{10}}{(\sqrt{5})^2} = \frac{2\sqrt{10}}{5}$$

∴ We get $2\sqrt{2} \div \sqrt{5} = 2\sqrt{10} \div 5$



Application : 27. Let us rationalise the denominator of (i) $6 \div \sqrt{7}$ (ii) $5\sqrt{2} \div 6\sqrt{3}$

$$(i) 6 \div \sqrt{7} =$$

$$(ii) 5\sqrt{2} \div 6\sqrt{3} = \frac{5\sqrt{2}}{6\sqrt{3}} = \frac{5\sqrt{2} \times \sqrt{3}}{6\sqrt{3} \times \sqrt{3}} = \frac{5\sqrt{6}}{6(\sqrt{3})^2} = \frac{5\sqrt{6}}{6 \times 3} = \frac{5\sqrt{6}}{18} = 5\sqrt{6} \div 18$$

Application : 28. Let us rationalise the denominator of (i) $\frac{4\sqrt{5}}{5\sqrt{3}}$ (ii) $\frac{3\sqrt{7}}{\sqrt{6}}$

[Let me do it myself]

Application : 29. Let us rationalise the denominator of

(i) $4 \div (3-\sqrt{2})$ (ii) $(\sqrt{5}+2) \div (\sqrt{3}-1)$ (iii) $(\sqrt{2}+\sqrt{3}) \div (\sqrt{2}-\sqrt{3})$

$$(i) 4 \div (3-\sqrt{2}) = \frac{4}{3-\sqrt{2}} = \frac{4(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{4(3+\sqrt{2})}{(3)^2 - (\sqrt{2})^2} = \frac{4(3+\sqrt{2})}{9-2} = \frac{(12+4\sqrt{2})}{7}$$

$$(ii) (\sqrt{5}+2) \div (\sqrt{3}-1) = \frac{\sqrt{5}+2}{\sqrt{3}-1} = \frac{(\sqrt{5}+2)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{\sqrt{15}+2\sqrt{3}+\sqrt{5}+2}{(\sqrt{3})^2 - (1)^2} \\ = \frac{\sqrt{15}+2\sqrt{3}+\sqrt{5}+2}{2}$$



$$\begin{aligned}
 \text{(iii)} \quad (\sqrt{2}+\sqrt{3}) \div (\sqrt{2}-\sqrt{3}) &= \frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} = \frac{(\sqrt{2}+\sqrt{3})(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})(\sqrt{2}+\sqrt{3})} = \frac{(\sqrt{2}+\sqrt{3})^2}{(\sqrt{2})^2 - (\sqrt{3})^2} \\
 &= \frac{2+3+2\sqrt{2} \times \sqrt{3}}{2-3} \\
 &= \frac{5+2\sqrt{6}}{-1}
 \end{aligned}$$



Application : 30 Let us rationalise the denominator of

(i) $(4+2\sqrt{3}) \div (2-\sqrt{3})$ (ii) $(\sqrt{5}+\sqrt{3}) \div (\sqrt{5}-\sqrt{3})$ [Let me do it myself]

Let us work out 9.2

- Let us find the product of $3^{\frac{1}{2}}$ and $\sqrt{3}$
 - Let us write what should be multiplied with $2\sqrt{2}$ to get the product 4.
 - Let us calculate the product of $3\sqrt{5}$ and $5\sqrt{3}$
 - If $\sqrt{6} \times \sqrt{15} = x\sqrt{10}$, then let us write by calculating the value of x.
 - If $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3}) = 25 - x^2$ be an equation, then let us write by calculating the value of x.
- Let us calculate the product :**

 - $\sqrt{7} \times \sqrt{14}$
 - $\sqrt{12} \times 2\sqrt{3}$
 - $\sqrt{5} \times \sqrt{15} \times \sqrt{3}$
 - $\sqrt{2}(3+\sqrt{5})$
 - $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$
 - $(2\sqrt{3}+3\sqrt{2})(4\sqrt{2}+\sqrt{5})$
 - $(\sqrt{3}+1)(\sqrt{3}-1)(2-\sqrt{3})(4+2\sqrt{3})$
- If $\sqrt{x}$ is the rationalising factor of $\sqrt{5}$, let us write by calculating what is the smallest value of x (where x is an integer)
 - Let us calculate the value of $3\sqrt{2} \div 3$
 - Let us write which factor should we multiply the denominator to rationalise the denominator of $7 \div \sqrt{48}$.
 - Let us calculate the rationalising factor of $(\sqrt{5}+2)$ which is also its conjugate surd.
 - If $(\sqrt{5}+\sqrt{2}) \div \sqrt{7} = \frac{1}{7}(\sqrt{35}+a)$, Let us calculate the value of a
 - Let us write a rationalising factor of the denominator of $\frac{5}{\sqrt{3}-2}$, which is not its conjugate surd.
- Let us write the conjugate surds of mixed quadratic surds $(9-4\sqrt{5})$ and $(-2-\sqrt{7})$
- Let us write two conjugate surds of each mixed quadratic surds of the followings.**

 - $\sqrt{5}+\sqrt{2}$
 - $13+\sqrt{6}$
 - $\sqrt{8}-3$
 - $\sqrt{17}-\sqrt{15}$

6. Let us rationalising the denominators of the following surds.

$$\begin{array}{lll} \text{(i)} \frac{2\sqrt{3}+3\sqrt{2}}{\sqrt{6}} & \text{(ii)} \frac{\sqrt{2}-1+\sqrt{6}}{\sqrt{5}} & \text{(iii)} \frac{\sqrt{3}+1}{\sqrt{3}-1} \\ \text{(iv)} \frac{3+\sqrt{5}}{\sqrt{7}-\sqrt{3}} & \text{(v)} \frac{3\sqrt{2}+1}{2\sqrt{5}-1} & \text{(vi)} \frac{3\sqrt{2}+2\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} \end{array}$$



7. Let us divide first by second and rationalise the divisor.

$$\text{(i)} 3\sqrt{2}+\sqrt{5}, \sqrt{2}+1 \quad \text{(ii)} 2\sqrt{3}-\sqrt{2}, \sqrt{2}-\sqrt{3} \quad \text{(iii)} 3+\sqrt{6}, \sqrt{3}+\sqrt{2}$$

8. Let us find the value of (i) $\frac{2\sqrt{5}+1}{\sqrt{5}+1} - \frac{4\sqrt{5}-1}{\sqrt{5}-1}$ (ii) $\frac{8+3\sqrt{2}}{3+\sqrt{5}} - \frac{8-3\sqrt{2}}{3-\sqrt{5}}$

Application : 31 Let us simplify : $\frac{3\sqrt{4.2}-2\sqrt{12}+\sqrt{20}}{3\sqrt{18}-2\sqrt{27}+\sqrt{45}}$

$$\begin{aligned} \frac{3\sqrt{4.2}-2\sqrt{12}+\sqrt{20}}{3\sqrt{18}-2\sqrt{27}+\sqrt{45}} &= \frac{3\sqrt{4.2}-2\sqrt{4.3}+\sqrt{4.5}}{3\sqrt{9.2}-2\sqrt{9.3}+\sqrt{9.5}} = \frac{3.2\sqrt{2}-2.2\sqrt{3}+2\sqrt{5}}{3.3\sqrt{2}-2.3\sqrt{3}+3\sqrt{5}} \\ &= \frac{6\sqrt{2}-4\sqrt{3}+2\sqrt{5}}{9\sqrt{2}-6\sqrt{3}+3\sqrt{5}} \\ &= \frac{2(3\sqrt{2}-2\sqrt{3}+\sqrt{5})}{3(3\sqrt{2}-2\sqrt{3}+\sqrt{5})} = \frac{2}{3} \end{aligned}$$

∴ Required result = $\frac{2}{3}$

Application : 32 Let us simplify : $\frac{3\sqrt{20}+2\sqrt{28}+\sqrt{12}}{5\sqrt{45}+2\sqrt{175}+\sqrt{75}}$ [Let me do it myself]

Application : 33 Let us simplify : $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} - \frac{3\sqrt{3}}{\sqrt{2}+\sqrt{5}} + \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{5}}$



$$\text{Given} = \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} - \frac{3\sqrt{3}}{\sqrt{2}+\sqrt{5}} + \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{5}} \quad \text{————— (i)}$$

$$\text{First part} = \frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{5}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{\sqrt{15}-\sqrt{10}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{\sqrt{15}-\sqrt{10}}{3-2} = \sqrt{15}-\sqrt{10}$$

$$\begin{aligned} \text{Second part} &= \frac{3\sqrt{3}}{\sqrt{2}+\sqrt{5}} = \frac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} = \frac{3\sqrt{3}(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{3\sqrt{15}-3\sqrt{6}}{5-2} = \frac{3(\sqrt{15}-\sqrt{6})}{3} \\ &= \sqrt{15}-\sqrt{6} \end{aligned}$$

$$\begin{aligned}\text{Third part} &= \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{5}} = \frac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}} = \frac{2\sqrt{2}(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = \frac{2\sqrt{10}-2\sqrt{6}}{(\sqrt{5})^2-(\sqrt{3})^2} = \frac{2(\sqrt{10}-\sqrt{6})}{5-3} \\ &= \frac{2(\sqrt{10}-\sqrt{6})}{2} \\ &= \sqrt{10}-\sqrt{6}\end{aligned}$$

∴ From (i) we get

$$\begin{aligned}\text{Given} &= (\sqrt{15}-\sqrt{10})-(\sqrt{15}-\sqrt{6})+(\sqrt{10}-\sqrt{6}) \\ &= \sqrt{15}-\sqrt{10}-\sqrt{15}+\sqrt{6}+\sqrt{10}-\sqrt{6} = 0\end{aligned}$$

∴ Required result = 0.



Application : 34 Let us simplify $\frac{5}{\sqrt{2}+\sqrt{3}} - \frac{1}{\sqrt{2}-\sqrt{3}}$ [Let me do it myself]

Application : 35 If $x = \sqrt{3} + \sqrt{2}$, let us calculate simplified values of the followings (i) $x - \frac{1}{x}$ (ii) $x^2 + \frac{1}{x^2}$ and (iii) $x^3 - \frac{1}{x^3}$

$$x = \sqrt{3} + \sqrt{2}$$

$$\therefore \frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2} = \frac{\sqrt{3}-\sqrt{2}}{3-2} = \sqrt{3}-\sqrt{2}$$

$$(i) \quad x - \frac{1}{x} = (\sqrt{3} + \sqrt{2}) - (\sqrt{3} - \sqrt{2}) = \sqrt{3} + \sqrt{2} - \sqrt{3} + \sqrt{2} = 2\sqrt{2}$$

$$(ii) \quad x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2 \times x \times \frac{1}{x} = (2\sqrt{2})^2 + 2 \cdot 1 = 8 + 2 = 10$$

$$(iii) \quad x^3 - \frac{1}{x^3} = \left(x - \frac{1}{x}\right)^3 + 3 \times x \times \frac{1}{x} \left(x - \frac{1}{x}\right) = (2\sqrt{2})^3 + 3 \times 1 \times 2\sqrt{2} = 16\sqrt{2} + 6\sqrt{2} = 22\sqrt{2}$$

Application : 36 If $x = \sqrt{3} + \sqrt{2}$ let us calculate simplified values of $\left(x - \frac{1}{x}\right)$, $\left(x^3 + \frac{1}{x^3}\right)$ and $\left(x^2 - \frac{1}{x^2}\right)$ [Let me do it myself]

Application : 37 If $x = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ and $y = \frac{\sqrt{3}-1}{\sqrt{3}+1}$,

(a) let us show that $\frac{x^2+y^2}{x^2-y^2} = \frac{7\sqrt{3}}{12}$ (b) let us calculate simplified value of $\frac{x^2-xy+y^2}{x^2+xy+y^2}$

(c) let us calculate simplified value of $\frac{x^2}{y} + \frac{y^2}{x}$ (d) let us calculate simplified value of $x^3 - y^3$

At first, let us calculate value of $x+y$, $x-y$ and xy

$$\begin{aligned}x+y &= \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3+1+2\sqrt{3}+3+1-2\sqrt{3}}{(\sqrt{3})^2-(1)^2} = \frac{8}{3-1} \\ &= \frac{8}{2} = 4\end{aligned}$$

$$\begin{aligned}\text{or, } x+y &= \frac{\sqrt{3}+1}{\sqrt{3}-1} + \frac{\sqrt{3}-1}{\sqrt{3}+1} \\ &= \frac{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{2[(\sqrt{3})^2 + (1)^2]}{(\sqrt{3})^2 - (1)^2} \quad [\because (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)] \\ &= \frac{2(3+1)}{3-1} = \frac{2 \times 4}{2} = 4\end{aligned}$$

$$\begin{aligned}x-y &= \frac{\sqrt{3}+1}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{(\sqrt{3}+1)^2 - (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{(3+1+2\sqrt{3}) - (3+1-2\sqrt{3})}{(\sqrt{3})^2 - (1)^2} \\ &= \frac{3+1+2\sqrt{3}-3-1+2\sqrt{3}}{3-1} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}\end{aligned}$$

$$\begin{aligned}\text{or, } x-y &= \frac{\sqrt{3}+1}{\sqrt{3}-1} - \frac{\sqrt{3}-1}{\sqrt{3}+1} \\ &= \frac{(\sqrt{3}+1)^2 - (\sqrt{3}-1)^2}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{4 \times \sqrt{3} \times 1}{(\sqrt{3})^2 - (1)^2} \quad [\because (a+b)^2 - (a-b)^2 = 4ab] \\ &= \frac{4\sqrt{3}}{3-1} = \frac{4\sqrt{3}}{2} = 2\sqrt{3}\end{aligned}$$

$$x \times y = \frac{(\sqrt{3}+1)}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}-1)}{(\sqrt{3}+1)} = 1$$

$$(a) \quad \frac{x^2+y^2}{x^2-y^2} = \frac{(x+y)^2 - 2xy}{(x+y)(x-y)} = \frac{(4)^2 - 2 \times 1}{4 \times 2\sqrt{3}} = \frac{14}{4 \times 2\sqrt{3}} = \frac{7 \times \sqrt{3}}{4\sqrt{3} \times \sqrt{3}} = \frac{7\sqrt{3}}{12}$$

$$(b) \quad \frac{x^2-xy+y^2}{x^2+xy+y^2} = \frac{(x+y)^2 - 3xy}{(x+y)^2 - xy} = \frac{4^2 - 3 \times 1}{4^2 - 1} = \frac{13}{15}$$

$$\text{or, } \frac{x^2-xy+y^2}{x^2+xy+y^2} = \frac{(x-y)^2 + xy}{(x-y)^2 + 3xy} = \frac{(2\sqrt{3})^2 + 1}{(2\sqrt{3})^2 + 3} = \frac{12+1}{12+3} = \frac{13}{15}$$

We understand that we have to know the value of any one of $(x+y)$ or $(x-y)$

$$(c) \quad \frac{x^2}{y} + \frac{y^2}{x} = \frac{x^3+y^3}{xy} = \frac{(x+y)^3 - 3xy(x+y)}{xy} = \frac{4^3 - 3 \times 4}{1} = 64 - 12 = 52$$

$$(d) \quad x^3-y^3 = (x-y)^3 + 3xy(x-y) = (2\sqrt{3})^3 + 3 \times 1 \times 2\sqrt{3} = 24\sqrt{3} + 6\sqrt{3} = 30\sqrt{3}$$



Application : 38 Let us write which number is greater between $(\sqrt{5}+\sqrt{3})$ and $(\sqrt{6}+\sqrt{2})$

$$(\sqrt{5}+\sqrt{3})^2 = (\sqrt{5})^2 + 2 \times \sqrt{5} \times \sqrt{3} + (\sqrt{3})^2 = 5 + 2\sqrt{15} + 3 = 8 + 2\sqrt{15}$$

$$(\sqrt{6}+\sqrt{2})^2 = (\sqrt{6})^2 + 2 \times \sqrt{6} \times \sqrt{2} + (\sqrt{2})^2 = 6 + 2\sqrt{12} + 2 = 8 + 2\sqrt{12}$$

i.e., $\sqrt{15} > \sqrt{12}$, $\therefore 8 + 2\sqrt{15} > 8 + 2\sqrt{12}$ $\therefore (\sqrt{5}+\sqrt{3})$ is greater between the two given numbers.



Let us work out 9.3

1. (a) If $m + \frac{1}{m} = \sqrt{3}$, let us calculate simplified values of (i) $m^2 + \frac{1}{m^2}$ and (ii) $m^3 + \frac{1}{m^3}$

(b) Let us show that, $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 2\sqrt{15}$

2. Let us simplify (a) $\frac{\sqrt{2}(2+\sqrt{3})}{\sqrt{3}(\sqrt{3}+1)} - \frac{\sqrt{2}(2-\sqrt{3})}{\sqrt{3}(\sqrt{3}-1)}$ (b) $\frac{3\sqrt{7}}{\sqrt{5}+\sqrt{2}} - \frac{5\sqrt{5}}{\sqrt{2}+\sqrt{7}} + \frac{2\sqrt{2}}{\sqrt{7}+\sqrt{5}}$

(c) $\frac{4\sqrt{3}}{2-\sqrt{2}} - \frac{30}{4\sqrt{3}-\sqrt{18}} - \frac{\sqrt{18}}{3-\sqrt{12}}$ (d) $\frac{3\sqrt{2}}{\sqrt{3}+\sqrt{6}} - \frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$

3. If $x=2$, $y=3$ and $z=6$, let us write by calculating the value of

$$\frac{3\sqrt{x}}{\sqrt{y}+\sqrt{z}} - \frac{4\sqrt{y}}{\sqrt{z}+\sqrt{x}} + \frac{\sqrt{z}}{\sqrt{x}+\sqrt{y}}$$

4. If $x = \sqrt{7} + \sqrt{6}$, let us calculate simplified values of (i) $x - \frac{1}{x}$ (ii) $x + \frac{1}{x}$ (iii) $x^2 + \frac{1}{x^2}$ (iv) $x^3 + \frac{1}{x^3}$

5. Let us simplify: $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}} + \frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$

If the simplified value is 14, let us write by calculating the value of x .

6. If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $b = \frac{\sqrt{5}-1}{\sqrt{5}+1}$, let us calculate the followings :

(i) $\frac{a^2+ab+b^2}{a^2-ab+b^2}$ (ii) $\frac{(a-b)^3}{(a+b)^3}$ (iii) $\frac{3a^2+5ab+3b^2}{3a^2-5ab+3b^2}$ (iv) $\frac{a^3+b^3}{a^3-b^3}$

7. If $x=2+\sqrt{3}$, $y=2-\sqrt{3}$, let us calculate the simplified values of

(a) (i) $x - \frac{1}{x}$ (ii) $y^2 + \frac{1}{y^2}$ (iii) $x^3 - \frac{1}{x^3}$ (iv) $xy + \frac{1}{xy}$

(b) $3x^2 - 5xy + 3y^2$

8. If $x = \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}-\sqrt{3}}$ and $xy=1$, let us show that $\frac{x^2+xy+y^2}{x^2-xy+y^2} = \frac{12}{11}$

9. Let us write which one is greater of $(\sqrt{7} + 1)$ and $(\sqrt{5} + \sqrt{3})$

10. **Very short answer type question (V.S.A.):**

(A) M.C.Q :

- (i) If $x = 2 + \sqrt{3}$, the value of $x + \frac{1}{x}$ is (a) 2 (b) $2\sqrt{3}$ (c) 4 (d) $2 - \sqrt{3}$
- (ii) If $p + q = \sqrt{13}$ and $p - q = \sqrt{5}$, then the value of pq is (a) 2 (b) 18 (c) 9 (d) 8
- (iii) If $a + b = \sqrt{5}$ and $a - b = \sqrt{3}$, the value of $(a^2 + b^2)$ is
(a) 8 (b) 4 (c) 2 (d) 1
- (iv) If we subtract $\sqrt{5}$ from $\sqrt{125}$, the value is
(a) $\sqrt{80}$ (b) $\sqrt{120}$ (c) $\sqrt{100}$ (d) none of this
- (v) The product of $(5 - \sqrt{3})(\sqrt{3} - 1)(5 + \sqrt{3})(\sqrt{3} + 1)$ is
(a) 22 (b) 44 (c) 2 (d) 11

(B) Let us write whether the following statements are true or false :

- (i) $\sqrt{75}$ and $\sqrt{147}$ are similar surds.
- (ii) $\sqrt{\pi}$ is a quadratic surd.

(C) Let us fill up the blank :

- (i) $5\sqrt{11}$ a _____ number (rational/irrational)
- (ii) Conjugate surd of $(\sqrt{3} - 5)$ is _____.
- (iii) If the product and sum of two quadratic surds is a rational number, then the surds are _____ surds.

11. **Short answer type (S.A.)**

- (i) If $x = 3 + 2\sqrt{2}$, let us write the value of $x + \frac{1}{x}$
- (ii) Let us write which one is greater of $(\sqrt{15} + \sqrt{3})$ and $(\sqrt{10} + \sqrt{8})$
- (iii) Let us write two mixed quadratic surds of which product is a rational number.
- (iv) Let us write what should be subtracted from $\sqrt{72}$ to get $\sqrt{32}$.
- (v) Let us write simplified value of $\left(\frac{1}{\sqrt{2} + 1} + \frac{1}{\sqrt{3} + \sqrt{2}} + \frac{1}{\sqrt{4} + \sqrt{3}} \right)$

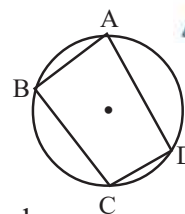
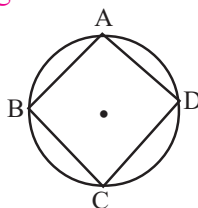
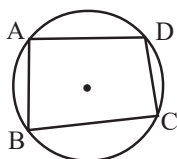
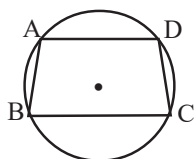
We will make some models on different geometrical topics. We, assembled at Mathematics laboratory of our school with many small and big sticks. Joining sticks we will make many small and big triangles and quadrilaterals of different measures and with those we will try to make some new models.



There are many circular rings in the laboratory of mathematics. Sahana did a funny work. She is trying to make a new model joining quadrilaterals made of sticks in the midst of the big circular ring.

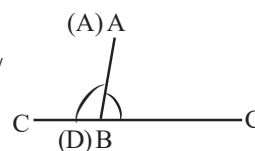
We see, that the quadrilaterals of all types and measures can not be fixed inside the circular ring.

- 1** Let us put the quadrilaterals which can be fixed inside the circular ring separately and try to find similarity among them.



We see the opening of the joint sticks fixed at the point A and C and putting them like the adjoining figure.

Two angles $\angle ABC$ and $\angle ADC$ are each other [complementary/supplementary]



Similarly two angles $\angle BAD$ and $\angle BCD$ are each other. [Let us write it myself].

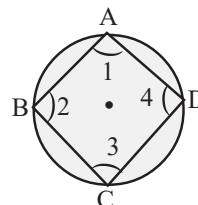
I draw a circle and cyclic quadrilateral on my copy and verifying by hands trial let us see what I shall get.

Hands on trial

- Drawing a circle on art paper I cut it off.
- Taking four points A, B, C and D on that circle and joining A, B; B, C; C, D and D, A we get the cyclic quadrilateral ABCD.
- I cut off the angles labelling with 1, 2, 3, 4.
- By putting two opposite angles of the four angles side by side we see that $\angle 1 + \angle 3 = \text{$ and $\angle 2 + \angle 4 = \text{$

[Let us write by drawing and cutting and verifying by hands on trial].

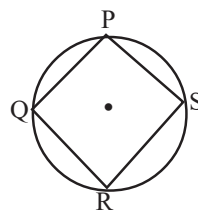
$\therefore$ By hands on trial we get, **opposite angles of a cyclic quadrilateral are supplementary.**





Rahul has drawn a cyclic quadrilateral PQRS. We are observing by measuring with protractor that $\angle P + \angle R = 180^\circ$ and $\angle Q + \angle S = 180^\circ$

[Let me verify it by drawing and measuring myself]



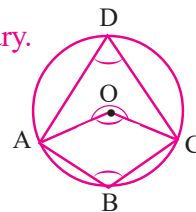
Let us prove with reason

Theorem : 38. The opposite angles of a cyclic quadrilateral are supplementary.

Given : ABCD is a cyclic quadrilateral of a circle with centre O.

To prove : $\angle ABC + \angle ADC = 2$ right angles
and $\angle BAD + \angle BCD = 2$ right angles

Construction : A, O and C, O are joined



Proof : The reflex angle $\angle AOC$ at the centre and the angle $\angle ABC$ on the circle are formed with the circular arc ADC.

$$\therefore \text{Reflex } \angle AOC = 2\angle ABC$$

$$\therefore \angle ABC = \frac{1}{2} \text{ reflex } \angle AOC \dots\dots\dots(i)$$

Again, $\angle AOC$ is the angle at the centre and $\angle ADC$ is the angle on the circle formed with the circular arc ABC.

$$\therefore \angle AOC = 2\angle ADC$$

$$\therefore \angle ADC = \frac{1}{2} \angle AOC \dots\dots\dots(ii)$$

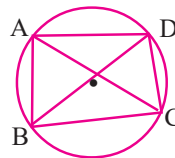
$$\begin{aligned} \therefore \text{From (i) and (ii), we get } \angle ABC + \angle ADC &= \frac{1}{2} \text{ reflex } \angle AOC + \frac{1}{2} \angle AOC \\ &= \frac{1}{2} (\text{reflex } \angle AOC + \angle AOC) \\ &= \frac{1}{2} \times 4 \text{ right angles} = 2 \text{ right angles} \end{aligned}$$

Similarly by joining B, O and D, O; it can be proved that, $\angle BAD + \angle BCD = 2$ right angles (proved)

Alternative proof :

Given : ABCD is a cyclic quadrilateral

To prove : $\angle ABC + \angle ADC = 2$ right angles
and $\angle BAD + \angle BCD = 2$ right angles



Construction : Two diagonals AC and BD are drawn.

Proof : $\angle ADB = \angle ACB$ [angles in the same segment of the circle]

Again, $\angle BAC = \angle BDC$ [angles in the same segment of the circle]

$$\begin{aligned} \text{Again, } \angle ADC &= \angle ADB + \angle BDC \\ &= \angle ACB + \angle BAC \end{aligned}$$

$$\therefore \angle ADC + \angle ABC = \angle ACB + \angle BAC + \angle ABC$$

$$\therefore \angle ADC + \angle ABC = 2 \text{ right angles } [\because \text{sum of three angles of a triangle is } 2 \text{ right angles}]$$

Similarly we can prove that, $\angle BAD + \angle BCD = 2$ right angles [Proved].



After proving the sum of two opposite angles to be equal to 2 right angles, it can easily be shown that the sum of the other two opposite angles is equal to 2 right angles by using the theorem of the sum of the four angles of a quadrilateral is equal to four right angles.



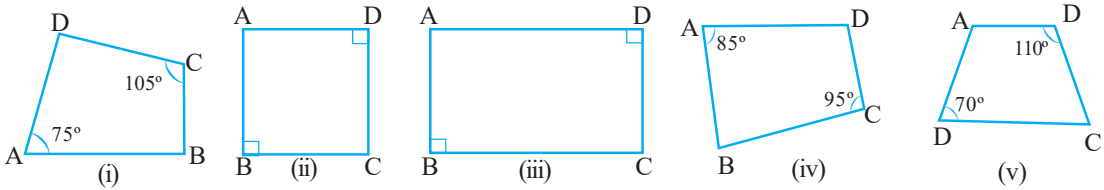
I draw any cyclic quadrilateral PQRS and let us prove with reason that,
 $\angle PQR + \angle PSR = 2 \text{ right angles}$ $\angle QPS + \angle QRS = 2 \text{ right angles}$ [Let me do it myself].

Is the converse theorem of this theorem possible?

i.e. If one pair of opposite angles of a quadrilateral be supplementary then the vertices of the quadrilateral will be concyclic.

Hands on trial

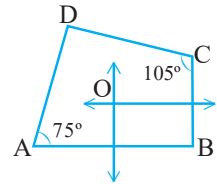
Joyita drew many quadrilaterals of which the sum of opposite angles is 2 right angles



(I) In picture no. (i) quadrilateral I drew two perpendicular bisectors of sides AB and BC and they intersect each other at the point O.

(II) The circle drawn with the point O as centre and OA as the length of radius, we see that the circle passes through the points A, B, C and .

$\therefore$ We get if a fixed circle passes through three non collinear points A, B and C then the circle passes through the point D.



We get by hands and trial, if the opposite angles of a quadrilateral be supplementary, then the vertices of the quadrilateral will be concyclic.

Like picture no. (ii), (iii), (iv) and (v) I draw any quadrilateral of which sum of opposite angles is 180° and similarly verify by hands and trial that the vertices of quadrilateral are concyclic.

Theorem : 39. If opposite angles of any quadrilateral be Supplementary then the vertices of the quadrilateral will be concyclic. [Proof is not included in the evaluation]

Given: Let PQRS is a quadrilateral of which $\angle PQR$ and $\angle PSR$ are supplementary

$$\angle PQR + \angle PSR = 2 \text{ right angles.}$$

To prove : The vertices of a quadrilateral i.e. the four points P, Q, R, S are concyclic

Construction : Only one circle can be drawn through three non collinear points P, Q, R. Let the circle does not pass through the point S. The circle intersects PS or extended PS at the point T. T, R are joined.

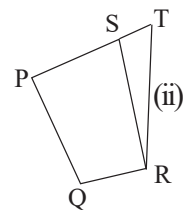
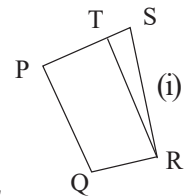
Proof : According to the construction PQRT is a cyclic quadrilateral.

$$\therefore \angle PQR + \angle PTR = 2 \times \text{right angles} \dots\dots(i)$$

$$\text{But } \angle PQR + \angle PSR = 2 \times \text{right angles} \{ \text{Given} \} \dots\dots(ii)$$

$$\text{From (i) and (ii) we get } \angle PQR + \angle PTR = \angle PQR + \angle PSR$$

$$\therefore \angle PTR = \angle PSR$$

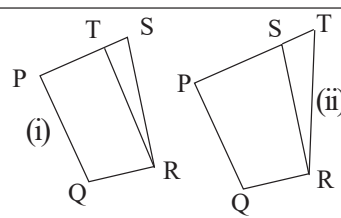


The exterior angles of a triangle can not be equal to interior opposite angle.

$\therefore \angle PTR = \angle PSR$ is possible when the points S and T will coincide.

$\therefore$ So, the circle passes through the points P, Q, R must also pass through the point S.

$\therefore$ The four points P, Q, R and S are concyclic.



Corollary : If any side of a cyclic quadrilateral be extended, the exterior angle so formed is equal to the interior opposite angle – Let us prove with reason.

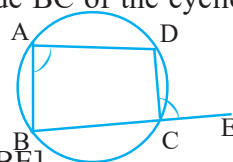
Given : The exterior $\angle DCE$ has been formed by extending the side BC of the cyclic quadrilateral ABCD upto the point E.

To prove : $\angle DCE =$ interior opposite $\angle BAD$

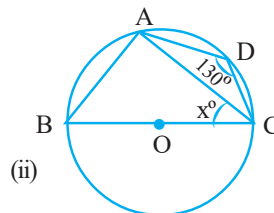
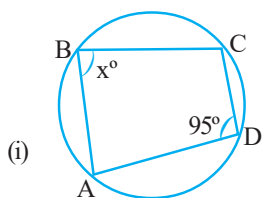
Proof : $\angle BAD + \angle BCD = 180^\circ$ [$\because$ ABCD is a cyclic quadrilateral]

Again, $\angle BCD + \angle DCE = 180^\circ$ [$\because$ DC stands on the line segment BE]

$\therefore \angle BAD + \angle BCD = \angle BCD + \angle DCE \quad \therefore \angle DCE = \angle BAD$ [Proved]



Application 1 : Let us see the following pictures of two circles with centre O and write by calculating the value of x° in each case.



(i) ABCD is a quadrilateral $\therefore \angle ABC + \angle ADC = 180^\circ$

$$\therefore x^\circ = \angle ABC = 180^\circ - \text{} = \text{$$

$x = \text{$ [Let me do it myself]

(ii) ABCD is a quadrilateral $\therefore \angle ABC + \angle ADC = 180^\circ$

$$\therefore \angle ABC = \text{$$

Again, $\angle BAC = \text{$ right angle $\therefore \angle BAC = 90^\circ$

$$\therefore x^\circ + \angle ABC + \angle BAC = 180^\circ$$

$$\therefore x^\circ + 50^\circ + 90^\circ = 180^\circ; \therefore x = \text{$$
 [Let me do it myself]

Application 2 : ABCD is a cyclic quadrilateral and O is the centre of that circle. If $\angle COD = 120^\circ$ and $\angle BAC = 30^\circ$. Let us write by calculating the values of $\angle BOC$ and $\angle BCD$.

The angles, subtended by the minor arc BC, are $\angle BOC$ at the centre and $\angle BAC$ on the circle.

$$\therefore \angle BOC = 2\angle BAC = \text{$$
 [Let me do it myself]

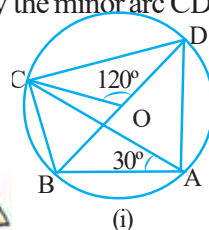
The angles, $\angle COD$ at the centre and $\angle CAD$ on the circle arc subtended by the minor arc CD

$$\therefore \angle CAD = \frac{1}{2} \angle COD = \frac{1}{2} \times 120^\circ = 60^\circ$$

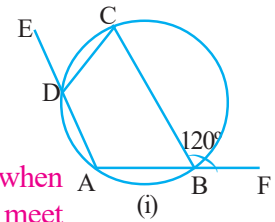
$$\therefore \angle BAD = 30^\circ + 60^\circ = 90^\circ$$

$\therefore$ In cyclic quadrilateral ABCD $\angle BCD + \angle BAD = 180^\circ$

$$\therefore \angle BCD = \text{$$
 [Let me do it myself]



Application 3 : The sides AD and AB of a cyclic quadrilateral ABCD in the picture beside are extended upto the points E and F respectively. If $\angle CBF = 120^\circ$, let us write by calculating the value of $\angle CDE$. [Let me do it myself]



Application 4 : ABCD is a cyclic quadrilateral. The sides AB and DC, when extended meet at the point P and the sides AD and BC, when extended, meet at the point Q. If $\angle ADC = 85^\circ$ and $\angle BPC = 40^\circ$, let us write by calculating the values of $\angle BAD$ and $\angle CQD$.

For cyclic quadrilateral ABCD, exterior $\angle PBC = \angle ADC = 85^\circ$

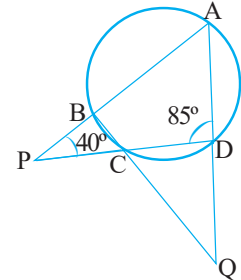
$$\therefore \text{In } \triangle BPC, \angle BCP = 180^\circ - (85^\circ + 40^\circ) = \boxed{}$$

Again, $\angle BAD = \text{exterior } \angle BCP = 55^\circ$

$$\therefore \text{In } \triangle CQD, \angle CQD + \angle DCQ = 85^\circ$$

$$\therefore \angle CQD = 85^\circ - \angle DCQ = 85^\circ - \angle BCP = \boxed{}$$

$$\therefore \angle BAD = 55^\circ \text{ and } \angle CQD = 30^\circ$$



Application 5 : Let us prove that a cyclic parallelogram is a rectangular figure

Given : ABCD is a cyclic parallelogram.

To prove : Quadrilateral ABCD is a rectangular figure.

Proof : ABCD is a parallelogram.

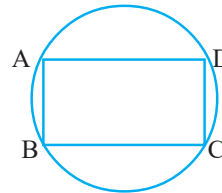
$$\therefore \angle ABC = \angle ADC$$

Again, ABCD is a cyclic quadrilateral

$$\therefore \angle ABC + \angle ADC = \boxed{}$$

$$\text{So, } 2\angle ABC = 180^\circ \therefore \angle ABC = 90^\circ$$

One angle of parallelogram is right angle. So, ABCD is a rectangular figure.



Application 6 : ABCD is a cyclic quadrilateral. The bisectors of $\angle DAB$ and $\angle BCD$ intersect the circle at the points X and Y. Let us prove that XY is a diameter of that circle.

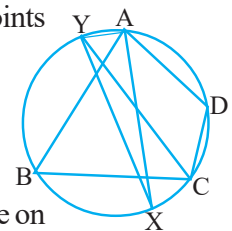
Given : The bisectors of $\angle DAB$ and $\angle BCD$ intersect the circle at the points X and Y.

To prove : XY is a diameter of the circle.

Construction : A, Y are joined

Proof : The angles $\angle YAB$ and $\angle YCB$ subtended by the minor arc YB are on the same segment of the circle.

$$\therefore \angle YAB = \angle YCB = \frac{1}{2} \angle BCD \dots\dots\dots(I) \quad [\because CY \text{ is bisector of } \angle BCD]$$



$$\text{Again, } \angle XAY = \angle XAB + \angle YAB$$

$$= \frac{1}{2} \angle BAD + \frac{1}{2} \angle BCD \text{ [From (I) we get, } \because AX \text{ is bisector of } \angle DAB]$$

$$= \frac{1}{2} (\angle BAD + \angle BCD) = \frac{1}{2} \times 180^\circ [\because ABCD \text{ is a cyclic quadrilateral}]$$

$$= 90^\circ$$

$\therefore \angle XAY$ is semicircular angle. $\therefore$ XY is a diameter



Application 7: Let us prove that a cyclic trapezium which is not a rectangle is an isosceles trapezium or rectangle and lengths of two diagonals are equal.

Given: ABCD is a cyclic trapezium of which $AD \parallel BC$

To prove: $AB = DC$ or ABCD is a rectangle and $AC = BD$

Proof: $\angle ADC + \angle DCB = 180^\circ$ [$\because AD \parallel BC$ and DC is transversal]

Again, $\angle BAD + \angle DCB = 180^\circ$ [$\because$ ABCD is a cyclic quadrilateral]

$\therefore \angle ADC + \angle DCB = \angle BAD + \angle DCB \quad \therefore \angle ADC = \angle BAD$ (I)

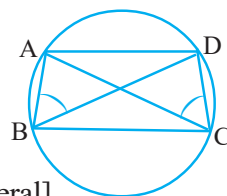
In $\triangle BAD$ and $\triangle ADC$, $\angle BAD = \angle ADC$ [From (I) we get]

$\angle ABD = \angle DCA$ [$\because$ Angles in the same segment]

AD is common side

$\therefore \triangle BAD \cong \triangle ADC$ [A-A-S congruence property]

$\therefore AB = DC \therefore$ ABCD is an isosceles trapezium or a rectangle and $AC = BD$ (Similar part of congruent triangle) **[Proved]**



Application 8: The middle points of two chords AP and AQ of a circle with centre O are R and S respectively. Let us prove that the four points O, R, A, S are concyclic.

Given : The middle points of two chords AP and AQ of a circle with centre O are R and S respectively.

To prove : The four points O, R, A, S are concyclic.

Construction : O, R and O, S are joined

Proof: OR bisects the chord AD $\therefore OR \perp AP$

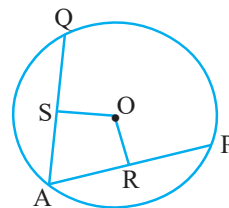
i.e. $\angle ARO = 90^\circ$

OS bisect the chord AQ $\therefore OS \perp AQ$

i.e. $\angle ASO = 90^\circ$

$\therefore \angle ARO + \angle ASO = 90^\circ + 90^\circ = 180^\circ$

So, a pair of opposite angles of quadrilateral AROS is supplementary. So four points O, R, A, S. are concyclic.



Application 9 : Let us prove that the quadrilateral formed by angle bisectors of a cyclic quadrilateral is also concyclic.

Given : A cyclic quadrilateral in which AR, BP, CP and DR are bisectors of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectively.

To prove : PQRS is a cyclic quadrilateral.

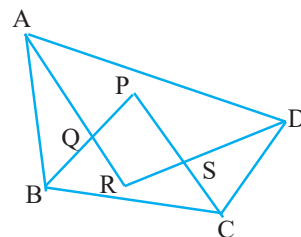
Proof: In $\triangle ARD$, $\angle ARD + \angle RDA + \angle DAR = 180^\circ$

$\therefore \angle ARD + \frac{1}{2} \angle A + \frac{1}{2} \angle D = 180^\circ$ (I)

Again, in $\triangle BPC$, $\angle BPC + \angle PCB + \angle CBP = 180^\circ$

$\therefore \angle BPC + \frac{1}{2} \angle C + \frac{1}{2} \angle B = 180^\circ$ (II)

From (I) and (II), we get $\angle ARD + \frac{1}{2} \angle A + \frac{1}{2} \angle D + \angle BPC + \frac{1}{2} \angle C + \frac{1}{2} \angle B = 180^\circ + 180^\circ$



$$\text{Or, } \angle ARD + \angle BPC + \frac{1}{2}(\angle A + \angle B + \angle C + \angle D) = 360^\circ$$

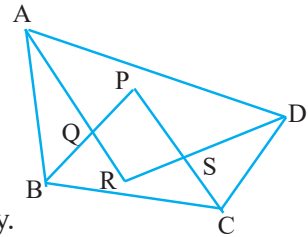
$$\text{Or, } \angle ARD + \angle BPC + \frac{1}{2} \times 360^\circ = 360^\circ$$

$$\text{Or, } \angle ARD + \angle BPC = 360^\circ - 180^\circ = 180^\circ$$

$$\therefore \angle QRS + \angle QPS = 180^\circ$$

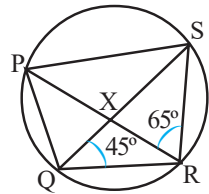
Thus, two opposite angles of quadrilateral PQRS are supplementary.

$\therefore$ Hence, PQRS is a cyclic quadrilateral



Let us Workout 10

- In the adjoining figure two diagonals of quadrilateral PQRS intersect each other at the point X in such a way that $\angle PRS = 65^\circ$ and $\angle RQS = 45^\circ$. Let us write by calculating values of $\angle SQP$ and $\angle RSP$.
- The side AB of a cyclic quadrilateral ABCD is extended to the point X and by measuring we see that $\angle XBC = 82^\circ$ and $\angle ADB = 47^\circ$. Let us write by measuring $\angle BAC$.
- Two sides PQ and SR of a cyclic quadrilateral PQRS are extended to meet at the point T. O is the centre of circle. If $\angle POQ = 110^\circ$, $\angle QOR = 60^\circ$, $\angle ROS = 80^\circ$. Let us write by measuring $\angle RQS$ and $\angle OTR$.
- Two circles intersect each other at the points P and Q. Two straight lines through the points P and Q intersect one circle at the points A and C respectively and the other circle at the points B and D respectively. Let us prove that $AC \parallel BD$.
- I drew a cyclic quadrilateral ABCD and the side BC is extended upto the point E. Let us prove that the bisectors of $\angle BAD$ and $\angle DCE$ meet in the circle.
- Mohit drew two straight lines through any external point X to a circle which intersects the circle at the points A, B and points C, D respectively. Let us prove with reason that $\triangle XAC$ and $\triangle XBD$ are equiangular.
- I drew two circles which intersect each other at the points G and H. Now I drew a straight line through the point G which intersects two circles at the points P and Q and the straight line through the point H parallel to PQ intersects the two circles at the points R and S. Let us prove that $PQ = RS$.
- I drew a triangle ABC of which $AB = AC$ and E is any point on the extended BC. If the circumcircle of ABC intersects AE at the point D. Let us prove that $\angle ACD = \angle AEC$.
- ABCD is a cyclic quadrilateral. Chord DE is the external bisector of $\angle BDC$. Let us prove that AE (Or extended AE) is the external bisector of $\angle BAC$.
- BE and CF are perpendiculars on the sides AC and AB of triangle ABC respectively. Let us prove that four points B, C, E, F are concyclic and from it let us prove that two angles of each of $\triangle AEF$ and $\triangle ABC$ are equal.
- ABCD is a parallelogram. A circle passing through the points A and B intersects the sides AD and BC at the points E and F respectively. Let us prove that the four points E, F, C, D are concyclic.
- ABCD is a cyclic quadrilateral. The two sides AB and DC are extended to meet at the point P and two sides AD and BC are extended to meet at the point R. The two circumcircles of $\triangle BCP$ and $\triangle CDR$ intersect at the point T. Let us prove that the points P, T, R are collinear.

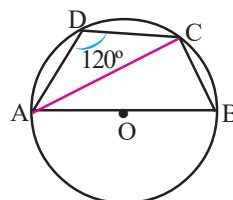


13. If the point O is the ortho-centre of triangle ABC. Let us prove that O is the incentre of the triangle.
14. I have drawn such a cyclic quadrilateral ABCD that AC bisected $\angle BAD$. Now AD is extended upto the point E in such a way that $DE = AB$. Let us prove that $CE = CA$.
15. Of two circles, one passes through the centre 'Q' of the other circle and the two circles intersect each other at the point A and B. The straight line passes through A intersects the circle with centre Q at the point P and intersects the circle with centre 'O' at the point 'R'. Joining P, B and R, B let us prove that $PR = PB$
16. Let us prove that any four vertices of a regular pentagon are concyclic.
17. **Very Short Answer (V.S.A)**

(A) (M. C. Q.) :

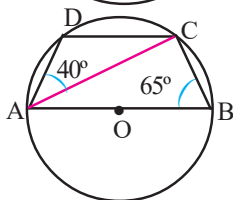
- (i) In the adjoining figure, O is the centre of circle and AB is a diameter. ABCD is a cyclic quadrilateral. If $\angle ADC = 120^\circ$, the value of $\angle BAC$ is

(a) 50° (b) 60° (c) 30° (d) 40°



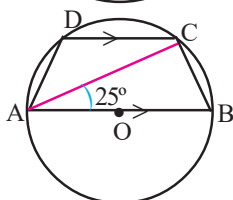
- (ii) In the adjoining figure, O is the centre of the circle and AB is a diameter. ABCD is a cyclic quadrilateral. If $\angle ABC = 65^\circ$, $\angle DAC = 40^\circ$ the value of $\angle BCD$ is

(a) 75° (b) 105° (c) 115° (d) 80°



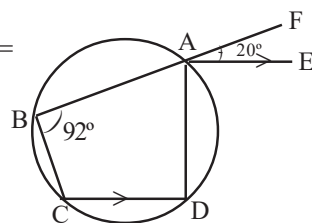
- (iii) In the adjoining figure, O is the centre of the circle and AB is a diameter. ABCD is a cyclic quadrilateral in which $AB \parallel DC$ and if $\angle BAC = 25^\circ$ then the value of $\angle DAC$ is

(a) 50° (b) 25° (c) 130° (d) 40°



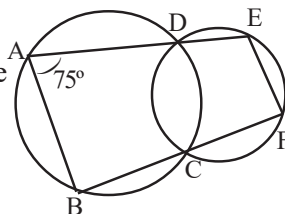
- (iv) In the adjoining figure, ABCD is a cyclic quadrilateral. BA is extended to the point F. If $AE \parallel CD$, $\angle ABC = 92^\circ$ and $\angle FAE = 20^\circ$, then the value of $\angle BCD$ is

(a) 20° (b) 88° (c) 108° (d) 72°



- (v) In the adjoining figure, two circles intersect each other at the points C and D. Two straight lines through the points D and C intersect one circle at the points A and B and the other circle at the points E and F respectively. If $\angle DAB = 75^\circ$, then the value of $\angle DEF$ is

(a) 75° (b) 70° (c) 60° (d) 105°



(B) Let us write true or false :

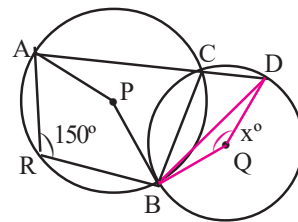
- (i) The opposite angles of a cyclic quadrilateral is complementary.
- (ii) If any side of a cyclic quadrilateral be extended in such a way that the exterior angle so formed is equal to the interior opposite angle.

(C) Let us fill in the blanks :

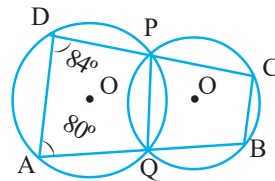
- (i) If the opposite angles of a quadrilateral be supplementary then the vertices of the quadrilateral will be _____.
- (ii) A cyclic parallelogram is a _____ figure.
- (iii) The vertices of a square are _____.

18. Short Answer (S.A):

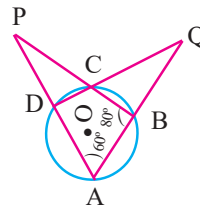
- (i) In the adjoining figure, two circles with centres P and Q intersect each other at the points B and C. ACD is a line segment. If $\angle ARB = 150^\circ$, $\angle BQD = X^\circ$, then let us find the value of X° .



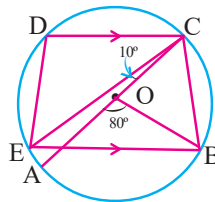
- (ii) In the adjoining figure, two circles intersect each other at the points P and Q. If $\angle QAD = 80^\circ$ and $\angle PDA = 84^\circ$, then let us find the values of $\angle QBC$ and $\angle BCP$.



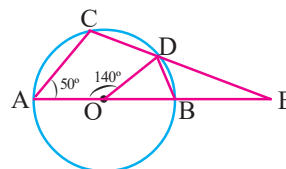
- (iii) In the adjoining figure, if $\angle BAD = 60^\circ$, $\angle ABC = 80^\circ$, then let us find the values of $\angle DPC$ and $\angle BQC$.



- (iv) In the adjoining figure, O is the centre of the circle and AC is its diameter. If $\angle AOB = 80^\circ$ and $\angle ACE = 10^\circ$, let us find the value of $\angle BED$.



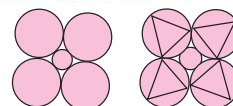
- (v) In the adjoining figure, O is the centre of the circle and AB is its diameter. If $\angle AOD = 140^\circ$ and $\angle CAB = 50^\circ$, let us find the value of $\angle BED$.



This year it has been decided in our school that each will make articles according to their own choice. I have decided that I shall embroider on a new cover of our table.

So, I drew a diagram on the cover of our table with the pencil as in the adjoining picture.

My brother did a funny work. He drew some chords in many circles drawn by me as in the picture beside.



1 We see in diagram the chords of the circle have formed a triangle in the circle. What is the relation between circle drawn like this and a triangle lies in the circle. (vertices of which lie on the circle)

The circle circumscribed a triangle which lie within the circle. So, the circle is **circumcircle** of the triangle.

The triangle which we shall get by adding any three points on any circle, then the circle is **circumcircle** of the triangle.

But if any triangle is given, then how shall we draw the circumcircle of this triangle?

I draw any triangle and try to draw the circumcircle of this triangle.



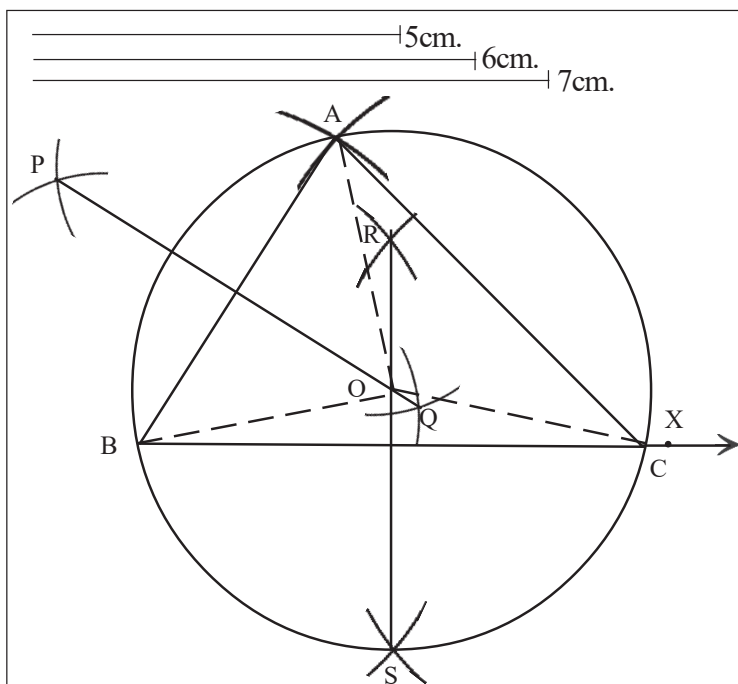
Construction : Construction of the circumcircle of any triangle.

By drawing a triangle with sides 5 cm, 6 cm and 7 cm respectively, let us draw a circumcircle of this triangle.

(i) At first I draw a triangle $\triangle ABC$ having the sides of 5 cm, 6 cm and 7 cm respectively.

(ii) [At first I shall find the centre of circumcircle for drawing circle circumscribing $\triangle ABC$. So, I shall draw perpendicular bisectors of any two sides of $\triangle ABC$]

Perpendicular bisectors PQ and RS of sides AB and BC of $\triangle ABC$ are drawn to intersect each other at the point O respectively.



(iii) The point O is the intersecting point of two perpendicular bisectors AB and BC.

The point O is equidistant from the points A, B and C. Having 'O' at the centre and the length of radius OA or OB or OC we draw a circle, which is the circumcircle of $\triangle ABC$.

Proof: O, A ; O, B ; O, C are joined.

O, is a point on the perpendicular bisector of AB.

$\therefore$ O is equidistant from A and B i.e. $OA = OB$

Similarly, we can prove that $OB = OC$

$\therefore OA = OB = OC$.

$\therefore$ The circle drawn with O as centre and OA as radius, passes through the points A, B and C of $\triangle ABC$.

$\therefore$ That circle is a circumcircle of $\triangle ABC$.



Let us draw a circumcircle of an equilateral triangle with sides of 4 cm. [Let me do it myself]

2 What is called the centre and radius of circumcircle of a triangle ?

The centre of circumcircle is called **circumcentre** and radius is called **circumradius**.

Let us do

11

1. The sides of a triangle with respect to circumcircle of a triangle are [radii / chords] of the circle.
2. The intersecting point of the perpendicular bisectors of two chords of any circle is
3. The distances of the vertices of a triangle from the point of intersection of the perpendicular bisectors of the sides are
4. The distance of any vertex from the point of intersection of the perpendicular bisectors of sides of triangle is the length of the [diameter / radius] of circumcircle of the triangle.

My friend Jahir has decided that he will embroider on a handkerchief like me. So by drawing different types of triangles on his copy he is trying to draw circumcircle. Jahir on his copy

(i) has drawn a triangle ABC of which $BC = 4$ cm, $\angle ABC = 60^\circ$, $\angle ACB = 70^\circ$

(ii) has drawn a triangle PQR of which $QR = 3.5$ cm, $\angle PQR = 90^\circ$ and $PR = 4.5$ cm

(iii) has drawn $\triangle XYZ$ of which $\angle XYZ = 120^\circ$, $\angle YZX = 30^\circ$ and $YZ = 3$ cm.



We see that no (i) triangle is an acute angled triangle.

I drew a circumcircle of an acute angled triangle ABC as triangle no (i)

We see, the circumcentre of an acute angled triangle ABC lies [within / outside] the triangular region.

Drawing a circumcircle of any acute angled triangle, I see that the circumcentre lies inside the triangular region

[Let me do it myself]

No (ii) triangle is a triangle which is drawn by Jahir.

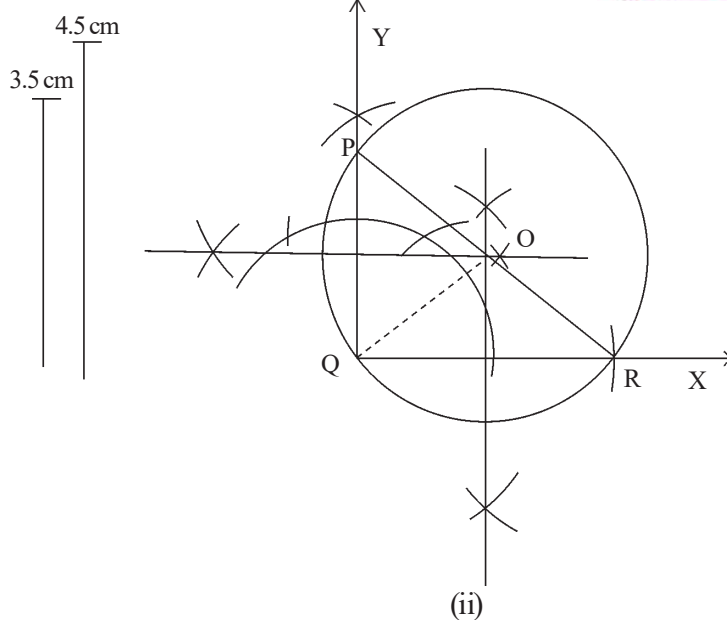
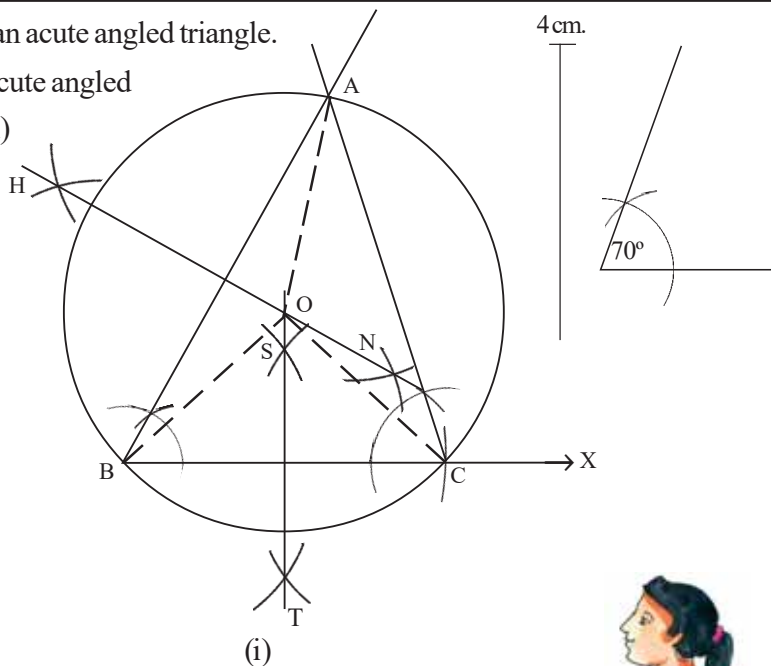
I drew a circumcircle of picture no (ii) triangle i.e. right angled triangle.

I see that the circumcentre O of right angled triangle PQR lies on hypotenuse and it lies on of the hypotenuse as $PO = OR$ [Where O is the circumcentre of $\triangle PQR$]

So, shall we draw the perpendicular bisectors of two sides of the triangle, to find out the circumcentre of the right angled triangle?

As the circumcentre of right-angled triangle is the mid-point of the hypotenuse, we shall get circumcentre bisecting the hypotenuse.

e.g. If we draw perpendicular bisector of hypotenuse PR of right-angled triangle $\triangle PQR$ we shall get circumcentre O. By drawing circumcircle of any right-angled triangle we see that the circumcentre is the mid point of the hypotenuse of the triangle. [Let me do it myself]



No (iii) triangle which is drawn by Jahir is triangle.

I draw circumcircle of no (iii) triangle i.e. I draw circumcircle of an obtuse-angled triangle.

I see that the circumcentre O of an obtuse-angled triangle lies [inside / outside] the triangular region.

By drawing circumcircle of any obtuse-angled-triangle, we see that the circumcentre lies outside the triangular region.

[Let me do it myself]

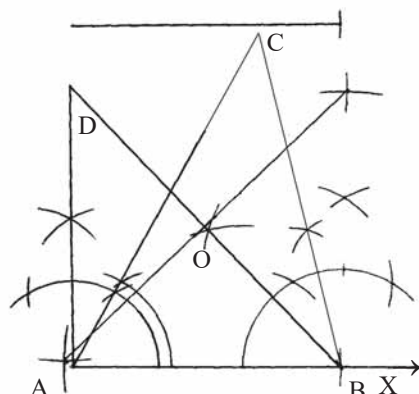
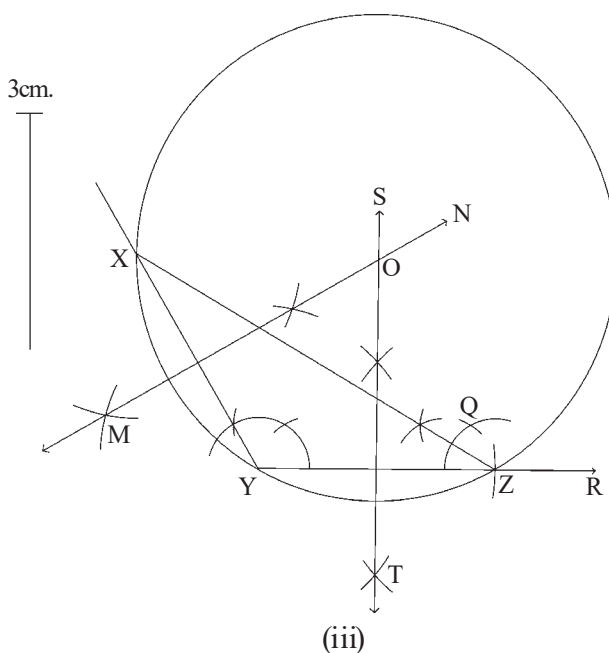
We get, i) if the triangle is acute angled, right-angled or obtuse-angled then the circumcentre lies the triangular region, on the hypotenuse or the triangular region respectively.

(ii) The circum-centre of a right-angled triangle is the mid-point of the hypotenuse.

I drew a triangle ABC of which $AB = 3.5$ cm, $\angle BAC = 60^\circ$ and $\angle ABC = 75^\circ$

On the triangle ABC drawn by me, Sahana has drawn a triangle ABD of which $\angle BAD = 90^\circ$, $\angle ABD = 45^\circ$ and the point D will lie on that side of AB in which the point C lies.

I draw circumcircle of right-angled triangle ABD and see whether the point C lies on the circle. [Let me do it myself]



Answer Hints : Let us determine the mid-point D of hypotenuse DB and a circle is drawn with 'O' as centre and 'DO' as length of radius and also draw a circumcircle of $\triangle ABD$.

We see that the circumcircle of $\triangle ABD$ passes through the point C of $\triangle ABC$.

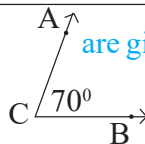
But let us write with reason by measuring the angle of a triangle why we get same circumcircle of $\triangle ABD$ and $\triangle ABC$. [Let me do it myself]



Application : 1. I draw a triangle ABC of which $BC = 6.5$ cm, $\angle ABC = 60^\circ$ and $\angle ACB = 70^\circ$. At the time of drawing the triangle we will draw the length of line segment 6.5 cm with scale and $\angle ACB = 70^\circ$ with the protractor. Again why will we draw 60° angle of triangle with pencil compass? Why we draw 60° angle of the triangle with pencil compass? At the time of construction there is neither protractor nor any symbolic scale with us.

i.e. at the time of drawing angle and line segment. We have only a pencil, a pencil compass and a scale without any symbol.

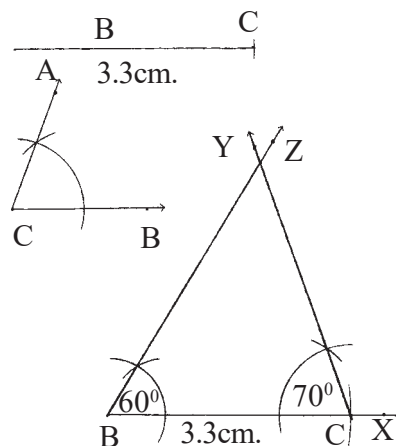
In question $\overline{BC}$ 3.3 cm and $\angle ACB = 70^\circ$ are given.



The angles which can be drawn with the help of pencil compass are not given in question.

So, from the ray BX the length of 3.3 cm is cut off making equal to the line segment BC which is drawn in question and an angle $\angle YCB = 70^\circ$ is drawn taking equal measurement of an angle $\angle ACB$ given in the question at the point 'C' on the line segment BC. Then an angle $\angle ZBC = 60^\circ$ is drawn at the point B with the help of a pencil compass. CY and BZ intersect each other at the point A. In this way a fixed triangle ABC is drawn.

As a line segment of length 3.3 cm and an angle measuring 70° are not given in any question paper we have to draw those two respectively with the help of a scale and protractor.



Let us work out

11.1

- Let us draw the following triangles. By drawing the circumcircle in each case, let us write the position of circum-centre and the length of circumradius by measuring it.
 - An equilateral triangle having each side of length 6 cm.
 - An isosceles triangle whose length of the base is 5.2 cm and each of the equal sides is 7 cm.
 - A right-angled triangle having two sides 4 cm and 8 cm length, containing right angle.
 - A right-angled triangle having length of 12 cm. hypotenuse and other side is of 5 cm length.
 - A triangle of which length of one side is 6.7 cm and the two angles adjacent to this side are 75° and 55° .
 - ABC is a triangle of which $BC = 5$ cm., $\angle ABC = 100^\circ$ and $AB = 4$ cm.

- Given $PQ = 7.5$ cm. $\angle QPR = 45^\circ$, $\angle PQR = 75^\circ$;

$PQ = 7.5$ cm. $\angle QPS = 60^\circ$, $\angle PQS = 60^\circ$;

Let us draw $\triangle PQR$ and $\triangle PQS$ in such a way that the points R and S lie on the same side of PQ, let us draw circum circle of $\triangle PQR$ and let us observe and write the position of the point S within, on and out side the circum circle . Let us find out its explanation.

- Given $AB = 5$ cm. $\angle BAC = 30^\circ$, $\angle ABC = 60^\circ$;

$AB = 5$ cm, $\angle BAD = 45^\circ$, $\angle ABD = 45^\circ$;

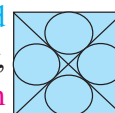
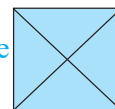
Let us draw $\triangle ABC$ and $\triangle ABD$ in such a way that the point C and D lie on opposite sides of AB. Let us draw the circle circumscribing $\triangle ABC$. Let us write the position of the point D with respect to circumcircle. Let us write by understanding what other characteristics we are observing here.

- We draw a quadrilateral ABCD having $AB = 4$ cm, $BC = 7$ cm $CD = 4$ cm, $\angle ABC = 60^\circ$ $\angle BCD = 60^\circ$. Let us draw the circle circumscribing $\triangle ABC$ and write what other characteristics we observe.
- Let us draw the rectangle PQRS having $PQ = 4$ cm, and $QR = 6$ cm. Let us draw the diagonals of the rectangle. Let us write by calculating the position of the centre of the circum-circle of $\triangle PQR$ and the length of circum radius without drawing. By drawing circumcircle of $\triangle PQR$, let us verify.
- If any circular picture is given, then how shall we find its centre? Let us find the centre of the circle in the adjoining figure.



I draw as in the picture mentioned beside with the pencil on the handkerchief of Jahir.

Uma has drawn many circles in the triangle as in the adjoining picture and has made a design.



Uma drew,

- We see that the circles in the triangle touches the three sides of the triangle which is drawn by Uma. What is called this type of circle?

The circle inscribed in a triangle touching three sides of triangle is called inscribed circle or incircle.

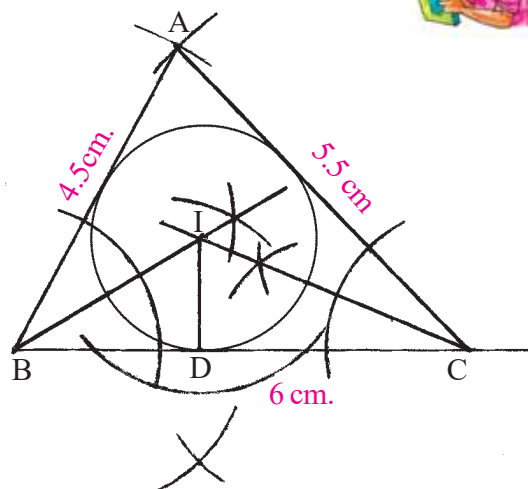
Construction : I draw a triangle of which three sides are 4.5 cm, 5.5 cm and 6 cm. Let us try to draw in-circle of this triangle.

Construction of incircle of a triangle.

I draw a triangle of which $BC = 6$ cm $CA = 5.5$ cm and $AB = 4$ cm. Let us draw in-circle of $\triangle ABC$.

Construction Procedure :

- We have drawn the bisectors BI and CI of the two angles $\angle ABC$ and $\angle ACB$ respectively intersecting each other at the points I.
- From the point I, a perpendicular to the side BC is drawn to meet BC at the point D.
- A circle is drawn with centre I and length of radius ID. That is the circle inscribed in $\triangle ABC$.



4 What is called the centre and radius of incircle of a triangle ?

The centre of incircle of triangle is called **Incentre** and radius is called **Inradius**.



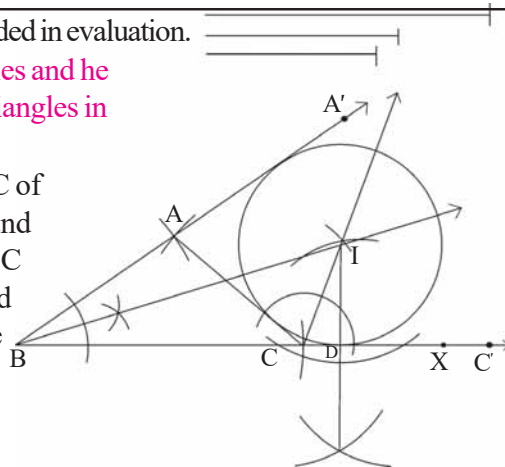
In any equilateral triangle, the bisector of any angle and from the vertex of that angle the perpendicular bisector on the opposite sides are the same straight lines. So, by drawing let us verify where the circumcentre and the incentre of an equilateral triangle will lie. [Let me do it myself]

By drawing circumcentre and incentre of an equilateral triangle we have seen, the circumcentre and incentre of an equilateral triangle are [same / different].

This part (drawing of an excircle) is not included in evaluation.

My friend Sajal has drawn different types of circles and he tried to draw circumcircle and incircle of those triangles in his copy.

Sajal did a funny work. He drew a triangle ABC of which $BC = 4.5$ cm $CA = 2.7$ cm., $AB = 3$ cm. and extended the sides BA and BC upto A and C respectively. He draws bisectors $\angle ABC$ and $\angle ACC$ which intersect each other at I.. From the point I he draws perpendicular ID on the extended side BC which intersects the side BC at the point D. He draws a circle with centre I and with radius ID, touching the extended sides BC, CA and BA.



What is called this type of circle ?

The circle that touches one side of a triangle and the other two extended sides is called **excircle**.

Point I is said to be **excentre** and radius is said to be **exradius**.

I draw any triangle and let us try to draw excircle of that triangle.

- Let me write myself how many excircle and incircle of a triangle can be drawn.
- Let me write myself how many points of a triangle are equidistant from the sides of a triangle.



Let us Work out

11.2

- Let us draw the following triangles and by drawing incircle of each circle, let us write by measuring the length of inradius in each case.
 - the lengths of three sides are 7 cm., 6 cm. and 5.5 cm.
 - the length of two sides are 7.6 cm., 6 cm. and the angle included by those two sides is 75°
 - the length of a side is 6.2 cm. and measures of two angles adjacent to this side are 50° and 75°
 - a triangle is a right-angled triangle of which two sides containing the right angle have the lengths of 7 cm. and 9 cm.
 - a triangle is a right-angled triangle of which the length of hypotenuse is 9 cm. and another side 6.5 cm.
 - The triangle is an isoscales triangle having the 7.8 cm. length of base and the length of each equal side of 6.5 cm. 10 cm.
 - a triangle is an isosceles triangle, having the 10 cm. length of base and each of the equal angle is 45°
 - Let us draw an equilateral triangle having 7 cm. length of each side. By drawing the circumcircle and incircle, let us find the length of circum radius and inradius and let us write whether there is any relation between them.

Every year we buy handfans of palm leaves from the fair. We use these handfans in our house. It is comfortable to have small handfans on the way of going to school. So, we have decided that we will make hand fans with the hard board left in the house.

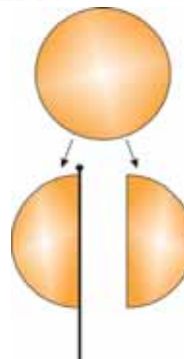


Sujit, the friend of my brother, has brought many small and big circular plates of hard board.

My brother folded into two equal parts of a hard board and cuts the semicircular plate along diameter of the circular plate and pasted a stick and paper with gum and made a hand fan like a picture mentioned beside.

1 But, when my brother is rotating the semicircular plate taking that stick as axis, then it looks mostly like the solid ball, What this figure is called?

Likewise the plate having rotated taking the diameter of semicircular plate as axis, solid figure which we found is **Sphere**.



We understand that the surface of the sphere is like surface of a ball.

∴ The number of surface of sphere is and this is a [curved surface/ plane surface]

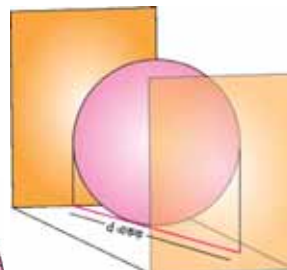
We easily made many hand fans with the help of circular plate of paper brought by Sujit, each of us took one hand fan. Now we have decided that we shall find out spherical solid objects used in house. My sister has brought us a big leather ball.

But now shall we get how much leather is required for making that leather ball i.e How shall we get curved surface area or whole surface area of sphere.



Measurement of the curved surface area or whole surface area by hands on trial.

(1) Let us take a spherical ball and put it in between two vertical hard boards just like the adjoining figure. Then after measuring the perpendicular distance between two vertical boards. We get the length of diameter of the ball. Hence we can easily evaluate the length of the radius of the ball. Let it be 'r' units of length.



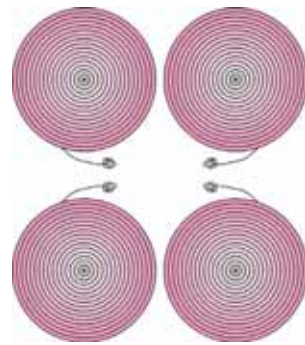
(2) Now we fix a pin on the ball.

(3) Then we attach a rope with the pin and it is rolled up in such a way that there is no empty place left and there should not be any overlapping on any part of the rope. [See the adjoining figure]



(4) Now we make the starting point and end point of the portion of the rope which is required to cover the whole surface of the ball.

- (5) Let us draw a number of circles on a white hard board with length of the radius r units.
- (6) Now we start to cover each of the circular regions one after another by the rope as it was done in the case of the ball [See the adjoining figure]



We see the portion of the rope between the two marked points can cover exactly four circular regions and we are left with no remaining length of the rope.

$$\begin{aligned}\therefore \text{Area of the surface of the sphere} \\ &= 4 \times (\text{Area of circular region of radius of length } r \text{ units}) \\ &= 4 \times \pi r^2 \text{ square units} \\ &= 4\pi r^2 \text{ square units}\end{aligned}$$

∴ We got by hands and trial

The Curved Surface area or Whole Surface area of sphere is $= 4\pi r^2$

We understand that the leather required $4\pi r^2$ square units for making leather ball with radius r units.

Application : 1. If the diameter of ball is 42 cm., let us calculate how much leather is required for making ball.

Diameter of the ball = 42 cm. $\therefore$ Radius of the ball = $\frac{42}{2}$ cm. = 21 cm.

$$\begin{aligned}\therefore \text{Whole surface area of the ball} &= 4\pi \times (21)^2 \text{ sq cm} \\ &= 4 \times \frac{22}{7} \times 21 \times 21 \text{ sq cm} \\ &= \boxed{} \text{ sq cm}\end{aligned}$$



$\therefore$ The ball contains 5544 sq cm leather.

Application : 2. The diameter of a sphere made of iron sheet is 14 cm., let us calculate What is the cost of colouring the sphere at Rs. 2.50 per square cm. **[Let me do it myself]**

2 How shall we get the quantity of stone is required for the ball if the spherical ball would be of the solid stone.

We shall get by determining the volume of sphere.

$$\text{Volume of sphere} = \frac{4}{3} \pi r^3 \text{ cubic unit}$$

[where length of radius of sphere is ' r ' units, the stone of the solid stone ball with radius r units $= \frac{4}{3} \pi r^3$ cubic unit]

Application : 3. If the length of diameter of a solid spherical ball is 14 cm. Let us write by calculating how much stone are there in the spherical solid ball.

The length of radius of the solid spherical ball of stone = $\frac{14}{2}$ cm. = 7 cm.

$$\begin{aligned}\therefore \text{solid spherical ball of stone contains stone} &= \frac{4}{3} \times \frac{22}{7} \times 7^3 \text{ cubic cm.} \\ &= \boxed{} \text{ cubic cm.}\end{aligned}$$



Application : 4. My spherical stone ball having the radius 0.7 dcm. length is completely immersed into the water of reservoir. Let us write by calculating how much water will be overflowed from the reservoir. **[Let me do it myself]**

Application : 5. If whole area of a sphere is 2464 sq metre, let us write by calculating the volume of the sphere.

Let the length of radius of sphere is r metre

According to the condition, $4\pi r^2 = 2464$

$$\text{or, } 4 \times \frac{22}{7} \times r^2 = 2464$$

$$\text{or, } r^2 = 2464 \times \frac{7}{22 \times 4} = 28 \times 7 = 7^2 \times 2^2$$

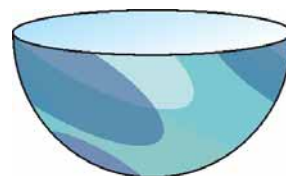
$$\therefore r = 14$$

$$\begin{aligned} \therefore \text{Volume of that sphere} &= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14 \text{ cubic metre.} \\ &= 11498.67 \text{ cubic metre.} \end{aligned}$$



3 There was a paperweight on the table in my reading room. My sister is measuring the length of radius of that paperweight. with scale.

We see that the paperweight is a solid hemispherical object of which number of plane is and number of curved surface is .



$$\begin{aligned} \text{The area of curved surface of this semicircular sphere} &= \frac{1}{2} \times \text{area of curved surface of sphere} \\ &= \frac{1}{2} \times 4\pi r^2 \text{ sq unit} \\ &= 2\pi r^2 \text{ sq unit} \\ &\text{[where the length of radius of semicircular sphere is } r \text{ units.]} \end{aligned}$$

And area of plane surface of solid semicircular solid object $= \pi r^2$ sq unit

$$\begin{aligned} \text{We understand that, if the length of radius of solid hemisphere is } r \text{ units, then the whole surface} \\ \text{area} &= (2\pi r^2 + \pi r^2) \text{ sq units} \\ &= 3\pi r^2 \text{ sq units} \end{aligned}$$

$\therefore$ Whole surface area of Solid Hemisphere $= 3\pi r^2$ sq unit

Application : 6. If the length of diameter of hemispherical solid object is 14 cm, let us write by calculating its whole surface area.

$$\begin{aligned} \text{Whole surface area} &= 3 \times \frac{22}{7} \times 14 \times 14 \text{ sq units} \\ &= \text{ } \text{sq units} \end{aligned}$$



Application : 7. If it requires 173.25 sq cm of sheet to make a hemispherical bowl then let us write by calculating the length of diameter of forepart of the bowl. **[Let me do it myself]**

[Answer hints : As the bowl is not a solid object, curved surface area of sheet will only be required.]

4 Now let us calculate how much stone is there in a solid hemispherical paperweight?

Let the length of radius of hemispherical paperweight be r units.

$$\begin{aligned}\therefore \text{Volume of a solid hemisphere} &= \frac{1}{2} \times \text{volume of sphere} \\ &= \frac{1}{2} \times \frac{4}{3} \pi r^3 \text{ cubic units} = \frac{2}{3} \pi r^3 \text{ cubic units} \\ \therefore \text{Volume of hemisphere} &= \frac{2}{3} \pi r^3 \text{ cubic units.}\end{aligned}$$

Application : 8. If the length of radius of solid hemispherical solid object is 14 cm, let us calculate how much quantity of stone is there in that object.

Stone contains in hemispherical paperweight is $= \frac{2}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$ cubic units = cubic units

Application : 9. If ratio of whole area of two spherical solid objects is 1:4, let us write by calculating the ratio of their volumes.

Let the lengths of radii of two spherical solid objects are r_1 units and r_2 units respectively.

$$\text{According to the condition } \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{1}{4} \quad \text{or} \quad \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 \therefore \frac{r_1}{r_2} = \frac{1}{2}$$



$$\therefore \frac{\text{Volume of first sphere}}{\text{Volume of 2nd sphere}} = \frac{\frac{4}{3} \pi r_1^3}{\frac{4}{3} \pi r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$\therefore$ The ratio of volumes of two spherical solid objects is 1:8

Application : 10. If the ratio of lengths of radii of two spheres is 1:2, let us write by calculating the ratio of their whole surface areas. **[Let me do it myself]**

Application : 11. If the numerical values of volume and curved surface area of a sphere are equal, let us write by calculating numerical value of radius of sphere

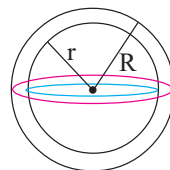
Let the length of radius of sphere be r units.

$\therefore$ Curved surface area of sphere $= 4\pi r^2$ sq units and volume $= \frac{4}{3} \pi r^3$ cubic units

According to the given condition, $\frac{4}{3} \pi r^3 = 4\pi r^2$ (i.e Numerical values are equal) $\therefore r = 3$ [$\because r \neq 0$]

$\therefore$ Numerical value of length of radius of sphere is 3 cm.

5 If the lengths of outer radius and inner radius of hollow sphere are R units and r units respectively, how much metal is there in a hollow sphere i.e. How shall we get the volume of metal which is required for making that hollow sphere.



The volume of metal is required to make that hollow sphere $= \frac{4}{3} \pi (R^3 - r^3)$ cubic units.

Application : 12. If two spheres with the radii of 1 cm. and 6 cm. lengths are melted and a hollow sphere with the thickness of 1 cm is made, let us write by calculating the outer curved surface area of the hollow sphere.

Let the length of outer radius of sphere is r cm.

$\therefore$ The length of inner radius of that sphere is $= (r-1)$ cm.

By the condition $\frac{4}{3} \pi r^3 - \frac{4}{3} \pi (r-1)^3 = \frac{4}{3} \pi (1)^3 + \frac{4}{3} \pi (6)^3$

$$\text{or, } \frac{4}{3} \pi \{r^3 - (r-1)^3\} = \frac{4}{3} \pi (1+216)$$

$$\text{or, } r^3 - r^3 + 3r^2 - 3r + 1 = 217$$

$$\text{or, } 3r^2 - 3r - 216 = 0$$

$$\text{or, } r^2 - r - 72 = 0$$

$$\text{or, } r^2 - 9r + 8r - 72 = 0$$

$$\text{or, } r(r-9) + 8(r-9) = 0$$

$$\text{or, } (r-9)(r+8) = 0$$

Either, $r - 9 = 0 \therefore r = 9$

or, $r + 8 = 0 \therefore r = -8$

$\therefore$ Since $r \neq -8$, as r is the length of radius i.e it can not be negative. $\therefore r = 9$

$\therefore$ The length of outer radius of new hollow sphere is 9 cm.

$\therefore$ Outer surface area $= 4 \times \frac{22}{7} \times 9 \times 9$ sq cm. = sq cm. **[let us write it by calculating]**



Let us work out 12

1. If the length of radius of a sphere is 10.5 cm., let us write by calculating the whole surface area of the sphere.
2. If the cost of making a leather ball is ` 431.20 at ` 17.50 per square cm., let us write by calculating the length of diameter of the ball.
3. If the length of diameter of the ball used for playing shotput in our school is 7 cm. let us write by calculating how many cubic cm of iron is there in the ball.
4. If the length of diameter of a solid sphere is 28 cm and it is completely immersed into the water, let us calculate the volume of water displaced by the sphere.
5. The length of radius of spherical gas balloon increases from 7 cm to 21 cm as air is being pumped into it let us find the ratio of surface areas of the balloon in two cases.
6. $127\frac{2}{7}$ sq cm. of sheet is required to make a hemispherical bowl. Let us write by calculating the length of diameter of the forepart of the bowl.
7. The length of radius of solid spherical ball is 2.1 cm; let us write by calculating how much cubic cm iron is there and let us find the curved surface area of the iron ball.

8. The length of diameter of solid sphere of lead is 14 cm. If the sphere is melted, let us write by calculating how many spheres with length of 3.5 cm. radius can be made.
9. Three spheres made of copper having the lengths of 3 cm. 4 cm. and 5 cm. radii are melted and a large sphere is made. Let us write by calculating the length of radius of the large sphere.
10. The length of diameter of base of a hemispherical tomb is 42 dm. Let us write by calculating the cost of colouring the upper surface of the tomb at the rate of ₹ 35 per square metre.
11. Two hollow spheres with the lengths of diameter 21 cm and 17.5 cm respectively are made from the sheets of the same metal. Let us calculate the volumes of sheets of metal required to make the two spheres.
12. The curved surface of a solid metallic sphere is cut in such a way that the curved surface area of the new sphere is half of that previous one. Let us calculate the ratio of the volumes of the portion cut off and the remaining portion of the sphere.
13. On the curved surface of the axis of a globe with the length of 14 cm. radius, two circular holes are made each of which has the length of radius 0.7 cm. Let us calculate the area of metal sheet surrounding its curved surface.
14. Let us write by calculating how many marbles with lengths of 1 cm. radius may be formed by melting a solid sphere of iron having 8 cm length of radius.
15. **Very short answer type question (V.S.A.)**

(A) (M.C.Q.) :

- (i) The volume of a solid sphere having the radius of $2r$ units length is
(a) $\frac{32\pi^3}{3}$ cubic unit (b) $\frac{16\pi^3}{3}$ cubic unit (c) $\frac{8\pi^3}{3}$ cubic unit (d) $\frac{64\pi^3}{3}$ cubic unit
- (ii) If the ratio of the volumes of two solid spheres is 1:8, the ratio of their curved surface areas is
(a) 1:2 (b) 1:4 (c) 1:8 (d) 1:16
- (iii) The whole surface area of a solid hemisphere with length of 7 cm radius is
(a) 588π sq cm. (b) 392π sq cm. (c) 147π sq cm. (d) 98π sq cm.
- (iv) If the ratio of curved surface areas of two solid spheres is 16:9, the ratio of their volumes is
(a) 64:27 (b) 4:3 (c) 27:64 (d) 3:4
- (v) If numerical value of curved surface area of a solid sphere is three times of its volume the length of radius is
(a) 1 unit (b) 2 unit (c) 3 unit (d) 4 unit

(B) Let us write whether the following statements are true or false.

- (i) If we double the length of radius of a solid sphere, the volume of sphere will be doubled.

- (ii) If the ratio of curved surface areas of two hemispheres is 4:9, the ratio of their lengths of radii is 2:3

(C) Let us fill in the blanks.

- (i) The name of solid which is composed of only one surface is _____ .
 (ii) The number of surfaces of a solid hemisphere is _____ .
 (iii) If the length of radius of a solid hemisphere is $2r$ units, its whole surface area is _____ πr^2 sq units.

16. Short answer type (S. A.)

- (i) The numerical values of volume and whole surface area of a solid hemisphere are equal, let us write the length of radius of the hemisphere.
 (ii) The curved surface area of a solid sphere is equal to the surface area of a solid right circular cylinder. The lengths of both height and diameter of cylinder are 12 cm. Let us write the length of radius of the sphere.
 (iii) Whole surface area of a solid hemisphere is equal to the curved surface area of the solid sphere. Let us write the ratio of lengths of radius of hemisphere and sphere.
 (iv) If curved surface area of a solid sphere is S and volume is V , let us write the value of $\frac{S^3}{V^2}$ [not putting the value of π]
 (v) If the length of radius of a sphere is increased by 50%, let us write how much percent will be increased of its curved surface area.

13

VARIATION

Rahul and I decided to make a road map indicating the position of the school. Market, Hospital, Doctor's chamber, river, Co-operative society, big pond, cultivated land in our village and to paste it on a big board in front of our club room.

So we both have started work today at summer noon taking different measures of art paper-big and small, pen, pencil, scissors, gum etc.



1 But how shall we take measurement of length of road in the map?

We have shown that 8km length of road of our village is equivalent to 1 cm in the map. Then 24 km length of road of our village will be equivalent to 3cm.

2 Considering the length of road of our village = x cm and length of road in the map = y cm. in the table below; then let us write what I shall get.

		$\times 3$	$\times 5$	$\times 7$	$\times 3.5$	$\times 5.5$
Length of road of village (x) [cm.]	800000	2400000	4000000	5600000	2800000	4400000
Length of road in the map (y) [cm.]	1	3	5	7	3.5	5.5

We see that according to scale of map the length of road of village is as multiple of the first so as to. multiple of length of road in map.

$$\text{i.e. } \frac{800000}{1} = \frac{2400000}{3} = \frac{4000000}{5} = \dots\dots\dots = \frac{x}{y}$$

i.e. There is no change of value of $\frac{x}{y}$

Let, $\frac{x}{y} = k, [k \neq 0] \therefore x = ky, [\text{here } k \text{ is non zero constant}]$



3 If the variables x and y are related to each other in such a way that $\frac{x}{y} = k$ (non zero constant) what will the relation between two variables be called?

If two interrelated variables x and y are related in such a way that, $\frac{x}{y} = k$ (non zero constant), it is called that x and y are in direct variation and non zero constant is said to be variation constant.

x and y are in **Direct variation** and it can be written as $x \propto y$ and non zero constant is said to be **Variation Constant**.

The length of road and length of road in map are x and y respectively which are related in direct variation i.e. $x \propto y$.

We are trying to draw map of road on a square art paper with length of 60 cm.

4 If the square art paper having the sides of 60 cm. length is covered with a coloured paper, on its four sides, then let us write by calculating how much length of coloured paper is required.

Coloured paper is required = 4×60 cm of length = 240 cm in length.

If the length of one side of square art paper is 50 cm, then the coloured paper is required to cover the art paper of cm. length. [Let me do it yourself]

Considering length of square art paper = x cm and perimeter of art paper is y cm; let us fill up the table and let us see what the relation is between x and y .

Length of a side of square art paper (x) [cm]	60	50	100	120
Perimeter of art paper (y) [cm]	240	200	400	Let me write it myself

In this case we see that the length of square is x and perimeter is y of art paper They are so inter related with each other that $\frac{x}{y} = \frac{1}{4}$ (non zero constant)

i.e. x and y are in direct variation.

$\therefore x \propto y$ and here value of variation constant is $\frac{1}{4}$



5 Similarly let us write by understanding that the number of pen (x) and total cost price of pen (y) are in direct variation. [Let me do it myself]

6 Let us write four examples related to two variables, where the variables are in direct variations. [Let me do it myself]

A	90	30	12	15	156
B	60	20	8	10	104

We have got values of interrelated variables A and B:

Application : 1. Let us find the variation relation between A and B and let us write the value of variation constant.

We see that if value of A increases or decreases, the value of B increases or decrease.

Again, $\frac{A}{B} = \frac{90}{60} = \frac{30}{20} = \frac{12}{8} = \frac{15}{10} = \frac{156}{104} = \frac{3}{2} \therefore A = \frac{3}{2} B$

$\therefore A \propto B$ and here value of variation constant is $\frac{3}{2}$

Hence the value of variation constant is positive. So if the value of A increases or decreases then the corresponding value of B will increase or decrease.

We have got the values of related variables P and Q

P	35	49	56	14
Q	15	21	24	6

7 Let us find if there is any variation relation between P and Q, [Let me do it myself]



Application : 2. The time of oscillation of a pendulum varies as the square root of its length. If the pendulum with length of 1 metre oscillates once in a second, let us write by calculating the length of the pendulum oscillating once in 2.5 seconds.

Let, t = the time of oscillation, l = length of pendulum.

$\therefore$ According to the condition $t \propto \sqrt{l}$

$\therefore t = k\sqrt{l}$ [k is non zero variation constant]

$\therefore$ If $t = 1$ second, $l = 1$ metre.

So, we get $1 = k\sqrt{1} \therefore k = 1 \therefore$ Value of variation constant is 1

So, we get $t = \sqrt{l}$

$\therefore$ If $t = 2.5$ seconds, $\sqrt{l} = 2.5$

$$l = 6.25$$

$\therefore$ 6.25 metre length of pendulum oscillating once in 2.5 second.

Application : 3. y varies directly with the square of x , and $y = 9$ when $x = 9$; let us express y in the form of x and if $y = 4$, let us write by calculating the value of x .

y varies directly with the square of x

So, $y \propto x^2 \therefore y = kx^2$ [where K is non zero constant]

Again, $y = 9$ when $x = 9$

So, $9 = k(9)^2 \therefore k = \frac{9}{9^2} = \frac{1}{9}$

So, $y = \frac{1}{9}x^2$ _____ (I)

putting $y = 4$ in equation (I) we got $4 = \frac{1}{9}x^2$

$$\text{or, } x^2 = 36 \therefore x = \pm 6$$

$\therefore$ If $y = 4$ then $x = \pm 6$

Application : 4. y varies directly with the square root of x and $y = 9$ when $x = 4$, let us write the value of variation constant; and let us express y in the form of x and if $y = 8$, let us write the value of x .

[Let me do it myself]

When I and Rahul were busy in making the map of the road of the village, my grandma had sent some laddus, made of coconut for us, in a steel tiffin box.

8 By counting we see that there are 24 coconut laddus in the tiffin box. We shall divide them equally without breaking any laddu, let us find how many laddus we shall get each.

We shall get each $24 \div 2 = 12$ laddus.

But Shibani too has joined with us in this activity.

So, we shall divide 24 laddus among 3 persons. Now each will get laddus.

But it was expected that three friends would come in club room. If they will come we shall be 6 persons in total.

So, if 24 laddus are divided among 6 persons then each will get laddus.



Variation-constant is always non zero by definition. So instead of k is "non zero variation-constant" one may write k is "variation-constant"

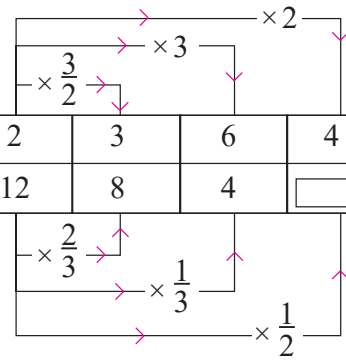


- 9 Let us write in the box below how many laddus each will get if 24 laddus sent by my grand mother are divided among various friends.

Number of friends (x)	2	3	6	4
The number of laddus (y) each would get	12	8	4	

∴ We see that the number of coconut laddus are unaltered, if the number of friends increases or decreases the number of laddus each gets will decrease or increase respectively. We see that $2 \times 12 = 3 \times 8 = 6 \times 4 = 4 \times 6 = x \times y$.

i.e. the value of $x \times y$ is unchanged. Let $xy = k$, (non zero constant)



- 10 If two interrelated variables x and y such that $xy = k$ (non zero constant), then what will be said the relation between two variables.

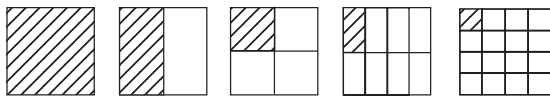
If two interrelated variables x and y such that $xy = k$ (non zero constant) then it is said x and y are in inverse variation. and written as $x \propto \frac{1}{y}$ and non zero constant is said to be **Variation Constant**.

Where $xy = k$ (non zero constant), and $k > 0$, then the value of one increases, corresponding value of the other decreases and the value of one decreases, corresponding value of the other increases.

If $xy = k$ (non zero constant), and $k < 0$, let us write what type of change of two variables will be.

The number of laddus and number of friends x and y respectively are in inverse variation.
i.e. $x \propto \frac{1}{y}$

- 11 My friend Sabina by folding many square art papers with length of 50 cm. side divided some of them equally. The remaining friends will draw on different parts of art paper and pasted on the board in the club room. I see the artpapers folded by Sabina and let us try to write the area of different parts in the table below.



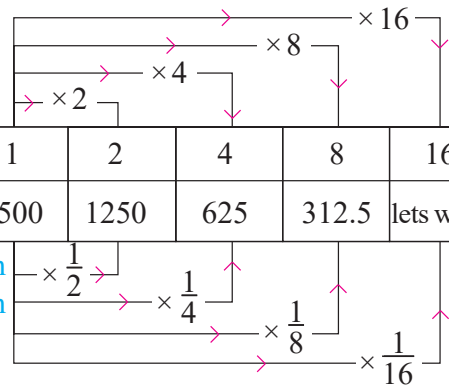
Part of square art paper (x)	1	2	4	8	16
Area of each folded part (y) [square cm]	2500	1250	625	312.5	lets write

We see that, if equal number (x) of part of paper with same measure of square increases, area (y) of each part will decrease.

$$\therefore 1 \times 2500 = 2 \times 1250 = 4 \times 625 = 8 \times 312.5 = 16 \times 156.25 = x \times y$$

i.e. there is no change of value of xy .

$$\therefore x \propto \frac{1}{y} \text{ and here value of variation constant is } 2500.$$



Application : 5. I arranged 36 buttons in a rectangular size and let us write by understanding in each case, if there is any inverse variation between buttons lying along length and those of breadth. [Let me do it myself]

Application : 6. Let us write two examples with inter related two variables where the variables are in inverse variation. **[Let me do it myself]**

Application : 7. We have got values of inter related variables x and y :

x	5	25	10	4	8
y	10	2	5	12.5	6.25

Let us determine if there is any variation relation between x and y and find the value of variation constant.

We see that $xy = 5 \times 10 = 25 \times 2 = 10 \times 5 = 4 \times 12.5 = 8 \times 6.25$

$\therefore xy = \text{Constant}$

$\therefore x \propto \frac{1}{y}$ and variation constant is 50.



Application : 8. The values of interrelated variables x and y are

x	3	2	6
y	18	27	9

Let us determine if there is any variation relation between x and y, **[Let me do it myself]**

Application : 9. In our factory 42 machines are required to produce a given number of iron articles in 63 days.

Let us calculate by applying the theory of variation how many machines would be required to produce the same number of iron articles in 54 days.

Let the number of days = D and number of machines = M

Volume of work remains constant, if the number of machine will increase, the number of days will decrease at the same rate.

$\therefore D$ and M are in inverse variation.

So, $D \propto \frac{1}{M} \therefore D = \frac{k}{M}$ [k = non zero variation constant]

If $D = 63$, $M = 42$

So, $63 = \frac{k}{42}$

or, $k = 63 \times 42$

$\therefore D = \frac{63 \times 42}{M}$ _____ (I)

$\therefore$ Putting $D = 54$ in equation (I)

$$54 = \frac{63 \times 42}{M}$$

$$\text{or, } M = \frac{63 \times 42}{54} \therefore M = 49$$

$\therefore$ 49 machines would be required to produces the same number of iron articles in 54 days.

Application : 10. Samir babu takes 2 hours to reach, the station from his house by driving a car at the speed of 60 km/hour. Let us calculate by applying the theory of variation, how much time will he take to reach the station from his house if he drives at the speed of 80 km/hour. **[Let me do it myself]**



[Answer Hints : Velocity and required time are in inverse variation to travel a fixed distance]

Application : 11. If $x \propto y$, then $y \propto x$ let us prove with reason.

Proof : $x \propto y \quad \therefore x = ky$ [k is non zero variation constant]

So, $y = \frac{1}{k}x = mx$, where $m = \frac{1}{k}$ is a non zero variation constant. $\therefore k$ is non zero variation constant.
 $\therefore y \propto x$

$\therefore$ So, we get if $x \propto y$, then $y \propto x$

Application : 12. If $x \propto y$ and $y \propto z$, let us prove that $x \propto z$.

Proof : $x \propto y \quad \therefore x = ky$ and $y \propto z \quad \text{so, } y = k'z$

Here, k and k' are two non zero variation constants.

Again, $x = ky = k(k'z) = kk'z = mz$ [Where $m = kk' = \text{non-zero constant}$] $\therefore x \propto z$

Application : 13. If $x \propto y$ let us prove that $x+y \propto x-y$

Proof : $x \propto y$ so, $x = ky$, where k is non zero variation constant.

$$\therefore \frac{x+y}{x-y} = \frac{ky+y}{ky-y} = \frac{y(k+1)}{y(k-1)} = \frac{k+1}{k-1} = n \quad [\text{where, } n = \text{non zero constant}]$$

$$\therefore x+y = n(x-y) \quad \therefore x+y \propto x-y$$

Application : 14. If $x \propto y$, let us prove that $x^n \propto y^n$

Proof : $x \propto y$ so, $x = ky$, where k is non zero variation constant.

$$x^n = k^n y^n = k_1 y^n \quad [k_1 = k^n = \text{non zero constant}] \quad \therefore x^n \propto y^n$$

Application : 15. If $x+y \propto x-y$, let us prove that $x \propto y$

Proof : $x+y \propto x-y$

$$\therefore x+y = k(x-y) \quad [\text{where } k \text{ is non zero variation constant}]$$

$$\frac{x+y}{x-y} = \frac{k}{1}$$

$$\text{or, } \frac{2x}{2y} = \frac{k+1}{k-1} \quad [\text{Applying componendo and dividendo}]$$

$$\text{or, } \frac{x}{y} = k_1 \quad [\text{where, } k_1 = \frac{k+1}{k-1} = \text{non zero constant}]$$

$$\text{or, } x = k_1 y \quad \therefore x \propto y$$

Application : 16. If $A^2 + B^2 \propto A^2 - B^2$, let us prove that $A \propto B$ [Let me do it myself]

Application : 17. If $x \propto y$ and $u \propto z$, let us prove that $xu \propto yz$ and $\frac{x}{u} \propto \frac{y}{z}$

Proof : $x \propto y$, so, $x = k_1 y$

$u \propto z$, so, $u = k_2 z$ [where k_1 and k_2 are non zero variation constants]

$$\therefore xu = k_1 y \cdot k_2 z = k_1 k_2 yz = myz \quad [\text{where } m = k_1 k_2 = \text{non zero constant}]$$

$$\therefore xu \propto yz$$

$$\frac{x}{u} = \frac{k_1 y}{k_2 z} = \left(\frac{k_1}{k_2} \right) \frac{y}{z} = n \frac{y}{z} \quad [\text{where } n = \frac{k_1}{k_2} = \text{non zero constant}]$$

$$\therefore \frac{x}{u} \propto \frac{y}{z}$$



Our friend Arko has made some triangular regions by cutting paper.

We measure length of base and height of triangular region and let us write by calculating their areas in the table given below.

Base of triangle (x) [cm]	8	14	13	22
Height of triangle (y) [cm]	10	7	15	12
Area of triangle (A) [sq cm]	$\frac{1}{2} \times 8 \times 10$	$\frac{1}{2} \times 14 \times 7$	$\frac{1}{2} \times 13 \times 15$	$\frac{1}{2} \times 22 \times 12$

We got $\frac{A}{x \times y} = \frac{\frac{1}{2} \times 8 \times 10}{8 \times 10} = \frac{\frac{1}{2} \times 14 \times 7}{14 \times 7} = \frac{\frac{1}{2} \times 13 \times 15}{13 \times 15} = \frac{\frac{1}{2} \times 22 \times 12}{22 \times 12} = \frac{1}{2}$

$\therefore A \propto x \times y$



12 Here we see that a variable A is in direct variation with the product of x and y. What this type of variation is called?

If a variable is in direct variation with the product of two or more variables, the first variable is said to be in **Joint Variation** with the other variables.

13 **Theorem on joint variation** (only statement)

If three variables x, y and z be such that $x \propto y$ when z is constant, $x \propto z$ when y is constant, then $x \propto yz$, when y and z both vary.

We understand that here area A of triangle is in joint variation with base (x) and height (y) of triangle.

In case of ideal gas, we get from the combination of Boyle's law and Charle's law,

$PV = RT$, R is a non zero constant.

$\therefore V = R \frac{T}{P}$

14 Here, is V in joint variation with T and $\frac{1}{P}$?

$V = R \cdot \frac{T}{P}$ [R= constant] in this relation we can say V is in joint variation with T and $\frac{1}{P}$.

Let us write by understanding in case of any work, total earning bears a joint variation with number of persons engaged in that work and number of days they worked. **[Let me do it myself]**

Application : 18. If 5 farmers can harvest jute, cultivated in 10 bighas of land in 12 days. Let us find by applying theory of variation how many farmers can harvest jute, cultivated in 18 bighas in 9 days.

Let numbers of farmers be A,

numbers of days be B

and area of land be C

Then numbers of farmers is in direct variation with area of land, when number of days remain constant.

$\therefore A \propto C$, when B is constant.

Again, if area of land remains constant, then the number of farmers is in the inverse variation with number of days. i.e. $\therefore A \propto \frac{1}{B}$, when C is constant.



∴ According to theorem on joint variation, $A \propto \frac{C}{B}$, when B and C both vary.

i.e $A = K \frac{C}{B}$ [where, K is non zero constant] _____ (I)

Given $A = 5$, $B = 12$ and $C = 10$

From (I) we get $5 = K \frac{10}{12}$

$$\text{or, } K = \frac{5 \times 12}{10} \quad \therefore K = 6$$

Now, putting the value of K in the equation (I) we get $A = 6 \frac{C}{B}$ _____ (II)

If $B = 9$ and $C = 18$ then, we get from (II), $A = \frac{6 \times 18}{9} = 12$

∴ Required number of farmers = 12.



Application : 19. If 5 men can cultivate in 10 bighas of land in 9 days, let us calculate by theory of variation how long will be taken by 25 men for cultivating 10 bighas of land. [Let me do it myself]

Application : 20. x varies directly with y (when z is constant) and x varies inversely with z (when y is constant) in such a way that when $y = 4$, $z = 5$, then $x = 3$. when $y = 16$, $z = 25$, then let us find the value of x.

$x \propto y$, where z is constant

$x \propto \frac{1}{z}$, where y is constant

∴ $x \propto \frac{y}{z}$, where y and z both vary.

So, $x = k \cdot \frac{y}{z}$ [where k is non zero variation constant] _____ (I)

Given, $x = 3$, $y = 4$ and $z = 5$

∴ We get from (I) $3 = k \frac{4}{5} \quad \therefore k = \frac{15}{4}$

Putting the value of K in equation No (I) we get $x = \frac{15y}{4z}$ _____ (II)

When $y = 16$, $z = 25$ then from (II) we get $x = \frac{15 \times 16}{4 \times 25} \quad \therefore x = \frac{12}{5} = 2\frac{2}{5}$

Application : 21. If $\frac{x}{y} \propto x+y$ and $\frac{y}{x} \propto x-y$ let us show that $x^2 - y^2 = \text{constant}$.

$$\frac{x}{y} \propto x+y \quad \therefore \frac{x}{y} = k_1(x+y) \text{ [where } k_1 \text{ is non zero variation constant]}$$

$$\text{Again, } \frac{y}{x} \propto x-y \quad \therefore \frac{y}{x} = k_2(x-y) \text{ [where } k_2 \text{ is non zero variation constant]}$$

$$\text{So, } \frac{x}{y} \times \frac{y}{x} = k_1(x+y) \times k_2(x-y)$$

$$\text{or, } 1 = k_1 k_2 (x^2 - y^2)$$

$$\text{or, } (x^2 - y^2) = \frac{1}{k_1 k_2}$$

$$\therefore x^2 - y^2 = \text{constant} \quad (\because k_1 \text{ and } k_2 \text{ constant, } \therefore \frac{1}{k_1 k_2} \text{ constant}) \quad \text{[Proved]}$$



Application : 22. $x^2 \propto yz$, $y^2 \propto zx$ and $z^2 \propto xy$ let us show that the product of the three constants of variation = 1

$$x^2 \propto yz \quad \therefore x^2 = k_1 yz$$

$$\text{Again, } y^2 \propto zx \quad \therefore y^2 = k_2 zx$$

$$\text{Again, } z^2 \propto xy \quad \therefore z^2 = k_3 xy$$

Where, k_1 , k_2 and k_3 are non zero variation constant.

$$\text{So, } x^2 \times y^2 \times z^2 = k_1 yz \times k_2 zx \times k_3 xy$$

$$\text{or, } x^2 y^2 z^2 = k_1 k_2 k_3 \cdot x^2 y^2 z^2$$

$$\therefore k_1 k_2 k_3 = 1 \quad \text{[Proved]}$$

Application : 23. $a \propto b$ and $b \propto c$ let us prove that $a^3 + b^3 + c^3 \propto 3abc$

$$a \propto b \quad \therefore a = k_1 b \quad [\text{Where } k_1 \text{ is a non zero variation constant}]$$

$$\text{Again, } b \propto c \quad \therefore b = k_2 c \quad [\text{Where } k_2 \text{ is a non zero variation constant}]$$

$$\text{So, } a = k_1 b = k_1 k_2 c$$

$$\frac{a^3 + b^3 + c^3}{3abc} = \frac{(k_1 k_2 c)^3 + (k_2 c)^3 + c^3}{3(k_1 k_2 c) \times (k_2 c) \times c} = \frac{c^3 (k_1^3 k_2^3 + k_2^3 + 1)}{3k_1 k_2^2 c^3} = \frac{k_1^3 k_2^3 + k_2^3 + 1}{3k_1 k_2^2} = \text{non zero constant.}$$

[$\because k_1$ and k_2 are non zero constants]

$$\therefore a^3 + b^3 + c^3 \propto 3abc$$

Application : 24. If $x \propto y$ and $y \propto z$, let us prove that $x^2 + y^2 + z^2 \propto xy + yz + zx$ [Let me do it myself]

Application : 25. The volume of any sphere varies directly as cube of its radius. If three solid spheres with radius of 3 cm, 4 cm and 5 cm are melted and a new sphere is made, and no volume is lost for melting, then in let us calculate by theory of variation the length of diameter of new sphere.

Let the volume of a sphere of radius 'r' cm be 'v' cubic cm.

$$\text{So, } v \propto r^3$$

$$\therefore v = kr^3 \quad [\text{here } k \text{ is non zero variation constant}]$$

$$\therefore \text{Volume of a sphere with radius of 3 cm is } = k \times 3^3 \text{ cubic cm.} = 27k \text{ cubic cm.}$$

$$\text{Volume of a sphere with radius of 4 cm is } = k \times 4^3 \text{ cubic cm.} = 64k \text{ cubic cm.}$$

$$\text{Volume of a sphere with radius of 5 cm is } = k \times 5^3 \text{ cubic cm} = 125k \text{ cubic cm.}$$

According to the question, volume of new sphere is $(27k + 64k + 125k)$ cubic cm = $216k$ cubic cm.

If the radius of the new sphere is R. then

$$k R^3 = 216k$$

$$\text{or, } R^3 = 216$$

$$\text{or, } R^3 = 6^3 \quad \therefore R = 6$$

$\therefore$ The length of radius of new sphere is 6 cm

$\therefore$ The length of diameter of new sphere is 6×2 cm = 12 cm



Application : 26. The total expenses of a hostel are partly constant and partly vary directly as the number of boarders. When the number of boarders are 120 and 100 the total expenses are ₹ 2000 and ₹ 1700 respectively. Let us write by calculating, total number of boarders if the total expense is ₹ 1880.

Let the expenses and number of boarders be ₹ x and y respectively.

Let $x = k_1 + B$, where expenses of hostel k_1 is partly constant and $B \propto y$

$\therefore B = k_2 y$, where k_2 is non zero variation constant.

By the condition $x = k_1 + k_2 y$

Given $y = 120, x = 2000 \quad \therefore 2000 = k_1 + 120k_2$ _____ (I)

Again, if $y = 100, x = 1700$ (Given) $\therefore 1700 = k_1 + 100k_2$ _____ (II)

(Subtracting) $300 = 20k_2$
or, $k_2 = \frac{300}{20} \therefore k_2 = 15$

$\therefore$ From (II) we get, $1700 = k_1 + 100 \times 15$

or, $k_1 = 1700 - 1500$

$\therefore k_1 = 200$

So, $x = 200 + 15y$ _____ (III)

$\therefore$ When $x = 1880$ then we get from equation (III)

$1880 = 200 + 15y$

or, $15y = 1880 - 200$

or, $15y = 1680$

or, $y = \frac{1680}{15} \therefore y = 112$

$\therefore$ If expenses of hostel is ₹ 1880 then the number of boarders is 112.

Application : 27. The volume of a sphere varies directly with the cube of length of its radius. The length of diameter of a solid sphere of lead is 14 cm. If the sphere is melted, let us find by applying theory of variation how many spheres having the length of 3.5 cm. radius can be made. (let the volume remain unchanged before or after melting.) **[Let me do it myself]**



Let us work out

13

1. Corresponding values of two variables A and B are :

A	25	30	45	250
B	10	12	18	100

If there is any relation of variation between A and B, let us determine it and write the value of variation constant.

2. Corresponding values of two variables x and y

x	18	8	12	6
y	3	$\frac{27}{4}$	$\frac{9}{2}$	9

If there is any relation of variation between x and y, let us write it by understanding.

3. (i) A taxi of Bipin uncle travels 14 km path in 25 minute. Let us calculate by applying theory of variation how much path he will go in 5 hours by driving taxi with same speed.
 (ii) A box of sweets is divided among 24 children of class one of our school, they will get 5 sweets each. Let us calculate by applying theory of variation how many sweets would each get, if the number of the children is reduced by 4.
 (iii) 50 villagers had taken 18 days to dig a pond. Let us calculate by using theory of variation how many extra persons will be required to dig the pond in 15 days.
4. (i) y varies directly with square root of x and $y = 9$ when $x = 9$. Let us find the value of x when $y = 6$.
 (ii) x varies directly with y and inversely with z . When $y = 4$, $z = 5$, then $x = 3$. Again, if $y = 16$, $z = 30$, let us write by calculating the value of x .
 (iii) x varies directly with y and inversely with z . When $y = 5$, $z = 9$ then $x = \frac{1}{6}$. Let us find the relation among three variables x , y and z and if $y = 6$ and $z = \frac{1}{5}$, let us write by calculating the value of x .
5. (i) If $x \propto y$, let us show that $x + y \propto x - y$
 (ii) $A \propto \frac{1}{C}$, $C \propto \frac{1}{B}$ let us show that $A \propto B$
 (iii) If $a \propto b$, $b \propto \frac{1}{c}$ and $c \propto d$, let us write the relation of variation between a and d .
 (iv) If $x \propto y$, $y \propto z$ and $z \propto x$ Let us find the relation among three constants of variation.
6. If $x + y \propto x - y$, let us show that
 (i) $x^2 + y^2 \propto xy$
 (ii) $x^3 + y^3 \propto x^3 - y^3$
 (iii) $ax + by \propto px + qy$ [where a, b, p, q are non zero constant]
7. (i) If $a^2 + b^2 \propto ab$, let us prove that $a + b \propto a - b$
 (ii) If $x^3 + y^3 \propto x^3 - y^3$, let us prove that $x + y \propto x - y$
8. If 15 farmers can cultivate 18 bighas of land in 5 days, let us determine by using the theory of variation the number of days required by 10 farmers to cultivate 12 bighas of land.
9. Volume of a sphere varies directly with the cube of its radius. Three solid spheres having length of $1\frac{1}{2}$, 2 and $2\frac{1}{2}$ metre diameter are melted and a new solid sphere is formed. Let us find the length of diameter of the new sphere. [let us consider that the volume of sphere remains same before and after melting]
10. y is a sum of two variables, one of which varies directly with x and another varies inversely with x . When $x = -1$, then $y = 1$ and when $x = 3$, then $y = 5$. Let us find the relation between x and y .
11. If $a \propto b$, $b \propto c$ let us show that $a^3b^3 + b^3c^3 + c^3a^3 \propto abc(a^3 + b^3 + c^3)$
12. To dig a well of x dcm deep. one part of the total expenses varies directly with x and other part varies directly with x^2 . If the expenses of digging wells of 100 dcm and 200 dcm depths are ₹ 5000 and ₹ 12000 respectively, let us write by calculating the expenses of digging a well of 250 dcm depth

13. Volume of a cylinder is in joint variation with square of the length of radius of base and its height. Ratio of radii of bases of two cylinders is 2 : 3 and ratio of their heights is 5 : 4, let us find the ratio of their volumes.
14. An agricultural Co-operative Society of village of Pachla has purchased a tractor. Previously 2400 bighas of land were cultivated by 25 ploughs in 36 days. Now half of the land can be cultivated only by that tractor in 30 days. Let us calculate by using the theory of variation, the number of ploughs work equally with one tractor.
15. The Volume of a sphere varies directly with cube of length of its radius and surface area of sphere varies directly with the square of the length of radius. Let us prove that the square of volume of sphere varies directly with cube of its surface area.

16. **Very short answer type question (V.S.A.)**

(A) M.C.Q.

- (i) $x \propto \frac{1}{y}$, then (a) $x = \frac{1}{y}$ (b) $y = \frac{1}{x}$ (c) $xy = 1$ (d) $xy = \text{non-zero constant}$.
- (ii) If $x \propto y$, then (a) $x^2 \propto y^3$ (b) $x^3 \propto y^2$ (c) $x \propto y^3$ (d) $x^2 \propto y^2$
- (iii) If $x \propto y$ and $y = 8$ when $x = 2$; if $y = 16$, then the value of x is (a) 2 (b) 4 (c) 6 (d) 8
- (iv) If $x \propto y^2$ and $y = 4$ when $x = 8$; if $x = 32$, then the value of y is (a) 4 (b) 8 (c) 16 (d) 32
- (v) If $y - z \propto \frac{1}{x}$, $z - x \propto \frac{1}{y}$ and $x - y \propto \frac{1}{z}$, sum of three variation constants is (a) 0 (b) 1 (c) -1 (d) 2

(B) Let us write whether the following statements are true or false :-

- (i) If $y \propto \frac{1}{x}$, $\frac{y}{x} = \text{non-zero constant}$.
- (ii) If $x \propto z$ and $y \propto z$ then $xy \propto z^2$

(C) Let us fill in the blanks :

- (i) If $x \propto \frac{1}{y}$ and $y \propto \frac{1}{z}$, then $x \propto$ _____
- (ii) If $x \propto y$, $x^n \propto$ _____
- (iii) If $x \propto y$ and $x \propto z$, then $(y + z) \propto$ _____

17. **Short answer type question (S.A.)**

- (i) If $x \propto y^2$ and $y = 2a$ when $x = a$; x and y let us find the relation between x and y .
- (ii) If $x \propto y$, $y \propto z$ and $z \propto x$, let us find the product of three non zero constants.
- (iii) If $x \propto \frac{1}{y}$ and $y \propto \frac{1}{z}$, let us find if there be any relation of direct or inverse variation between x and z .
- (iv) If $x \propto yz$ and $y \propto zx$, let us show that z is a non zero constant.
- (v) If $b \propto a^3$ and a increases in the ratio of 2 : 3, let us find in what ratio b will be increased.

14

PARTNERSHIP BUSINESS

My elder sister wants to run a business of selling exercise copies binding them in a room of the ground floor of our house. Two friends of my elder sister, Tirtha and Tanbir, want to join this business. So my elder sister, Tirtha and Tanbir have started the business of selling exercise copies by investing `10000, `12000, `11000 respectively.



1 What is called this type of business when two or more persons jointly agree to run a business? When two or more persons together agree to form some organisations for running jointly a business the partnership business is one of them. In partnership business money invested by each partner is the capital of their own share.



They make a profit of `9900 at the end of the year in the business of my elder sister. But how will they divide this profit.

2 Partnership business generally builds up on some principles and the principles are decided on the basis of discussion among the partners. These are —

- (i) **Capital** : The business partners collect the total amount of capital by investing equally or collect the capital at a unanimous ratio.
- (ii) Distribution of profit are shared according to a joint decision.
 - (a) May be equally divided among the partners.
 - (b) May be shared on the basis of ratio of their investment.
 - (c) May be shared on the basis of another unanimous agreement. If the terms and conditions of the method of profit/loss are not clearly mentioned in the agreement, it is assumed that the profit/loss so distributed among the partners on the basis of the ratio of their investment.
- (iii) **Honorary allowance to manage a business** : If a few or all the partners invest their energy and time to run a business then the rate of their honorary allowance is clearly mentioned in the contract. The profit is shared only after the disbursement of these allowances.

Application 1 : My elder sister and her two friends have started a partnership business and at first the ratio of their investment for sharing profit which they will get is to be determined.

∴ The ratio of disbursement of profit will be,

money of elder sister : money of Tirtha : money of Tanbir = 10000 : 11000 : 12000 = 10 : 11 : 12

∴ In the profit of `9900, my elder sister will get =  $\frac{10}{10+11+12} \times 9900 = \text{` } \boxed{}$

In the profit of `9900, Tirtha will get =  $\frac{11}{33} \times 9900 = \text{` } 3300$

In the profit of `9900, Tanbir will get =  $\frac{12}{33} \times 9900 = \text{` } 3600$



Application 2 : Sulekha, Joynal and Shibu started a partnership business with capitals ` 5000, ` 4500 and ` 7000 respectively. If the profit is ` 11550, at the end of the year, let us write by calculating the profit share by each of them. [Let me do it myself]



Application 3 : Mitadidi, Sahanabibi and Amal uncle of our village started a business of jam-jelly with capitals of ` 15000, ` 10000 and ` 17500. But at the end of the year loss is ` 4250. Let us write by calculating how much more each of them will have to pay.

In partnership business the ratio of investment of Mitadidi, Sahanabibi and Amal uncle = 15000 : 10000 : 17500 = 6 : 4 : 7

Each partner shared their part of loss in the ratio of their investment.

∴ In the loss of ` 4250, Mitadidi will pay = ` $\left(\frac{6}{6+4+7} \times 4250\right)$ = `

In the loss of ` 4250, Sahanabibi will pay = ` $\left(\frac{4}{17} \times 4250\right)$ = `

In the loss of ` 4250, Amal uncle will pay = ` [Let me do it myself]

Application 4 : Mariya and Shayan started a partnership business with capital of ` 25000 and 35000 respectively with the conditions that

(i) $\frac{1}{3}$ rd of the total profit at first will be equally divided among them (ii) Later, the remaining profit will be divided in the ratio of their capitals.



If the profit at the end of the year is ` 36000. Let us find the share of profit each will get.

(i) $\frac{1}{3}$ of total profit will be equally divided i.e. `  $\left(36000 \times \frac{1}{3}\right)$  = ` will be divided equally among Mariya and Shyan.

(ii) Remaining `  $(36000 - 12000)$  = ` 24000 will be divided in the ratio of their capitals.

Ratio of capitals of Mariya and Shayan = 25000 : 35000 = 5 : 7

∴ In the profit of ` 24000 Mariya will get = ` $\left(\frac{5}{5+7} \times 24000\right)$ = `

In the profit of ` 24000 Shayan will get = ` $\left(\frac{7}{5+7} \times 24000\right)$ = `

∴ Mariya gets total = `  $(12000 \div 2 + 10000)$  = `

Shayan gets total = `  $(12000 \div 2 + 14000)$  = `

∴ According to agreement Mariya will get ` 16000 and Shayan will get ` 20000 of the profit of ` 36000.



Application 5 : Three retired persons invested ` 19500, ` 27300 and ` 15600 respectively to set up a leather factory and after a year they had a profit of ` 43200. If they will divided $\frac{2}{3}$ of this profit equally among themselves and of the remaining in the ratio of their capitals. Let us find the share of each one of them. [Let me do it myself]

Application 6 : Abhra, Tanbir, Amrita and Tathagata started a partnership business together with capitals of ` 15000, ` 21000, ` 30000 and ` 45000 respectively with the condition that



- (i) Abhra and Tanbir each partner will manage the business for a period of 6 months and each will get a share of 0.25 of the profit by dividing equally.
 - (ii) The remaining profit is to be divided among four of them in the ratio of their capitals. If the annual profit is ` 27232, let us write the share of each.
- (i) $(0.25 \text{ part of } 27232) = ` \boxed{}$ will be equally divided among Abhra and Tanbir.
 - (ii) $(27232 - 6808) = ` \boxed{}$ will be divided among them in the ratio of their capitals.

Ratio of capitals of Abhra, Tanbir, Amrita and Tathagata = 15000 : 21000 : 30000 : 45000
= 5 : 7 : 10 : 15

∴ Remaining part of profit of ` 20424 will be divided among them in the ratio of their capitals.

$$\text{Abhra will get} = \frac{5}{5+7+10+15} \times 20424 = ` \boxed{}$$

Similarly in ` 20424, let us write by calculating how much money will Tanbir, Amrita and Tathagata get.

$$\therefore \text{Abhra will get total} = (6808 \div 2 + \frac{5}{37} \times 20424) = ` \boxed{}$$

$$\text{Tanbir will get total} = (6808 \div 2 + \frac{7}{37} \times 20424) = ` \boxed{}$$

Amrita will get ` 5520 and Tathagata will get ` 8280.

Application 7 : Jaya aunty has started a small business of selling handicrafts with a capital of ` 10000. After 6 months Sulekha didi joined the business of Joya aunty with capitals of ` 14000. They make a profit of ` 5100. Let us write by calculating how much profit they will get.

In this case here we see that the capitals of different partners were not invested for equal period of time to run the business. So, in this case how shall we find their ratio of capitals?



3 This type of partnership business is called mixed partnership business.

There are two types of partnership business (1) Simple (2) Compound.

Simple : If the capitals of all the partners are invested for the same time, then such business is called a simple partnership business.

Compound : If the capitals of partners are invested for different time period, then such a business is called a compound partnership business.

Here, at first calculate the equivalent capital (with respect to time) of each partner.

But we see that in compound partnership business how shall we get equivalent capital with respect to time.

At first the time for which the capital invested in this business of Joya aunty and Sulekha didi will be settled for equal period.

Let Joya aunty sells object of ` 10000 in 1 month and gets profit of ` x

She sells object of ` 10000 in 2 months and get profit of ` 2x.

∴ She sells object of ` 10000 in 12 months and get profit of ` 12x

Jaya aunty has to make a profit equals to ` 12x in a month she should have to invest  $12 \times 10000 = 120000`$

Again similarly if in one month, Sulekha didi has to make a profit equals to that she earns the profit investing ` 14000, for 6 months, she should have invested  $(6 \times 14000) = ` 84000$.

We should get the equivalent capitals of Jaya aunty and Sulekha didi with respect to 1 month having taken equal time in both cases.

4 In compound partnership business the time for which the capital is invested must be taken for equal period i.e. we should therefore calculate the equivalent capitals in respect of 1 month. So, in such cases, the individual capital is multiplied by the numerical value of the time of investment and the profit is distributed according to the ratio of these products.

∴ The ratio of capitals of Jaya aunty and Sulekhadidi = 120000 : 84000 = 10 : 7

The profit is divided in the ratio of capitals. i.e. profit is divided in the ratio 10 : 7

∴ In profit of ₹ 5100 Jaya aunty will get = ₹ $\left(\frac{10}{10+7} \times 5100\right)$ = ₹

In profit of ₹ 5100 Sulekhadidi will get = ₹ $\left(\frac{7}{10+7} \times 5100\right)$ = ₹

Application 8 : Manisha has started a business with capital of ₹ 3750. After 6 months Rajat joined the business with a capital of ₹ 15000. If at the end of the year there was a loss of ₹ 6900. Let us write by calculating the money each must pay to make up the loss. [Let me do it myself]



Application 9 : Aminabibi, Ramenbabu and Ishita aunty of our village started a partnership business on first January of last year with capitals of ₹ 50000, ₹ 60000 and ₹ 70000 respectively. On first April, Ramenbabu invested ₹ 10000 more money but on 1st June Ishita aunty, withdrew ₹ 10000. If the total profit upto 31 December was ₹ 39240, Let us write by calculating the profit share of each one of them on the basis of the ratio of their capitals.

Aminabibi has invested ₹ 50000 for 12 months, if in a month she has to make a profit equals to that she gained in 12 months on that amount, she should invest ₹ (50000×12) i.e. ₹ 60000.

Similarly, Ramenbabu has invested ₹ 60000 for 3 months and ₹ $(60000 + 10000)$ i.e. 70000 for 9 months. Thus his investment on a monthly scale should be ₹ $\{(60000 \times 3) + (70000 \times 9)\}$ = ₹ 810000

Again Ishita aunty has invested ₹ 70000 for 5 months and ₹ $(70000 - 10000)$ = ₹ 60000 for 7 months.

Thus her investment on a monthly scale should be = ₹ $\{(70000 \times 5) + (60000 \times 7)\}$ = ₹ 770000

The ratio of capitals of Aminabibi, Ramenbabu and Ishita aunty

$$= 600000 : 810000 : 770000 = 60 : 81 : 77$$

∴ The proportional part of share of Aminabibi, Ramenbabu and Ishita aunty in the ratio of their capitals are $\frac{60}{218}$, $\frac{81}{218}$ and $\frac{77}{218}$ respectively.

Profit share is divided in the ratio of their capitals.

∴ In the profit of ₹ 39240, Aminabibi will get = ₹ $\left(\frac{60}{218} \times 39240\right)$ = ₹ 10800

In the profit of ₹ 39240, Ramenbabu will get = ₹ [Let me do it myself]

In the profit of ₹ 39240, Ishita aunty will get = ₹ [Let me do it myself]



Application 10 : Nivedita and Uma have started a business with capitals ₹ 3000 and ₹ 5000 respectively. After 6 months Nivedita invested ₹ 4000 more but after 6 months Uma withdrew ₹ 1000. If the profit at the end of the year is ₹ 6175, let us write by calculating the profit share of each of them. [Let me do it myself]

Application 11 : Sabba, Dipak and Pritha have started a business together with capitals of ₹ 6000, ₹ 5000 and ₹ 9000. After a few months Sabba has invested ₹ 3000 more. The profit is ₹ 3000 at the end of year and Pritha has got profit share of ₹ 1080. Let us find the period for which Sabba's capital of ₹ 3000 has been invested.

Let Sabba has invested ₹ 3000 more after x months

∴ Sabba has invested ₹ 6000 for x months and ₹ $(6000 + 3000) = ₹ 9000$ has been invested in the business for remaining $(12 - x)$ months.

If Sabba would want to earn the profit in one month that she would earn at the end of the year, Sabba should require to invest

$$\begin{aligned} & ₹ \{6000 \times x + 9000 \times (12 - x)\} \\ & = ₹ (108000 - 3000x) \end{aligned}$$



Again, Dipak and Pritha invested ₹ 8000 and ₹ 9000 respectively for 12 months.

∴ Thus to earn in one month the profit equals to the amount they would get as profit at the end of the year Dipak should require to invest ₹ 8000×12 and Pritha ₹ 6000×12

∴ The ratio of capitals of Sabba, Dipak and Pritha

$$\begin{aligned} & = (108000 - 3000x) : 8000 \times 12 : 9000 \times 12 \\ & = (108 - 3x) : 96 : 108 \\ & = (36 - x) : 32 : 36 \end{aligned}$$

Profit share will be divided in the ratio of their capitals.

$$\therefore \text{Profit share of Pritha} = ₹ 3000 \times \frac{36}{36 - x + 32 + 36} = ₹ \frac{3000 \times 36}{104 - x}$$

By the condition,

$$\frac{3000 \times 36}{104 - x} = 1080$$

$$\text{or, } 104 - x = \frac{3000 \times 36}{1080} = 100 \quad \therefore x = 4$$

∴ Sabba had invested ₹ 3000 after 4 months in the business.

Let us see by calculating

14

1. I and my friend Mala together have started a business with capitals of ₹ 15000 and ₹ 25000 respectively. If we make a profit of ₹ 16,800 in a year, let us see that the profit share shall we each get?
2. Priyam, Supriya and Bulu have opened a small shop of grocery shop with capitals of ₹ 15000, ₹ 10000 and ₹ 25000 respectively. But after a year there was a loss of ₹ 3000. Let us write by calculating what each must pay to make up the loss.

3. Shobha and Masud together bought a car for ₹ 250,000 and sold it for ₹ 262,500. If Sobha paid $1\frac{1}{2}$ times more than Masud, let us write by calculating their shares of profit.
4. Three friends started a partnership business by investing ₹ 5,000, ₹ 6,000 and ₹ 7,000 respectively. After running the business for one year, they found that there is a loss of ₹ 1,800. They decided to pay to make up that loss to undisturb their capitals. Let us write by calculating the amount they have to pay.
5. Dipu, Rabeya and Megha started a small business by investing the capitals of ₹ 6,500, ₹ 5,200, ₹ 9,100 respectively and just after one year they make a profit of ₹ 14,400. If they had divided $\frac{2}{3}$ rd of the profit equally among themselves and the remaining in the ratio of their capitals, let us find the profit share of each.
6. Three friends started a business by investing ₹ 8,000, ₹ 10,000 and ₹ 12,000 respectively. They also took an amount as bank loan. At the end of year they made a profit of ₹ 13,400. After paying the annual bank instalment of ₹ 5,000 they divided the remaining profit among themselves in the ratio of their capitals. let us write by calculating the profit share of each.
7. Three friends took loans of ₹ 6,000, ₹ 8,000 and ₹ 5,000 respectively from a co-operative bank on the condition that they would not have to pay interest, if they would repay their loan within two years. They invested the money to purchase 4 cycle rickshaws. After two years they made a profit of ₹ 30,400 excluding all the expenses. They divide the profit among themselves in the ratio of their capitals and paid back their individual loan amount to the bank. Let us write by calculating the amount of their individual share and the ratio of their shares.
8. Three friends invested ₹ 12,000, ₹ 15,000 and ₹ 11,000 respectively to purchase a bus. The first person is a driver and the other two are conductors. They decided to divide $\frac{2}{5}$ th of the profit among themselves in the ratio of 3 : 2 : 2 according to their work and the remaining in the ratio of their capitals. If they earn ₹ 29,260 in one month, let us find the share of each of them.
9. Pradipbabu and Aminabibi started a business by investing ₹ 24,000 and ₹ 30,000 respectively at the beginning of a year. After 5 months Pradipbabu invested ₹ 4,000 more. If the yearly profit was ₹ 27,716, let us write by calculating the share of each of them.
10. Niyamat chacha and Karabi didi have started a partnership business together by investing ₹ 30,000 and ₹ 50,000 respectively. After 6 months Niyamat chacha has invested ₹ 40,000 more but Karabi didi withdrew ₹ 10,000 for personal need. If the profit at the end of the year is ₹ 19,000, let us write by calculating the profit share of each of them.
11. Srikant and Soiffuddin invested ₹ 24,000 and ₹ 30,000 respectively at the beginning of the year to purchase a mini bus to run it on a route. After 4 months, their friend Peter joined them with a capital of ₹ 8,100. Shrikant and Soiffuddin have withdrawn that money in the ratio of their capitals. Let us write by calculating the share of each if they make a profit of ₹ 39,150 at the end of the year.

12. Arun and Ajoy started a business jointly by investing ₹ 24000 and ₹ 30000 respectively at the beginning of the year. But after a few months Arun invested ₹ 12000 more. After a year, the profit was ₹ 14030 and Arun received the profit share of ₹ 7130. Let us find after how many months did Arun invest money in that business.
13. Three clay modellers from Kumartuli collectively took a loan of ₹ 100000 from a co-operative bank to set up a modelling workshop. They made a contract that after paying back the annual bank instalment of ₹ 28100, they would divide half of the profit among themselves in terms of the number of working days and the other half will be equally divided among them. Last year they each worked 300 days, 275 days and 350 days respectively and made a profit of ₹ 139100. Let us write by calculating the share of each in this profit.
14. Two friends invested ₹ 40000 and ₹ 50000 respectively to start a business. They made a contract that they would divide 50% of the profit equally among themselves and the remaining profit in the ratio of their capitals. Let us write the share of profit of the first friend if it is ₹ 800 less than that of the 2nd friend.
15. Puja, Uttam and Meher started a partnership business with capitals of ₹ 5000, ₹ 7000 and ₹ 10000 respectively with the conditions that
(i) Monthly expense for running business is ₹ 125 (ii) Puja and Uttam each will get ₹ 200 for keeping the accounts. If the profit is ₹ 6960 at the end of the year, let us write by calculating the profit share each would get,.
16. **Very short Answer : (V.S.A.)**
(A) M.C.Q :
- (i) The capitals of three friends in a partnership business are ₹ 200, ₹ 150 and ₹ 250 respectively. After some time the ratio of their profit share will be
(a) 5:3:4 (b) 4:3:5 (c) 3:5:4 (d) 5:4:3
- (ii) Suvendu and Nousad started a business with capitals of ₹ 1500 and ₹ 1000. After a year there was a loss of ₹ 75. then the loss of Suvendu is
(a) ₹ 45 (b) ₹ 30 (c) ₹ 25 (d) ₹ 40
- (iii) Fatima, Shreya and Smita started a business by investing total ₹ 6000. After a year Fatima, Shreya and Smita get profit share of ₹ 50, ₹ 100 and ₹ 150 respectively. Smita invested in this business
(a) ₹ 1000 (b) ₹ 2000 (c) ₹ 3000 (d) ₹ 4000
- (iv) Amal and Bimal started a business. Amal invested ₹ 500 for 9 months and Bimal invested some money for 6 months. If they make a profit of ₹ 69 in a year and Bimal gets profit share of ₹ 46. The capital of Bimal in the business is
(a) ₹ 1500 (b) ₹ 3000 (c) ₹ 4500 (d) ₹ 6000

- (v) Pallabi invested ₹ 500 for 9 months and Rajiya invested ₹ 600 for 5 months in a business. The ratio of their profit share will be
 (a) 3:2 (b) 5:6 (c) 6:5 (d) 9:5

(B) Let us write whether the following statements are true or false :

- (i) At least 3 persons are needed in partnership business.
 (ii) Ratio of capital of Raju and Ashif in a business is 5 : 4 and if Raju gets profit share of ₹ 80 of total profit, Ashif will get profit share of ₹ 100.

(C) Let us fill in the blanks :

- (i) Partnership business is _____ type.
 (ii) Without any other conditions in partnership business if the capitals of all the partners are invested for the same time, then such a business is called _____.
 (iii) Without any other conditions in partnership business, if the capitals of all the partners are invested for different time period, then such a business is called _____

17. Short answer type question : (S.A.)

- (i) In partnership business the ratio of capitals of Samir, Idrish and Antony are as $\frac{1}{6} : \frac{1}{5} : \frac{1}{4}$; If they make a profit of ₹ 3700 at the end of the year, let us write by calculating profit share of Antony.
 (ii) If in partnership business the ratio of capitals of Pritha and Rabeya is 2 : 3 and the ratio of Rabeya and Jesmin is 4 : 5, let us write by calculating the ratio of capitals of Pritha, Rabeya and Jesmin.
 (iii) The total profit is ₹ 1500 in a partnership business of two persons. If the capital of Rajib is ₹ 6000 and profit is ₹ 900, let us calculate how much was the capital of Abtab.
 (iv) The ratio of capitals of three persons is 3 : 8 : 5, and the profit of 1st person is ₹ 60 less of the 3rd person, let us calculate the total profit in this business.
 (v) Jayanta, Ajit and Kunal started partnership business investing ₹ 15000. At the end of the year, Jayanta, Ajit and Kunal received ₹ 800, ₹ 1000 and ₹ 1200 respectively as profit share. Let us calculate the amount of Jayanta's capital that was invested in the business.

Last Sunday we along with our brothers and sisters of our house went to the village fair. In this fair we have bought many idols of porcelain, bangles, pickle, jelebi, whistle, bamboo sticks and toys made of cane and different hand made decorative pieces.



My brother bought a toy of bamboo stick as in the adjoining picture, of which the circular plate rotates in the air.

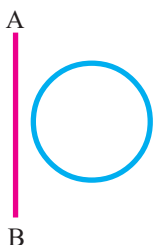


I have a circular ring. I have decided that I shall try to make this type of toy.

I putting the stick of bamboo in different positions near to the circular ring and I got three possibilities below.

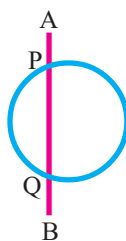


(i)



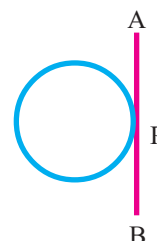
The stick AB and the circular ring have no common point at all

(ii)



The stick AB and the circular ring have two common points P and Q

(iii)



There is only one common point 'P' of the stick AB and the circular ring

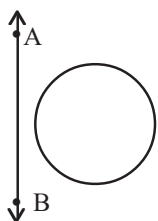
We understand in no., (i) the stick AB does not intersect the circular ring. Again in no. (ii) the stick AB intersects the circular ring.

1 But in no. (iii) in what position are the stick AB and the circular ring?

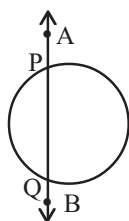
The stick AB touches the circular ring at the point P.

Like me, my friend Sumedha similarly drew in what positions a straight line and a circular ring can possibly be to each other in her copy.

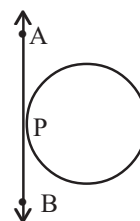
(i)



(ii)



(iii)



We see, in picture no. (i) straight line AB does not intersect the circle.

Again, in picture no. (ii) the straight line AB intersects the circle at the points and .



2 But in the picture no. (ii) and (iii) what this straight line AB of the circle is called?

In picture no (ii) AB is a secant of a circle and PQ is a corresponding chord of secant AB. We see that in picture no (iii) the common point of circle and straight line AB is .

∴ The straight line AB touches the circle at the point P.

3 But in picture no. (iii) what the straight line AB of the circle is called?

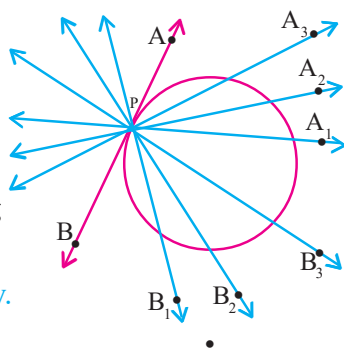
In picture no. (iii) the straight line AB is a tangent of the circle at the point P and the point P is point of contact.

We understand that if the end points of corresponding chord of secant of circle meet each other, the secant will be a tangent of circle

Let us verify by hands and trial

A circular wire is attached with a straight wire PA_1 at a point 'P' of the circular wire. Then the wire is gradually rotated about centring the point 'P' in two directions.

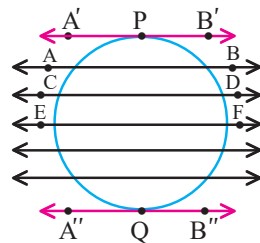
We see the positions of secant $PA_1, PA_2, PA_3 \dots$ or $PB_3, PB_2, PB_1 \dots$ is rotated, when the two end points of its corresponding chord coincide, then the secant of circle is .



Rajat has drawn a circle and a secant AB of it on his copy.

4 Let us try to draw parallel tangent of secant of that circle.

I draw a circle in my copy and with the help of a scale I draw two straight lines AB and CD at the two ends of the scale. Secant CD is parallel to the secant AB. Drawing one or more secants with the scale parallel to secant AB of the circle, we get secants A'B' and A''B'', these are the tangents of a circle at the points P and Q and there can not be more than two tangents parallel to secant AB.



We made a different toy like before with the help of circular ring and stick.

Moreover, we formed different circular rings of white hard board with which we shall make different toys.

Ritam has pasted a circular ring on his colour paper of which the centre is at O

Hands on trial

I draw a tangent at any point of this circle by hands on trial and let us see what I shall get

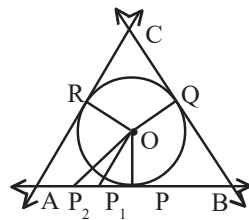
- (i) We take any three points P, Q and R respectively on a circle.
- (ii) Folding the paper I draw three tangents AB, BC and CA respectively at the points P, Q and R like the adjoining picture which intersect each other at the points A, B and C.

- (iii) We get three radii OP, OQ and OR.

- (iv) Except the point P, we took any points P_1, P_2 on AB. From $\triangle OP_1P$ and $\triangle OP_2P$ we see that $OP_1 > OP$ and $OP_2 > OP$.

∴ We see that OP is the smallest of all the line segments drawn from the centre of the circle to the tangent AB. ∴ $OP \perp AB$

By hands on trial we get tangent at any point of a circle is perpendicular to the radius that passes through the point of contact.



Let us prove with reason

Theorem 40. The tangent to a circle at any point on it is perpendicular to the radius passes through the point of contact.

Given : AB is a tangent at the point P of a circle with centre O and OP is a radius through the point P.

To prove : OP and AB are perpendicular to each other i.e. $OP \perp AB$

Construction : Any other point Q is taken on the tangent AB, O, Q are joined.

Proof : Any other point on AB except P is outside the circle;

$\therefore$ OQ intersects the circle at a point.

Let R be the point of intersection.

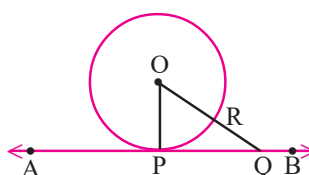
$\therefore OR < OQ$ [$\because$ R is a point between O, Q]

Again, $OR = OP$ [$\because$ radii of same circle]

$\therefore OP < OQ$

$\therefore$ The point Q is any point on AB, $\therefore$ OP is the least of all the line segments drawn from the centre O to the tangent AB. Again least distance is perpendicular distance.

$\therefore OP \perp AB$ (proved)



Let us prove converse theorem of 40 with reason

Application : 1 The perpendicular drawn on radius at the end point of radius of a circle will be a tangent to the circle at the end point of radius.

Given : In a circle with centre O and OP is a radius and AB is perpendicular on OP at the point P.

To prove : Straight line AB is a tangent at the point 'P' of a circle with centre O.

Proof : Let AB is not a tangent at the point P of a circle with centre O.

We draw a tangent CD at the point 'P' of circle with centre O.

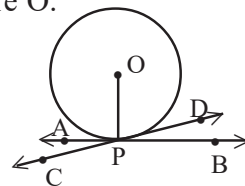
$\therefore$ CD is the tangent at the point P of circle with centre O and OP is radius through the point P, So, OP is perpendicular on CD. $\therefore \angle OPD = 90^\circ$

Again, $\angle OPB = 90^\circ$ (given) [$\because$ OP is perpendicular on AB]

$\therefore \angle OPD = \angle OPB$ i.e. the straight lines CD and AB will coincide each other.

So, straight line CD is not a tangent at the point P of circle with centre O.

$\therefore$ Straight line AB is the tangent at the point P of a circle with centre O.



Application : 2 Let us prove that only one tangent can be drawn at any point on the circle.

[Hints : Only one perpendicular can be drawn on radius through that point.]

Application : 3 Let us prove that perpendicular drawn to the tangent of the circle at the point of contact passes through the centre.

[Hints : Only one perpendicular can be drawn at any point on a straight line]

Application : 4 In the adjoining figure, AT is a tangent at the point A of the circle with centre O. Let us express x in terms of y.

In a circle with centre O, AT is tangent at A and OA is the radius through the point of contact.

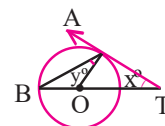
$\therefore \angle OAT = 90^\circ$; Again, $\angle ATO = x^\circ$

Again, for $\triangle AOT$ $\angle AOT = 90^\circ - x^\circ$ _____ (i)

Again, for $\triangle AOB$ $\angle OAB = \angle OBA = y^\circ$ [$\because$ OA and OB are radii of same circle $\therefore OA = OB$]

In $\triangle AOB$ exterior $\angle AOT = \angle OAB + \angle OBA = 2y^\circ$ _____ (ii)

$\therefore$ From (i) and (ii) we get $2y^\circ = 90^\circ - x^\circ$ or $x^\circ = 90^\circ - 2y^\circ \therefore x = 90 - 2y$



Application : 5 We drew a circle with centre O, of which one diameter is AB and the tangent at the point A is PAQ; RS is a chord parallel to the tangent PAQ, let us prove with reason that AB, is perpendicular bisector of RS.



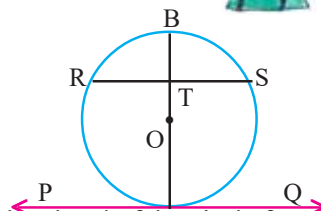
Proof : Let AB intersects RS at the point T. PAQ is the tangent at the point A and AB is a diameter $\therefore AB \perp PQ$

Again $PQ \parallel RS$ [given]

$\therefore AB \perp RS$ i.e. $OT \perp RS$

$\therefore T$ is the mid-point of RS [$\because$ The perpendicular drawn on the chord of the circle from its centre bisects the chord]

$\therefore AB$ is perpendicular bisector of RS



Application : 6 Let us prove that the two tangents at the extremities of a diameter of any circle are parallel to each other. [Let me do it myself]

Let us work out 15.1

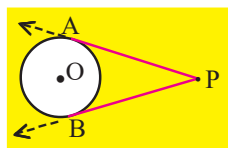
1. Masum has drawn a circle with centre 'O' of which AB is a chord. I draw a tangent at the point B which intersects extended AO at the point T. If $\angle BAT = 21^\circ$, let us write by calculating the value of $\angle BTA$.
2. XY is a diameter of a circle. PAQ is a tangent to the circle at the point 'A'. The perpendicular drawn on the tangent of the circle from X intersects PAQ at Z. Let us prove that XY is a bisector of $\angle YXZ$.
3. I drew a circle having PR as a diameter. I draw a tangent at the point P and a point S is taken on the tangent of the circle in such a way that $PR = PS$. If RS intersects the circle at the point T. Let us prove that $ST = RT = PT$.
4. Two radii OA and OB of a circle with centre O are perpendicular to each other. If two tangents drawn at the point A and B intersect each other at the point T, let us prove that $AB = OT$ and they bisect each other at a right-angle.
5. Two chords AB and AC of the larger of two concentric circles touch the other circle at points P and Q respectively. Let us prove that, $PQ = \frac{1}{2} BC$.
6. X is a point on the tangent at the point A lies on a circle with centre O. A secant drawn from a point X intersects the circle at the points Y and Z. If P is a mid-point of YZ, let us prove that XAPO or XAOP is a cyclic quadrilateral
7. P is any point on diameter of a circle with centre O. A perpendicular drawn on diameter at the point O intersects the circle at the point Q. Extended QP intersects the circle at the point R. A tangent drawn at the point R intersects extended OP at the point S. Let us prove that $SP = SR$
8. Rumela drew a circle with centre O of which QR is a chord. Two tangents drawn at the points Q and R intersect each other at the point P. If QM is a diameter, let us prove that $\angle QPR = 2 \angle RQM$.
9. Two chords AC and BD of a circle intersect each other at the point O. If two tangents drawn at the points A and B intersect each other at the point P and two tangents drawn at the points C and D intersect at the point Q, let us prove that $\angle P + \angle Q = 2\angle BOC$.

To day we have decided that we shall try to draw a tangent to a circle from any point by hands on trial and we shall know the number of tangents that can be drawn.

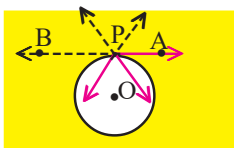


Hands on trial

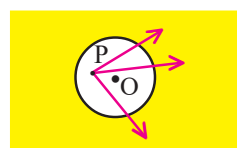
- (i) We take three coloured card board.
- (ii) Drawing three circles with same measure with centre O on a white paper and cut off the circular region and pasted them on this coloured paper.
- (iii) In three boards three pins are attached at the point P like picture below outside the circular region, on the circular region and inside the circular region respectively and a pin is attached with the one end of this thread and other end rotating in different directions like picture mentioned below. I tried to draw tangent of circle with thread.



(i)

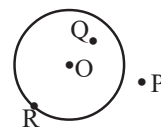


(ii)



(iii)

We see that in picture no. (i) i.e. tangents can be drawn to a circle from any point outside it
In picture no. (ii) i.e. tangent can be drawn to a circle from a point lying on the circle.
In picture no. (iii) No tangent can be drawn to a circle from any point lying inside the circle.
In a circle with centre O, P is an external point and Q is an interior point and a point R lying on circle like the adjoining figure.



Let us verify by hands on trial that how many tangents can be drawn from the points P, Q and R to that circle.

∴ By hands on trial we understood that two tangents can be drawn from an external point of that circle.

Let us prove with reason

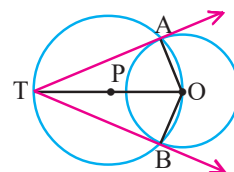


Application : 7 Two tangents can be drawn to a circle from an external point.

Given : T is any external point for a circle with centre O.

To prove : Two tangents can be drawn from T to the circle with centre O.

Construction : T, O are joined, A circle is drawn with TO as diameter. T is an external point and O is an interior point of a circle with centre O. So the circle drawn taking TO as diameter intersects the circle with centre O at two points. Let the two points of intersection be A and B; T, A; T, B; O, A; O, B are joined.



Proof : Each of $\angle OAT$ and $\angle OBT$ is an angle in a semicircle

$$\therefore \angle OAT = \angle OBT = 1 \text{ right angle}$$

$$\text{i.e. } OA \perp AT \text{ and } OB \perp BT$$

TA and TB are perpendiculars on radii OA and OB at the points A and B respectively.

∴ TA and TB are tangents to the circle with centre O at the points A and B respectively.

We get two tangents can be drawn to a circle from an external point. [proved]



We understand that, from any [interior/external] point, no tangent can be drawn to the circle
[Let me do it myself]

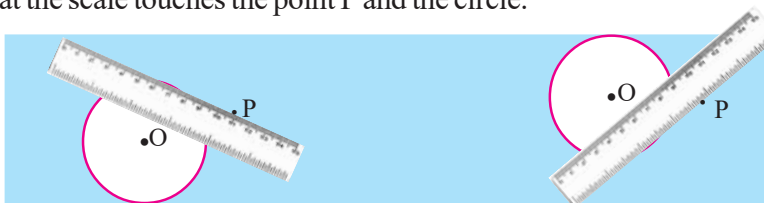
Application : 8 I drew two tangents PA and PB to a circle with centre O from an external point P which touches the circle at the points A and B respectively. Let us prove that $\angle APB + \angle AOB = 180^\circ$ [Let me do it myself]

We proved by hands on trial and with reason that two tangents can be drawn to a circle from an external point.

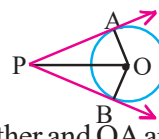
But let us verify by hands on trial what relation will be there between two tangents.

Hands on trial

- (1) I drew a circle with centre O on a paper and took any point P outside the circle.
- (2) Now a scale is put on paper in such a way like picture below in two directions of centre of circle that the scale touches the point P and the circle.



- (3) Folding two sides of the paper, we get two points of contact A and B from point P of a circle with centre O and A, P; B, P; O, P; O, A and O, B are joined and we get two tangents AP and BP.
- (4) Folding twice the paper along OP we see that AP and BP coincide each other and OA and OB coincide each other.



$\therefore$ By hands on trial, we get $AP = BP$ and $\angle AOP = \angle BOP$.

By hands on trial we get if two tangents are drawn to a circle from a point outside it, then the line segments joining the points of contact and the exterior are equal and they subtend equal angles at the centre.

[Similar drawing another circle, let us verify it by hands on trial]

But what the length from the external point P to the point of contact A is called?

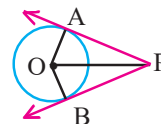
The length from the external point P to the point of contact A is said to be **length of tangent** PA from the point P.

Let us prove with reason

Theorem 41, If two tangents are drawn to a circle from a point outside it, then the line segments joining the point of contact and the exterior point are equal and they subtend equal angles at the centre.

Given : P be a point outside the circle with centre O. From the point P two tangents PA and PB are drawn, whose points of contact are A and B respectively. O, A; O, B; O, P are joined, this PA and PB subtend $\angle POA$ and $\angle POB$ at the centre respectively.

To prove that : (i) $PA = PB$ (ii) $\angle POA = \angle POB$



Proof : PA and PB are tangents and OA and OB are the radii through the points of contact.

$\therefore OA \perp PA$ and $OB \perp PB$

In the right angled triangles POA and POB, $\angle OAP = \angle OBP$ (each is 1 right angle)
hypotenuse OP is common side and $OA = OB$ (radii of the same circle)

$\therefore \triangle PAO \cong \triangle PBO$ [R-H-S axiom of congruency]

$\therefore PA = PB$ (corresponding sides) [(i) proved]

and $\angle POA = \angle POB$ (corresponding angles) [(ii) proved]



From this theorem we get more, $\angle APO = \angle BPO$ (corresponding angles).

$\therefore$ OP, bisects $\angle APB$.

Application : 9 I drew a circle with centre 'O' of which length of radius is 6 cm. A point P is 10 cm. away from the centre O of a circle. A tangent PT is drawn. Let us write by calculating the length of the tangent PT.

OT is radius and PT is a tangent of circle with centre O.

In right-angled triangle POT. $\therefore OT \perp PT$

$$\therefore PT^2 = PO^2 - OT^2 \text{ [we get from Pythagoras theorem]}$$

$$\text{or, } PT^2 = \{(10)^2 - (6)^2\} \text{ sq.cm.}$$

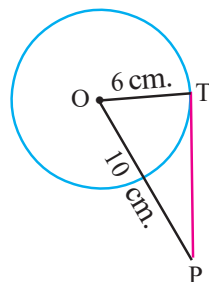
$$= (100 - 36) \text{ sq.cm.}$$

$$= 64 \text{ sq.cm.}$$

$$\therefore PT = \sqrt{64} \text{ cm.}$$

$$= 8 \text{ cm.}$$

$\therefore$ We get, length of tangent PT is 8 cm.



Application : 10 If I draw a circle with centre O, a point P is 26 cm. away from the centre of the circle and the length of the tangent drawn from the point P to the circle is 10 cm., let us write by calculating the length of radius of the circle.

$$PT = 10 \text{ cm.}, PO = 26 \text{ cm.}$$

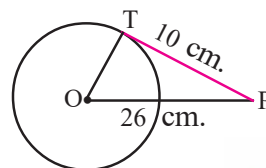
$\therefore$ From right-angled triangle POT, we get,

$$PO^2 = PT^2 + OT^2 \text{ [we get from Pythagoras theorem]}$$

$$\therefore (26\text{cm.})^2 = (10\text{cm.})^2 + OT^2$$

$$\text{or, } OT^2 = (26\text{cm.})^2 - (10\text{cm.})^2 = \boxed{}$$

$$\therefore OT = \boxed{} \text{ cm. [Let me do it myself]}$$



Application : 11 Two radii OA and OB of a circle with centre O make an angle 130° between them in the adjoining picture. Two tangents drawn at the points A and B intersect at the point T, let us write by calculating the values of $\angle ATB$ and $\angle ATO$.

Two tangents drawn at the points A and B of a circle intersect each other at the point T.

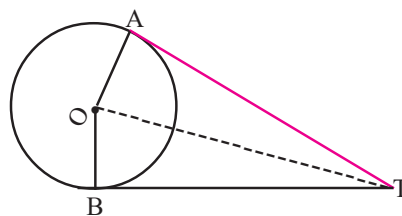
$\therefore$ AT and BT are the two tangents through the point T and OA and OB are the radii through the points of contact.

$$\therefore OA \perp AT \text{ and}$$

$$\therefore \angle ATB + \angle AOB = 360^\circ - (90^\circ + 90^\circ) \\ = 180^\circ$$

$$\therefore \angle ATB = 180^\circ - 130^\circ = 50^\circ$$

$$\text{Again, OT, bisects } \angle ATB. \text{ So, } \angle ATO = \frac{50^\circ}{2} = 25^\circ$$



Let us do 21.1

- (1) Let us write by calculating the length of tangent from a point at a distance of 13 cm. from the centre of circle to the point of contact of a tangent to a circle having 5 cm. length of radius.
- (2) If the distance of a point P situated at a distance of 13 cm. from the centre of a circle with centre O and length of tangent drawn through the point P is 12 cm., let us write by calculating the length of radius of the circle.
- (3) Two tangents PA and PB are drawn through an external point A to a circle with centre O. If $\angle AOB = 120^\circ$, let us write by calculating the values of $\angle APB$ and $\angle APO$

Application : 12 Two tangents have been drawn from an external point A to a circle with centre O which touch the circle at the points B and C respectively. Let us prove that AO is perpendicular bisector of BC.

Given : Two tangents AB and AC have been drawn from an external point A which touch the circle at the points B and C respectively. A, O are joined. AO intersects BC at the point D.

To prove : AO is the perpendicular bisector of BC

Proof : AB and AC are tangents of a circle with centre O. So, $AB = AC$

and AO has bisected an angle $\angle BAC$. i.e., $\angle BAD = \angle CAD$

In $\triangle ABD$ and $\triangle ACD$, $AB = AC$

$\angle BAD = \angle CAD$ and AD is common side

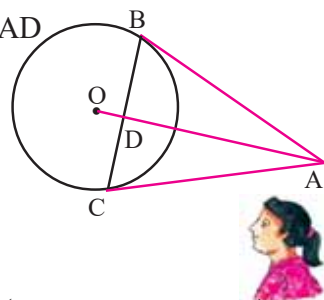
$\therefore \triangle ABD \cong \triangle ACD$ (S-A-S property of congruency)

So, $BD = CD$ (corresponding sides)

and $\angle BDA = \angle CDA$ (corresponding angles)

Again, $\angle BDA + \angle CDA = 180^\circ$ or, $2 \angle BDA = 180^\circ$ ($\because \angle BDA = \angle CDA$)

$\therefore \angle BDA = 90^\circ \therefore AD \perp BC \therefore AD$, is perpendicular bisector of BC (**proved**)



Let us do 21.2

- (1) Let us prove that the line segment joining a point outside a circle and the centre bisects the angle included by two tangents drawn from the external point.
- (2) Let us prove that the bisector of an angle included by two tangents to a circle from a point outside it passes through the centre.
- (3) Let us prove that if two tangents drawn to a circle at two points on it intersect each other, then the lengths of the line segments from the point of intersection to the points of contact are equal.

Today we have decided that we shall draw a circular picture of our own environment. Niladri can draw good picture. He drew a circular field on our school's black board.

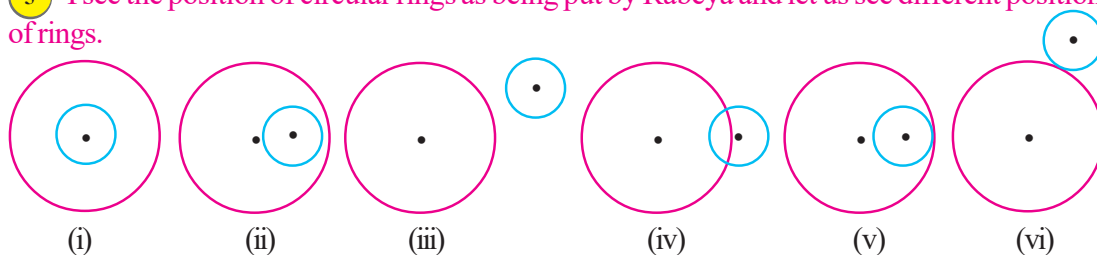


But a path equally width is running all around outside the circular field of our village. I draw a picture of circular field with path.



We see that, we get two circles having same centre i.e. we get two concentric circles. Here Rabeya is trying to make a new thing putting two circular ring in different positions.

5 I see the position of circular rings as being put by Rabeya and let us see different position of rings.

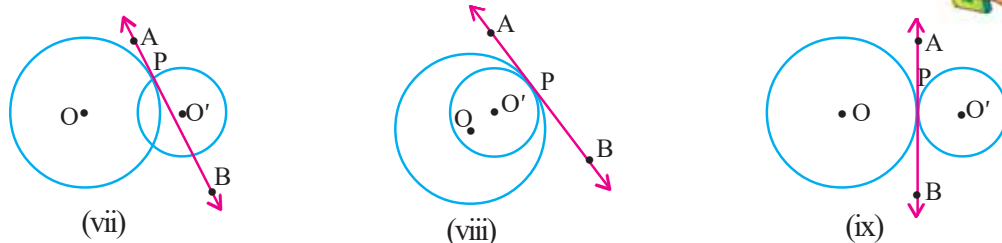


We see that in picture no. (i) two circles but, in picture no. (ii) two circles are not concentric circles, in picture no. (iii) two circles do not intersect each other, in picture no. (iv) the two circles intersect each other at two points.

6 If tangents are drawn at the point of contact, then what type of tangent we shall get let us see it by drawing picture.

But we see, in picture no. (v) and (vi) two circles touch each other.

7 How shall we understand that two circle touch each other?



There is only one intersecting point P of two circles with centre O and O' in the same plane and if the tangent of circle with centre O touches the circle with centre O' at that point, then it is called the two circles touch each other at the point P. In picture no. (vii) two circles do not touch each other. In picture no. (viii) and (ix) two circles touch each other.

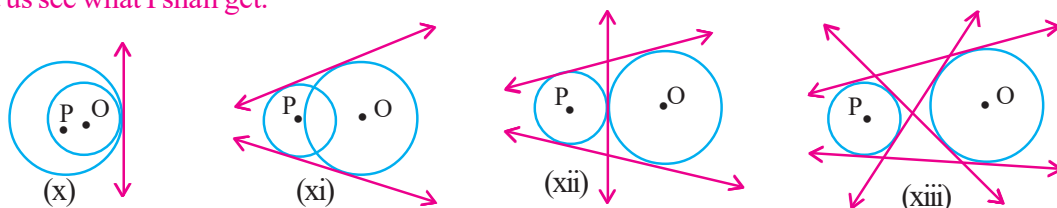
In picture no. (viii) two circle **touch internally** each other at the point P.

AB is a **common tangent** of two circles.

Again, in picture no. (ix) two circle **touch externally** each other at the point P. AB is a of the two circles. [Let me do it myself]

We understand that, if a straight line touches each of two circles, then the straight line is called a **common tangent** of two circles.

8 Let us draw a common tangent of two circles in different ways on my exercise book and let us see what I shall get.



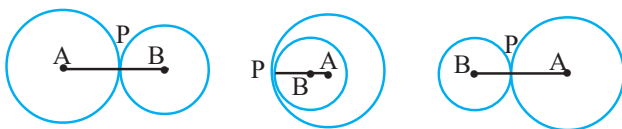
We see that in different positions of two circles sometimes 1 or 2 or 3 or sometimes four common tangents are drawn.

9 But sometimes we see that the two circles are situated on the same side of the common tangents, again some other times the two circles are situated on the opposite sides of the common tangents. What this type of tangent is called?

If two circles lie on same side of a common tangent the tangent is said to be direct common tangent and if two circles lie on opposite side of common tangent, the common tangent is said to be transverse common tangent.

We understand that the tangents of pictures no. (x) and (xi) are direct common tangents. But in picture no. (xii) there are two direct common tangents and 1 common tangent. Again in picture no. (xiii) there are direct common tangent and transverse common tangents. [let us write by seeing the picture]

10 But if two circles touch each other, then let us verify by drawing picture, the position of point of contact and centre of circle.



From the picture we see that two centres of circle and point of contact lie on same straight line.



I draw any two circles which touch each other and I see that two centers of circles and point of contact are collinear. [Let me do it myself]

Let us prove with reason

Theorem 42 : If two circles touch each other, then the point of contact will lie on the line segment joining the two centres.

Given : Two circles with centres A and B touch each other at the point P.

To prove : A, P and B are collinear.

Construction : A, P and B, P are joined

Proof : Two circles with centres A and B touch each other at a point P.

$\therefore$ There exists a common tangent at the point P.

Let ST is the common tangent, which touches both the circle at the point P.

$\therefore$ ST is the tangent of circle with centre A and AP is the radius through the point of contact

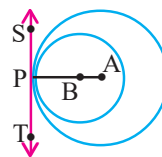
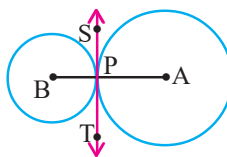
$\therefore AP \perp ST$

Again, ST is the tangent to the circle with centre B and BP is the radius through the point of contact.

$\therefore BP \perp ST$

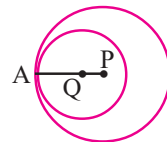
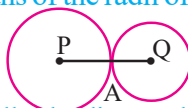
$\therefore$ AP and BP are both perpendiculars to ST at the same point P.

$\therefore$ AP and BP lie on the same straight line i.e., A, P and B are collinear.



Let us prove with reason that if two circles touch each other, the straight line through the centre of one circle and the point of contact passes through the centre of other circle. [Let me do it myself]
If two circles touch each other externally, the distance between the centres of two circles is equal to [sum/different] of the lengths of the radii of the circles.

In pictures beside $PQ = PA + AQ$



Again, if two circles touch each other internally, the distance between the centers of two circles is equal to the difference of the length of the radii of two circles. In picture beside $PQ = \text{---} - \text{---}$ [Let me right it myself]

Application : 13 I draw a circle with centre O of which AB is a diameter. Two parallel tangents are drawn at the points A and B of diameter AB of the circle to intersect the tangent, drawn to the circle at another point T of the circle, at the points P and Q respectively. Let us prove that $\angle POQ = 90^\circ$

Given : AB is a diameter of circle with centre O. Two parallel tangents are drawn at points A and B. Tangent drawn to the circle at another point T intersects the tangents which drawn at the points A and B, at the points P and Q respectively.

To prove : $\angle POQ = 90^\circ$

Construction : O, P; O, Q and O, T are joined

Proof : Tangents drawn at points A and T of a circle with centre O intersect at P.

$\therefore$ PO, is the internal bisector of $\angle APT$

$$\therefore \angle TPO = \frac{1}{2} \angle APT \text{ --- (i)}$$

Similarly, tangents drawn at points T and B intersect at Q.

$$\therefore \angle TQO = \frac{1}{2} \angle BQT \text{ --- (ii)}$$

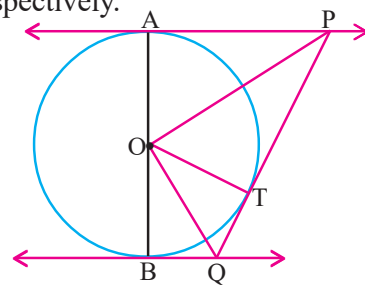
Again, $AP \parallel BQ$ and PQ is the transversal

$$\therefore \angle APT + \angle BQT = 180^\circ$$

$$\therefore \angle TPO + \angle TQO = \frac{1}{2} \angle APT + \frac{1}{2} \angle BQT \text{ [We get from (i) and (ii)]}$$

$$= \frac{1}{2} (\angle APT + \angle BQT) = \frac{1}{2} \times 180^\circ = 90^\circ$$

$\therefore$ For ΔPOQ remaining $\angle POQ = 90^\circ$ [Proved]



Application : 14 If a quadrilateral ABCD is circumscribed about a circle with centre O, let us prove that $AB + CD = BC + DA$

Given : A quadrilateral ABCD is circumscribed about a circle with centre O

Let sides of quadrilateral AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively.

To prove : $AB + CD = BC + DA$

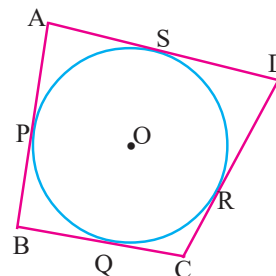
Proof : AS and AP are two tangents to a circle with centre O, drawn from the exterior point A, So, $\therefore AS = AP$

Similarly, $BP = BQ$, $CQ = CR$ and $DR = DS$

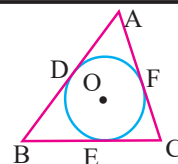
$$AB + CD = AP + BP + CR + DR$$

$$= AS + BQ + CQ + DS = (AS + DS) + (BQ + CQ) = AD + BC$$

[Proved]



Application : 15 In the adjoining figure, the incircle of $\triangle ABC$ touches the sides AB, BC and CA at the points D, E and F respectively. Let us prove that $AD + BE + CF = AF + CE + BD = \frac{1}{2}$ (perimeter of $\triangle ABC$) [Let me do it myself]



Application : 16 Two circles with centres P and Q touch each other externally at the point A. A direct common tangent to the two circles touches the two circles at R and S respectively. Let us prove that

- (i) direct common tangent drawn at the point A bisects the line segment RS at the point T.
- (ii) $\angle RAS = 1$ right angle
- (iii) If PT and QT intersect AR and AS at points C and B respectively, then ABTC is a rectangular figure.

Given : Two circles with centre P and Q touch each other externally at the point A. A direct common tangent to the two circles touches the two circles at the points R and S respectively. Tangent drawn at the point A intersects RS at the point T. PT intersects AR at C and QT intersects AS at B.

To prove : (i) AT bisects RS

- (ii) $\angle RAS = 1$ right angle
- (iii) ABTC is a rectangular figure.

Proof : Tangent drawn at the point A intersects RS at the point T. TR and TA are two tangents from the point T to a circle with centre P.

$$\therefore TR = TA$$

$$\text{Similarly, } TS = TA \therefore TR = TS$$

$\therefore$ AT bisects RS [(i) is Proved]

$$\text{Again, for } \triangle ATR, TR = TA \therefore \angle TAR = \angle TRA$$

$$\text{Similarly, } \angle TAS = \angle TSA$$

$$\therefore \angle RAS = \angle TAR + \angle TAS = 90^\circ$$

$$[\because \angle TRA + \angle TAR + \angle TAS + \angle TSA = 180^\circ]$$

$$\therefore 2(\angle TAR + \angle TAS) = 180^\circ; \therefore \angle TAR + \angle TAS = 90^\circ \quad \text{[(ii) is Proved]}$$

Again, PT is the bisector of $\angle RTA$ and QT is the bisector of $\angle ATS$

$\therefore PT \perp TQ$ i.e. $\angle PTQ = 1$ right-angle, Similarly, $QT \perp SA$

$PT \perp RA$ and $QT \perp SA$,

$$\therefore \angle ACT = \angle ABT = 1 \text{ right-angle}$$

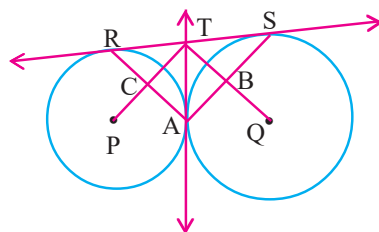
In quadrilateral ABTC, $\angle ACT = \angle ABT = 90^\circ$

$$\text{and } \angle CAB = 90^\circ \therefore \angle CTB = 90^\circ$$

$\therefore$ ABTC is a parallelogram ($\because$ opposite angles of a quadrilateral are equal)

Again one angle of parallelogram ABTC is 1 right angle

$\therefore$ ABTC is a rectangular figure [(iii) Proved]

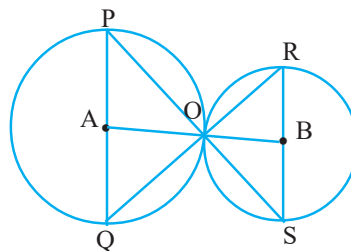


Application : 17. Sumita draws two circles which touch each other externally at the point O. If PQ and RS are diameters of the two circles and they are parallel, let us prove that P, O and S are collinear.

Given : Two circles touch each other externally at the point O. Diameters PQ and RS of two circles are parallel.

To prove : P, O and S are collinear.

Construction : Let the centres of the two circles be A and B. O, A; O, B; O, P; O, Q; O, R; O, S are joined.



Proof: A and B are the centres of two circles.

The two circles touch each other externally at O.

$\therefore$ A, O and B are collinear.

In $\triangle AOP$, $AP = AO$, $\therefore \angle APO = \angle AOP$

Again, $\angle PAO + \angle APO + \angle AOP = 180^\circ$

$\therefore 2\angle AOP = 180^\circ - \angle PAO$

Similarly, we get from $\triangle BOR$, $2\angle ROB = 180^\circ - \angle RBO$

$\therefore 2\angle AOP + 2\angle ROB = 360^\circ - (\angle PAO + \angle RBO)$

$= 360^\circ - 180^\circ$ [$\because PQ \parallel RS$ and AB is transversal]
 $= 180^\circ$

$\therefore \angle AOP + \angle ROB = 90^\circ$

$\therefore \angle POR = 180^\circ - (\angle AOP + \angle ROB) = 180^\circ - 90^\circ = 90^\circ$

$\angle POR + \angle ROS = 90^\circ + 90^\circ = 180^\circ$ ($\because \angle ROS$ is an angle in a semi circle

$\therefore \angle ROS = 90^\circ$)

$\therefore \angle POS = 180^\circ$

$\therefore$ P, O and S points are collinear. **[Proved]**



Let us see by calculating 15.2

1. An external point is situated at a distance of 17 cm from the centre of a circle having 16 cm. diameter, let us determine the length of the tangent drawn to the circle from the external point.
2. The tangents drawn at points P and Q to a circle intersect at A. If $\angle PAQ = 60^\circ$, let us find the value of $\angle APQ$.
3. AP and AQ are two tangents drawn from an external point A to a circle with centre O, P and Q are points of contact. If PR is a diameter, let us prove that $OA \parallel RQ$.
4. Let us prove that for a quadrilateral circumscribed about a circle, the angles subtended by any two opposite sides at the centres are supplementary to each other.
5. Let us prove that a parallelogram circumscribed by a circle is a rhombus.
6. Two circles drawn with centres A and B touch each other externally at C, O is a point on the tangent drawn at C, OD and OE are tangents at the points D and E drawn to the two circles of centres A and B respectively. If $\angle COD = 56^\circ$, $\angle COE = 40^\circ$, $\angle ACD = x^\circ$ and $\angle BCE = y^\circ$. Let us prove that $OD = OC = OE$ and $y - x = 8$

7. Two circles with centres A and B touch each other internally. Another circle touches the larger circle internally at the point X and the smaller circle externally at the point Y. If O be the centre of that circle, let us prove that $AO + BO$ is constant.
8. Two circles have been drawn with centre A and B which touch each other externally at the point O. I draw a straight line passing through the point O intersects the two circles at P and Q respectively. Let us prove that $AP \parallel BQ$.
9. Three equal circles touch one another externally. Let us prove that the centres of the three circles form an equilateral triangle.
10. Two tangents AB and AC drawn from an external point A of a circle touch the circle at the point B and C. A tangent drawn to a point X lies on minor arc BC intersects AB and AC at the points D and E respectively. Let us prove that perimeter of $\triangle ADE = 2 AB$.

11. **Very short answer type. (V.S.A.)**

M.C.Q.

- (i) A tangent drawn to a circle with centre O from an external point A touches the circle at the point B. If $OB = 5$ cm, $AO = 13$ cm, then the length of AB is
(a) 12 cm. (b) 13 cm. (c) 6.5 cm. (d) 6 cm.
- (ii) Two circles touch each other externally at the point C. A direct common tangent AB touch the two circles at the points A and B. Value of $\angle ACB$ is
(a) 60° (b) 45° (c) 30° (d) 90°
- (iii) The length of radius of a circle with centre O is 5 cm. P is a point at the distance of 13 cm from the point O. The length of two tangents are PQ and PR from the point P. The area of quadrilateral PQOR is
(a) 60 sq cm. (b) 30 sq cm. (c) 120 sq cm. (d) 150 sq cm.
- (iv) The lengths of radii of two circles are 5 cm and 3 cm. The two circles touch each other externally. The distance between two centres of two circles is
(a) 2 cm. (b) 2.5 cm. (c) 1.5 cm. (d) 8 cm.
- (v) The lengths of radii of two circles are 3.5 cm and 2 cm. The two circles touch each other internally. The distance between the centres of two circles is
(a) 5.5 cm. (b) 1 cm. (c) 1.5 cm. (d) none of these

(B) Let us write whether the following statements are true or false :

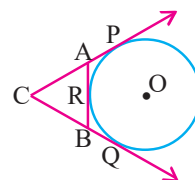
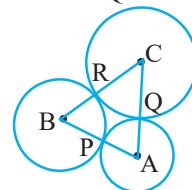
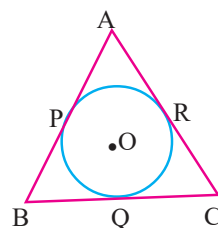
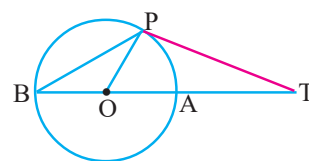
- (i) P is a point inside a circle. Any tangent drawn on the circle does not pass through the point P.
- (ii) There are more than two tangents can be drawn to a circle parallel to a fixed line.

(C) Let us fill in the blanks.

- (i) If a straight line intersects the circle at two points, then the straight line is called _____ of circle.
- (ii) If two circles do not intersect or touch each other, then the maximum number of common tangents can be drawn is _____.
- (iii) Two circles touch each other externally at the point A. A common tangent drawn to two circles at the point A is _____ common tangent (direct/transverse)

12. **Short answer question (S.A.)**

- (i) In the adjoining figure O is the centre and BOA is a diameter of the circle. A tangent drawn to a circle at the point P intersects the extended BA at the point T. If $\angle PBO = 30^\circ$, let us find the value of $\angle PTA$.
- (ii) In the adjoining figure, $\triangle ABC$ circumscribed a circle and touches the circle at the points P, Q, R. If $AP = 4$ cm, $BP = 6$ cm, $AC = 12$ cm and $BC = x$ cm, let us determine the value of x .
- (iii) In the adjoining figure, three circles with centres A, B, C touch one another externally. If $AB = 5$ cm, $BC = 7$ cm and $CA = 6$ cm, let us find the length of radius of circle with centre A.
- (iv) In the adjoining figure, two tangents drawn from external point C to a circle with centre O touch the circle at the points P and Q respectively. A tangent drawn at another point R of a circle intersects CP and CA at the points A and B respectively. If $CP = 11$ cm. and $BC = 7$ cm, let us determine the length of BR.
- (v) The lengths of radii of two circles are 8 cm and 3 cm and distance between two centres is 13 cm. let us find the length of a common tangent of two circles.



16

RIGHT CIRCULAR CONE

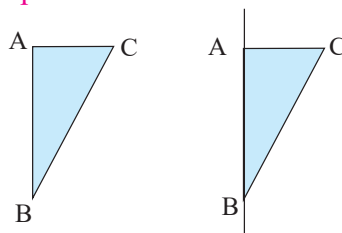
In the last month, we have made many hand fans by fixing a thin stick on a half-circular cardboard.

Again we have observed that we get globe-shaped solids if these hand fans are revolved forcibly by taking the sticks as axis.

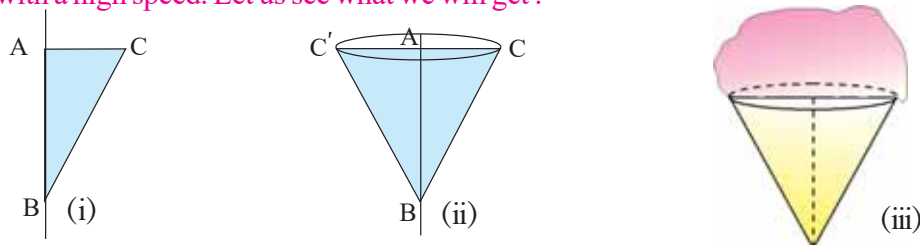


- 1 But in similar way, let us see, what we shall get if a right-angled triangular cardboard is revolved with respect to a stick by fixing it along its perpendicular or base.

My friend Aslam has cut out a right-angled triangular region ABC, as in the adjoining figure from a coloured rectangular cardboard, of which its $\angle A$ is a right-angle. I have fixed a very thin stick with a side AB adjacent to right angle of this right-angled triangular cardboard ABC and I have got a figure like the adjoining figure.



- 2 Now taking the stick AB as axis and let us revolve the triangular cardboard around the axis with a high speed. Let us see what we will get :



We observe that by revolving the right-angled triangle ABC around its axis having taken the side AB as axis, a shape of solid is formed like a cone of icecream.

- 3 What is the solid having this kind of shape called ?

The solid of this kind of shape is called right-circular cone.

- 4 What is the circle formed with AC by revolving the right-angled triangle ABC with respect to the side AB?

The circle formed by AC of the figure (ii) is called the base of the right-circular cone. The radius of the circular base of the cone is AC and the centre of the circular base is A.

- 5 What is the point B of the right circular cone in this fig. (ii) is called?

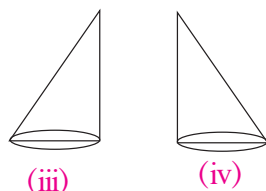
The point B of the figure (ii) is called the apex of the right circular cone. Again the perpendicular AB on the circular base is called the Height of the right circular cone and BC is called the Slant height.

The number of surfaces of a closed cone is . One is plane and other is [curved/plane] [Let me write it myself]



- 6 I write the names of 4 solid objects which are used in our house and whose shapes are like right circular cones.

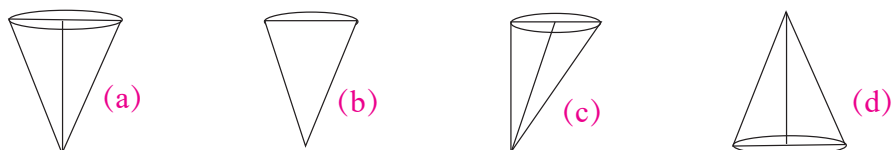
My sister has made two circular cones with paper as the adjoining figures.



- 7 Are the cones of the figures (iii) and (iv) right circular cones?

The cones of the figures (iii) and (iv) are not right circular cones. Because the line-segments joining the apex of the cones and the centres of their circular regions are not perpendicularly situated on their bases.

- 8 Let us observe the following figures of cones and let us write by understanding which are right circular cones among them :



But here, we shall treat the 'Cone' as 'right circular cone'.

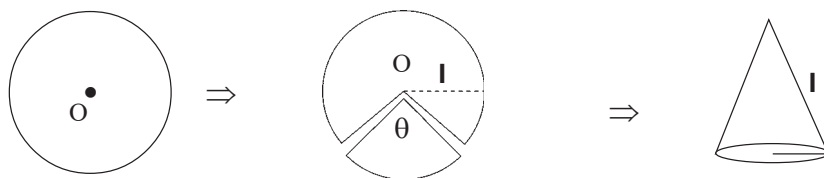
I shall try to make a right circular cone. But how shall I get the quantity of paper to be required to make a cone?

I shall get it by determining the curved surface area of the right circular cone.

Hands on trial

Let us make the right circular cone and determine its curved surface area.

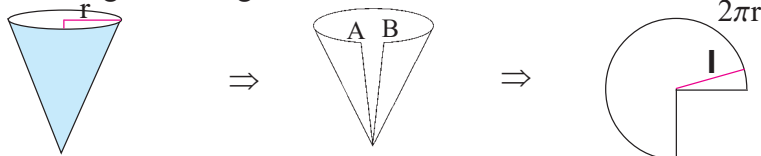
1. I fixed up a white art paper on a wooden board.
2. Now I cut out a circular region by drawing a circle with a radius of 1 unit length on that art paper.
3. From that circular region I cut out any sector.
4. Now I got a hollow right circular cone by fixing the two extreme radii with gum from that cut off sector.



Hands on trial Let us determine the curved surface area of that cone.

(1) By taking a right circular cone, like the following figure, I have unfolded it by cutting along its slant height.

Let the slant height of the right circular cone = l unit and radius = r unit



∴ By unfolding a right circular cone, I have got a sector.

The slant height of the cone = The length of the radius of the circular region = l .

The radius of the base of the cone = r

∴ The circumference of the base of the cone = $2\pi r$ =

The length of the sector.

∴ The curved surface area of a right circular cone

= Area of sector

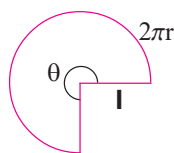
= $\frac{\text{length of arc}}{\text{circumference of circle}} \times \text{Area of Circle}$

$$= \frac{2\pi r}{2\pi l} \times \pi l^2 \text{ sq. unit} = \pi r l \text{ sq. unit}$$

[Area of sector = $\frac{\theta}{360} \times \pi l^2$, where sector makes an angle θ° at the centre of the circular region.]

Again, the length of the arc and the angle subtended by the arc at the centre of a circular region are

$$\text{proportional} \therefore \frac{\theta}{360} = \frac{2\pi r}{2\pi l}$$



∴ By hands on trial, we have got, the curved surface area of a right circular cone = $\pi r l$

[Where, the radius of the base of the cone is r and the slant height = l]

To make a right circular cone with one face open, $\pi r l$ sq. unit paper is required, where $r =$

and $l =$ [Let me write it myself]

Application : 1. If the length of the diameter of the base and the slant height of the right circular cone with one face open, which I have made are 14cm and 12cm respectively. Let us calculate the quantity of the paper that has been required to make this cone.

The length of the radius of the base of the cone = $\frac{14}{2}$ cm. = 7 cm.

∴ The curved surface area of the cone = $\frac{22}{7} \times 7 \times 12$ sq. cm = 264 sq. cm.

Application : 2. Let us calculate the curved surface area of a right circular cone whose radius of the base is 1.5m and slant height is 2m. [Let me do it myself]

Application : 3. If the area of the base of a cone is $78\frac{4}{7}$ sq.cm and slant height is 13cm., then let us calculate its curved surface area.

Let the length of the radius of the base of the cone be r cm.

According to the condition, $\frac{22}{7} r^2 = 78\frac{4}{7}$ ∴ $r =$

∴ Curved surface area of the cone = sq.cm. [Let me do it myself]



Application : 4. If the circumference of the base of a cone is $\frac{660}{7}$ cm. and the slant height is 25cm, then let us calculate the curved surface area of the cone. [Let me do it myself]

9 But if I make a closed right circular cone, then let me calculate the quantity of artpaper which is to be required to make it.

Let the radius of the base of the right circular cone be r unit and the slant height be l unit.

$\therefore$ The required quantity of art paper = The curved surface area of the cone + The area of the base of the cone

$$= \pi r l + \pi r^2 \text{ sq unit}$$

$$= \pi r (l + r) \text{ sq unit}$$

The total surface area of the cone = $\pi r (r + l)$ sq. unit

Application : 5. Let us write the total surface area of a cone whose length of the radius of the base is 1.5cm. and slant height is 2.5 cm.

$$\begin{aligned} \text{The total surface area of the cone} &= \frac{22}{7} \times 1.5 (1.5 + 2.5) \text{ sq cm.} \\ &= \frac{22}{7} \times \frac{15}{10} \times 4 \text{ sq cm.} = \boxed{} \text{ sq cm.} \end{aligned}$$

Application : 6. Let us write the total surface area of a cone whose diameter of the base is 20cm. and slant height is 2.5 cm. [Let me do it myself]

Application : 7. I have made a right circular cone whose diameter of the base is 12cm. and slant height is 10cm. Let me write by calculating, the height of the cone.

The length of the radius of the base of right circular cone = $\frac{12}{2}$ cm. = 6 cm.

In the figure, the radius of the base of the cone is OA, slant height is AB and height is OB

$$OA = 6 \text{ cm. } AB = 10 \text{ cm., } \quad \text{The height of the cone} = OB$$

$\therefore$ In right angled triangle OAB, $AB^2 = OB^2 + OA^2$ (From Pythagoras theorem)

$$\text{or, } OB^2 = AB^2 - OA^2$$

$$\text{or, } OB = +\sqrt{AB^2 - OA^2} \quad [\because \text{height can not be -ve}]$$

$$= \sqrt{(\text{Slant height})^2 - (\text{radius})^2}$$

$$= \sqrt{(10)^2 - 6^2} \text{ cm.}$$

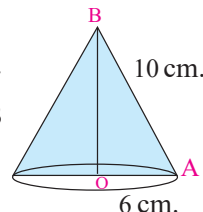
$$= \sqrt{64} \text{ cm.}$$

$$= 8 \text{ cm.}$$

$\therefore$ The height of the cone is 8cm.

$$\therefore \text{Height of a cone} = \sqrt{(\text{Slant height})^2 - (\text{radius})^2}$$

Application : 8. Let us calculate the slant height of a right circular cone whose circumference is $\frac{660}{7}$ cm and height is 20 cm. [Let me do it myself]

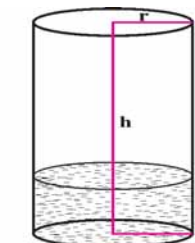
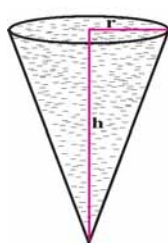


I have seen a glass flask with the shape of a right circular cone in the science laboratory of our school. This flask is filled with water. But Let us see how we shall get the quantity of water that can be hold in that right circular cone shaped flask. Let us try to measure it by hands on trial.

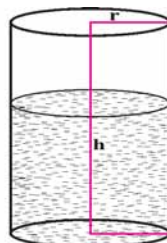


Hands on trial

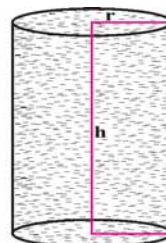
- (1) I took an empty right circular cylindrical pot and an empty right circular conical pot with the same diameter of the base and with same height.
- (2) Now, after completely filling the conical pot with water, that water is poured into a cylindrical pot.



First time $\frac{1}{3}$ part is filled up.



Second time $\frac{2}{3}$ part is filled up.



Thired time it is completely filled up.

We are observing that the cylindrical pot is filled completely by pouring the conical pot **thrice**.

So, the volume of cylinder = $3 \times$ volume of cone

$$\therefore \text{Volume of cone} = \frac{1}{3} \times \text{volume of cylinder} = \frac{1}{3} \pi r^2 h$$

$\therefore$ By hands on trial, we have got, **volume of cone** = $\frac{1}{3} \pi r^2 h$ [where r = length of the radius of base and h = height of cone]

$\therefore$ The water in the right circular conical flask of our school laboratory is = $\frac{1}{3} \pi r^2 h$

Application : 9. If the height of a corical flask of the science laboratory of the school is 4dcm. and slant height is 5 dcm, then let us calculate the quantity of water that can be hold in the flask.

Let, the length of the radius of base of conical flask be r dcm.

$$\therefore (5)^2 = r^2 + (4)^2$$

$$\text{or, } r^2 = 5^2 - 4^2 = 9 \therefore r = \pm 3$$

But the length of the radius can not be negative. $\therefore r \neq -3 \therefore r = 3$

$$\therefore \text{The quantity of water that can be hold in that flask} = \frac{1}{3} \times \frac{22}{7} \times 3 \times 3 \times 4 = \boxed{} \text{ cubic dcm.}$$

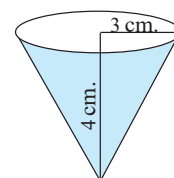
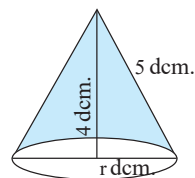
[Let me do it myself]

Application : 10. If the circumference of the base of right circular come is 2.2 m. and height is 45 dcm., then let us write by calculating, the volume of it. **[Let me do it myself]**

Application : 11. The lengths of the two sides adjacent to right angle of a right-angled triangle are 4 cm and 3cm.

Let us write by calculating, the curved surface area, total surface area and volume of the solid formed by completely revolving the triangle once by taking the longer side adjacent to right angle as axis.

Hints : The height of the right circular cone formed will be 4cm. and radius will be 3 cm.



Application : 12. The slant height of a right circular cone is 7 cm. and the total surface area of it is 147.84 sq.cm. Let us determine the base radius of the cone.

Let the length of the base radius of the cone = r cm.

$\therefore$ The total surface one of the cone = $\frac{22}{7} r(r+7)$ sq.cm.

$$\text{or, } \frac{22}{7} r(r+7) = 147.84$$

$$\text{or, } r(r+7) = \frac{14784}{100} \times \frac{7}{22} = \frac{1176}{25}$$

$$\text{or, } 25r^2 + 175r - 1176 = 0$$

$$\text{or, } 25r^2 + 280r - 105r - 1176 = 0$$

$$\text{or, } 5r(5r + 56) - 21(5r + 56) = 0$$

$$\text{or, } (5r + 56)(5r - 21) = 0$$

$$\text{Either, } 5r - 21 = 0 \text{ or, } 5r = 21 \text{ or, } r = \frac{21}{5}$$

$$\therefore r = 4.2$$

$$\text{or, } 5r + 56 = 0 \text{ or, } 5r = -56$$

$$\therefore r = \frac{-56}{5}$$

But the length of the radius can not be negative.

$$\therefore r \neq \frac{-56}{5}$$

$$\text{So, } r = 4.2$$

$\therefore$ The length of base radius of the cone is 4.2 cm.

Application : 13. If the ratio of the heights of the two right circular cone is 1:3 and the ratio of the lengths of their radii is 3:1, then let us show by calculating, that the ratio of the volumes of two cones will be 3:1.

Let the height of 1st cone be x unit, the height of 2nd cone be $3x$ unit, where x is common multiple. Again the length of the radius of 1st cone is $3y$ unit and the length of the radius of 2nd cone is y unit, where y is common multiple.

$$\text{Therefore, } \frac{\text{Volume of 1st cone}}{\text{Volume of 2nd cone}} = \frac{\frac{1}{3} \times \pi \times (3y)^2 \times x}{\frac{1}{3} \times \pi \times (y)^2 \times 3x} = \frac{9}{3} = \frac{3}{1} \quad \therefore \text{he ratio of the volumes of two cones is } 3:1$$

Application : 14. If the ratio of the heights of two right circular cone is 2:3 and the ratio of the lengths of their radii is 3:5. Then let us write by calculating, the ratio of the volumes of two cones.

[Let me do it myself]

Application : 15. The area of the base of a right circular conical tent is 13.86 sq.m. For making the tent, the tripal with the cost of `5775 is required and if the price of 1 sq.m. tent is `250, then let us determine the height of the tent. Let us write by calculating, the air in the tent in litre.

Let, the base radius of the tent be r m, height be h m, and slant height be l m.

Area of the base is 13.86 sq. m.

$$\text{According to the condition, } \frac{22}{7} \times r^2 = 13.86 \quad \therefore r = \boxed{} \quad [\text{Let me calculate it myself}]$$

$\therefore$ The length of the radius is 2.1 m.

$$\text{The quantity of the tripal in `5775 at the rate of `250 per metre} = \frac{5775}{250} \text{ sq.m} = 23.1 \text{ sq.m.}$$

$$\therefore \text{Curved (lateral) surface area} = \pi r l \text{ sq.m} = 23.1 \text{ sq.m}$$

$$\text{or, } \frac{22}{7} \times \frac{21}{10} \times l = 23.1 \quad \therefore l = \frac{7}{2} \quad [\text{Let me calculate if myself}]$$

$$\therefore \text{Slant height} = \frac{7}{2} \text{ m} = 3.5 \text{ m}$$

$$\therefore h = +\sqrt{l^2 - r^2} \quad [\because \text{height can not be negative}]$$

$$= \sqrt{(3.5)^2 - (2.1)^2} = \sqrt{(3.5+2.1) \times (3.5-2.1)} = \boxed{} \quad [\text{Let me write it myself}] \quad \therefore \text{Height} = 2.8 \text{ m.}$$

$$\therefore \text{The air occupied in the tent} = \frac{1}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.8 = \boxed{} \text{ m}^3 = 12936 \text{ dcm}^3 = 12936 \text{ litre}$$



Let us work out 16

1. I have made a closed right circular cone whose length of the base radius is 15cm and slant height is 24cm. Let us calculate the curved surface area and total surface area of that cone.
2. Let us determine the volume of the cone when, (i) base area is 1.54 sq.m. and height is 2.4m. (ii) the length of base diameter is 21m and slant height is 17.5m.
3. Amina has drawn a right-angled triangle whose lengths of two sides adjacent to right angle are 15 cm and 20 cm. Let us determine the curved surface area, total surface area and volume of the solid which is formed by taking the side of length 15cm. which is formed by completely revolving the triangle once around the side of the triangle with the length of 15cm, having been taken as an axis.
4. If the height and slant height of a cone are 6cm. and 10cm. respectively, let us determine total surface area and volume of the cone.
5. If the volume of a right circular cone is $100\pi \text{ cm}^3$ and height is 12cm., then let us write by calculating, the slant height of the cone.
6. 77 sq.m. tarpauline is required to make a right circular conical tent. If the slant height of the tent is 7 m., then let us write by calculating, the base area of the tent.
7. The diameter of the base area of a right circular cone is 21 m. and height is 14m. Let us calculate the expenditure to colour the curved surface at the rate of ` 1.50 per sq.m.
8. The length of the base diameter of a wooden toy of conical shape is 10cm. The expenditure for polishing whole surfaces of the toy at the rate of ` 2.10 per  $\text{cm}^2$  is ` 429. Let us calculate the height of the toy. Let us also determine the quantity of wood which is required to make the toy.
9. The quantity of iron sheet to make a boya of right circular conical shape is $75\frac{3}{7} \text{ m}^2$. If the slant height of it is 5m., then let us write, by calculating, the volume of air in the boya and its height. Let us determine the expenditure to colour the whole surface of the boy a at the rate of ` 2.80 per m^2 . [The width of the iron-sheet not to be considered while calculating.]
10. In a right circular conical tent 11 persons can stay. For each person 4m^2 space in the base and 20m^3 air are necessary. Let us determine the height of the tent put up exactly for 11 persons.
11. The external diameter of a conical- coronet made off thermocol is 21 cm. in length. To wrap up the outer surface of the coronet with foil, the expenditure will be ` 57.75 at the rate of 10p. per cm^2 . Let us write by calculating, the height and slant height of the coronet.
12. A heap of wheat is in the shape of a right circular cone, its base diameter is 9 m. and height is 3.5 m. Let us determine the total volume of wheat. Let us calculate the minimum quantity of plastic sheet to be required to cover up this heap of wheat [suppose $\pi = 3.14$, $\sqrt{130} = 11.4$]
13. **(V.S.A.)**
(A) (M.C.Q) :
(i) If the slant height of a right circular cone is 15cm. and the length of the base diameter is 16 cm., then the lateral surface area of the cone is
(a) $60\pi \text{ cm}^2$. (b) $68\pi \text{ cm}^2$. (c) $120\pi \text{ cm}^2$. (d) $130\pi \text{ cm}^2$.

- (ii) If the ratio of the volumes of two right circular cones is 1:4 and the ratio of their radii of their bases is 4:5, then the ratio of the heights is
(a) 1:5 (b) 5:4 (c) 25:16 (d) 25:64
- (iii) Keeping the radius of a right circular cone same, if the height of it is increased twice, the volume of it will be increased by
(a) 100% (b) 200% (c) 300% (d) 400%
- (iv) If each of radius and height of a cone is increased by twice of its length, then the volume of it will be
(a) 3 times (b) 4 times (c) 6 times (d) 8 times of the previous one.
- (v) If the length of the radius of a cone is $\frac{r}{2}$ unit and slant height of it is $2l$ unit, then the total surface area is
(a) $2\pi r(l+r)$ sq. unit (b) $\pi r(l + \frac{r}{4})$ sq. unit (c) $\pi r(l+r)$ sq. unit (d) $2\pi rl$ sq. unit

(B) Let us write whether the following statements are true or false :

- (i) If the length of base radius of a right circular cone is decreased by half and its height is increased by twice of it then the volume remains same.
- (ii) The height, radius and slant height of a right circular cone are always the three sides of a right-angled triangle.

(C) Let us fill in the blanks :

- (i) AC is the hypotenuse of a right-angled triangle ABC, the radius of the right circular cone formed by revolving the triangle once around the side AB as axis is—
- (ii) If the volume of a right circular cone is V cubic unit and the base area is A sq. unit, then its height is_____.
- (iii) The lengths of the base radii and the heights of a right circular cylinder and a right circular cone are equal. The ratio of their volumes is_____.

14. (S.A.)

- (i) The height of a right circular cone is 12cm. and its volume is $100\pi \text{ cm}^3$. Let us write the length of the radius of the cone.
- (ii) The curved surface area of a right circular cone is $\sqrt{5}$ times of its base area. Let us write the ratio of the height and the length of radius of the cone.
- (iii) If the volume of a right circular cone is V cubic unit, base area is A sq. unit and height is H unit, then let us write the value of $\frac{AH}{V}$.
- (iv) The numerical values of the volume and the lateral surface area of a right circular cone are equal. If the height and the radius of the cone are H unit and r unit respectively, then let us write the value of $\frac{1}{h^2} + \frac{1}{r^2}$.
- (v) The ratio of the lengths of the base radii of a right circular cylinder and a right circular cone is 3:4 and the ratio of their heights is 2:3; let us write the ratio of the volumes of the cylinder and the cone.

17

CONSTRUCTION OF TANGENT TO A CIRCLE

This year, in summer holidays the repairing work of our village club-room will be done.

The subscriptions are collected from all families of our locality. Each family has given subscription according to their ability. The iron grill has to be fixed up in front verandah of the club-room. So all the little ones of the locality, together will select the design of the grill.

Going to the iron shop we have selected a design of the grill like this—

On returning home, Maya tried to draw the design of the grill in her copy.

We observe that there are many circles and line-segments in our selected design. But how are the circles and line-segments in the design?

Some line-segments have touched the circles in the design, i.e. some line-segments are **tangents** of the circles.

How many tangents can be drawn in a circle? Let us observe by drawing it.

At a point on the circle, [1/2] tangent/tangents can be drawn. But from the point outside the circle [1/2] tangent/tangents can be drawn on the circle.

Construction : I draw a circle with a radius of 2.5 cm length and I draw a tangent on any point of the circle.

I have drawn a circle with its centre at O and radius of 2.5 cm length and A is any point on it; I draw a tangent of the circle with centre O at the point A.

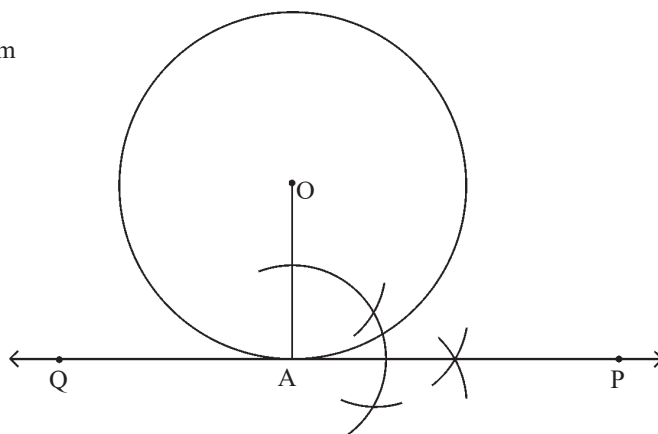
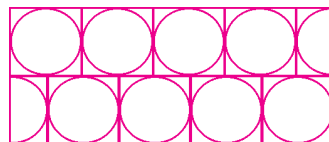
Rule of Construction :

- (i) I joined O, A.
 - (ii) I drew a perpendicular PQ 2.5 cm on OA at the point A with a pencil compass.
- ∴ PQ is the tangent at the point A of circle with centre O

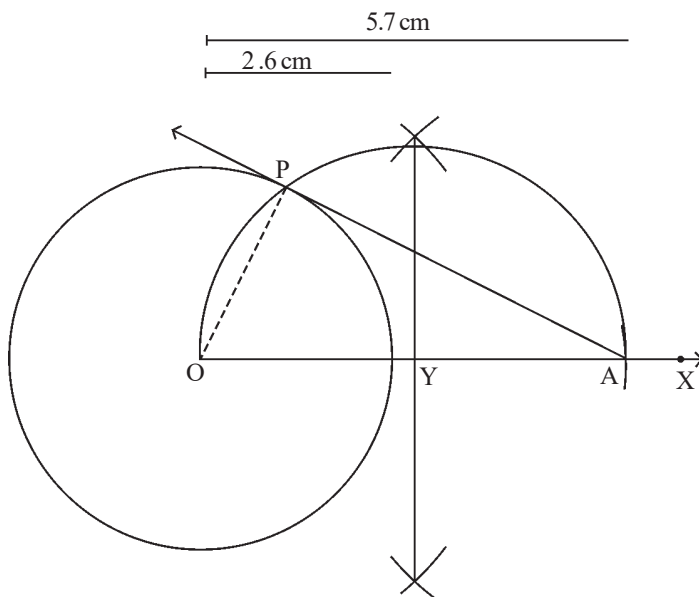
Proof : The radius OA of the circle with centre O intersects the circle at the point A and $PQ \perp OA$.

∴ PQ, is the tangent of the circle with centre O at the point A.

[∵ Any radius of a circle intersects at the point on the circle, the perpendicular drawn on that radius at that point is the tangent of the circle.]



Construction : I draw a circle with radius of 2.6 cm. length and I draw a tangent on this circle from an external point at a distant of 5.7 cm from the centre of that circle.



Rule of Construction :

- (i) I have drawn a circle with centre O and with radius of 2.6 cm. length. I cut off a line-segment OA of 5.7 cm length from any ray OX.
- (ii) I joined O, A.
- (iii) I bisect the line-segment OA at the point Y.
- (iv) Taking Y at centre and the radius of YO length. I draw a half circle which intersects the circle with centre O at the point P.
- (v) By joining A, P; I extended it.

$\therefore$ AP is the tangent of the circle with centre O from an external point A.

Proof : I join O, P.

$\angle OPA$ is a semi-circular angle. So $\angle OPA = 90^\circ$, OP is the radius of the circle with centre O and $OP \perp PA$. So, AP is a tangent of the circle with centre O.

If the distance of the external point A from the centre O of the circle is twice of the radius, then the mid-point Y of OA lies on the circle with centre O.



I have proved by hands on trial and with reason that, from any external point of a circle, two tangents can be drawn. I try to draw it myself.

Construction : I draw a circle with a radius of 3 cm length. I draw two tangents of the circle from an external point at a distance of 7 cm away from the centre.



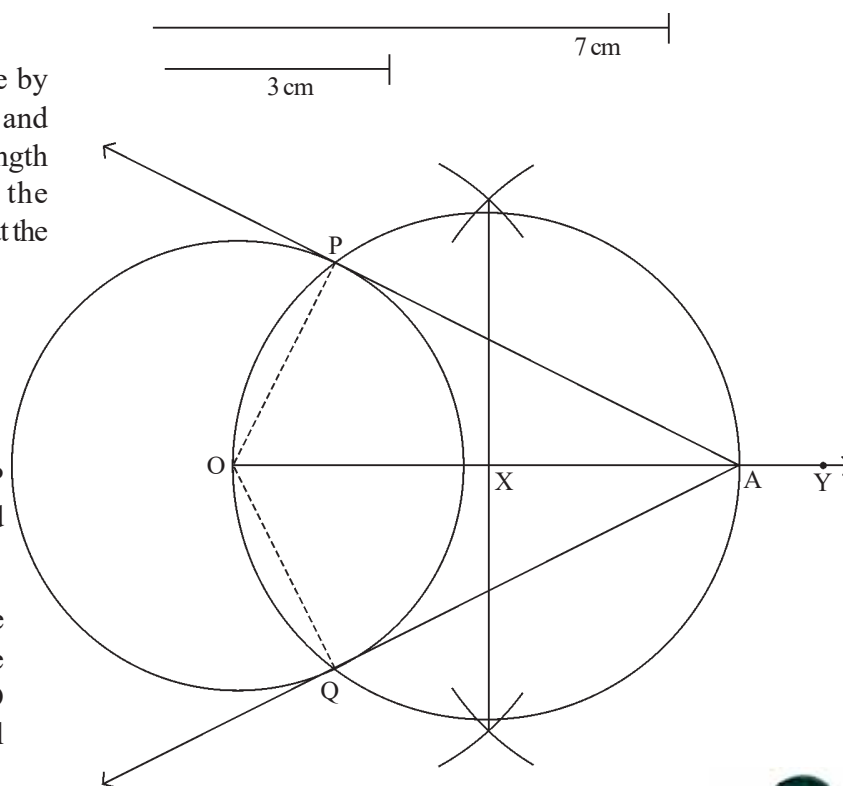
Rule of Construction :

- (i) I have drawn a circle with centre O whose radius is 3 cm. and I cut off the line-segment OA from the ray OY at a distance of 7 cm. from the centre O.
- (ii) I bisect the line-segment OA. Let the mid-point of OA be X.

- (iii) I draw a circle by taking X as centre and the radius of XO length which intersects the circle with centre O at the points P and Q.

- (iv) By joining A, P and A, Q I extended them.

$\therefore \vec{AP}$ and $\vec{AQ}$ are two tangents of the circle with centre O from the external point A.



Proof :

O, P are joined. $\angle OPA$ semi-circular angle.

$$\therefore \angle OPA = 90^\circ$$

Since OP is the radius of the circle with centre O and $OP \perp AP$, so $\vec{AP}$ is a tangent drawn on the circle with centre O from an external point P.

Similarly, $\vec{AQ}$ is another tangent of the circle with centre O.

Construction : I draw a circle with a radius of 2.5 cm length and I draw two tangents on the circle from an external point which is 10 cm away from the centre and I write the lengths of two tangents by measuring them. [let me do it myself]



1. Let us draw a circle with a radius of 3.2 cm length. Let us draw a tangent of the circle on any point of it.
2. By drawing a line-segment AB as a radius of 3 cm length. I draw a circle with centre A and a tangent of this circle at the point B.
3. I draw a circle with radius of 2.5 cm length. I take an external point at a distance of 6.5 cm. from the centre. From this external point I draw a tangent of the circle and measure the length of this tangent with a scale.
4. I draw a circle with a radius of 2.8 cm. length. I take a point which is at a distance of 7.5 cm from the centre. From that external point I draw two tangents of the circle.
5. PQ is a chord of a circle with centre O. I draw the tangents of the circle at the points P and Q.
6. I draw a line-segment XY of 8 cm. length and draw a circle by taking XY as a diameter. I draw the tangents of the circle at the points X and Y and write the relation between these two tangents.
7. By drawing any circle I draw two perpendicular diameters in it. I draw four tangents of the circle at four end points of two diameters and write the type of quadrilateral that is formed.
8. By drawing an equilateral triangle ABC with the sides of 5 cm. length, a circumcircle of it is drawn. Let us draw the tangents of that circumcircle at the points A, B and C.
9. I draw a circumcircle by drawing an equilateral triangle ABC with the sides of 5 cm. length. I draw a tangent of the circle at the point A and take a point P on it such that $AP = 5$ cm. From the point P, I draw another tangent of the circle and I write by observing the point at which this tangent has touched the circle.
10. O is a point on the line-segment AB and I draw a perpendicular PQ on AB at the point O. I draw two circles with centres A and B and radii of AO and BO length respectively and write the name of PQ in respect to these two circles. I draw other two tangents of the two circles from the point P.
11. P is a point on a circle with centre O. I draw a tangent of the circle at the point P and cut off a part 'PQ' equal in length to the radius of the circle, from the tangent. I draw another tangent QR of the circle from the point Q and write the value of $\angle PQR$ by measuring it with a protractor.

18

SIMILARITY

In this year, I have got a chance to play hockey for our district. For this reason, I have to fill up some forms and my photo is necessary there. My brother and I went to Prasanta uncle's digital store along with Mithundidi taking my recent photograph.



I told him to make 3 copies of photos from this photograph.

What size of the photo is needed ? Is it stamp size, passport size or post-card size ? What type will it be ? Explain it a little bit.



Stamp size



Passport size



Post-card size

I observe that the shapes of the photos are same but their sizes are different.

1 What relation do you find in the photos ?

These photos are similar to each other.

2 Will my passport size photo of 5 years ago and my recent passport photo be similar?

Let us see the two adjoining photos.



The two photos are not similar. Because, though the sizes of two photos are same, their shapes are different.

3 But will the two congruent figures be similar ?

Since the shape and size in the two congruent figures are equal, So the two congruent figures are always ☐ [similar/not similar]. [Let me write it myself]

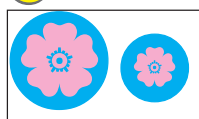
That is, if the sizes of more than one figures are different, but the shapes are same, then the figures will be similar to each other. So, congruent figures will always be similar, but similar figures may not be congruent.



On returning home, I have known that passport size photo is necessary and I gave an order to the shop accordingly.

On returning home, my brother drew many pictures and put the same pictures in separate groups.

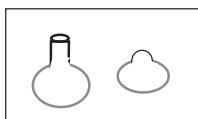
4 I observe whether the pictures in each group are similar or not.



(i)



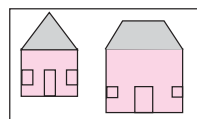
(ii)



(iii)



(iv)



(v)

I am observing that the pictures of the groups (i), (ii) and (iv) are ☐ to each other. But the pictures of the groups (iii) and (v) are not similar to each other.

- 5 I draw more than one squares of different sizes and see whether the squares are similar or not.



I am observing that, the squares ABCD, A'B'C'D' and A''B''C''D'' are similar to each other.

Again, I am observing, $\angle A = \angle A' = \angle A''$

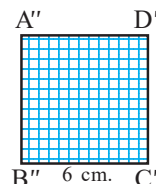
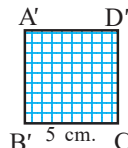
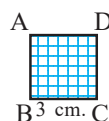
$\angle B = \angle B' = \angle B''$

$\angle C = \angle C' = \angle C''$

$\angle D = \angle D' = \angle D''$

$$\text{and } \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{DA}{D'A'} = \frac{3}{5}$$

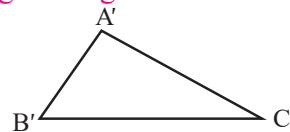
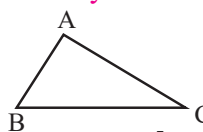
$$\frac{AB}{A''B''} = \frac{BC}{B''C''} = \frac{CD}{C''D''} = \frac{DA}{D''A''} = \frac{3}{6}$$



- 6 I draw two similar triangles ABC and A'B'C' and I see by measuring their angles and sides that

$\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$

$$\text{and } \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'} \quad [\text{Let me justify it by drawing myself}]$$

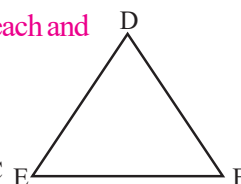
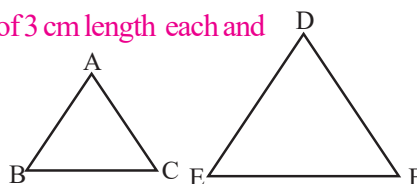


- 7 I draw two equilateral triangles one of which has sides of 3 cm length each and another has sides of 5 cm length each.

I am observing that, $\angle A = \angle D = 60^\circ$, $\angle B = \angle E = 60^\circ$,

$\angle C = \angle F = 60^\circ$

$$\text{and } \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{3}{5}; \text{ So, two triangles are similar.}$$



- 8 But will the two triangles be congruent?

I am observing that two triangles are not congruent.



i.e, if two triangles are congruent they are similar. But if they are similar, they are not congruent.

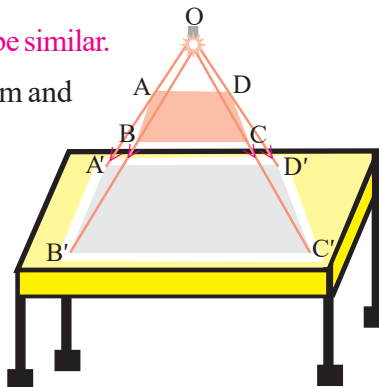
I am observing also that, if the three angles of a triangle are equal to the three angles of another triangle, then they may not be always congruent. That is, by A-A-A, the two triangles may not be always congruent.

I justify by hands on trial when the two polygonal figures will be similar.

- (1) I fixed a bulb at the point O near to the ceiling of the room and put a white art paper on a table just below it.

- (2) Now I put a polygonal cardboard (suppose quadrilateral ABCD) under the bulb and parallel to the table.

- (3) Now, if the bulb is put on, the shadow A'B'C'D' of the quadrilateral ABCD will fall on the art paper of the table and by joining with pencil I have got the quadrilateral A'B'C'D' which is similar to the quadrilateral ABCD. It is possible because light moves along straight line. A' is on the ray OA, B' is on the ray OB, C' is on the ray OC, D' is on the ray OD.



By measuring the angles and the sides of the quadrilaterals ABCD and A'B'C'D', I am observing that,

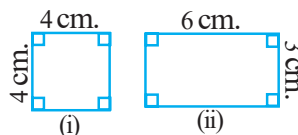
$$\angle A = \angle A', \angle B = \angle B', \angle C = \angle C', \angle D = \angle D', \text{ and } \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{DA}{D'A'}$$

[Let me justify similarly by hands on trial]

∴ By hands on trial, I have got that the two polygons with same number of sides will be similar if,

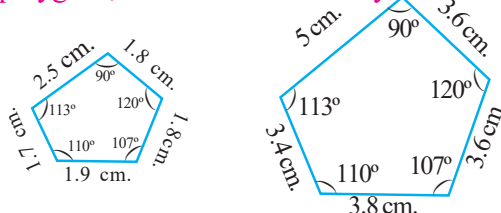
- (i) Their corresponding angles are equal and
- (ii) Corresponding sides are proportional

Saheli has drawn two quadrilaterals, let us see whether they are similar or not.



I am observing that, the two quadrilaterals (i) and (ii) drawn by Saheli are not similar. Because, though the corresponding angles of the quadrilateral are equal, the ratios of the corresponding sides are not equal.

Will a square and a rhombus always be similar? By drawing I write it with reason. [Let me do it myself]
Palash has drawn two polygons, let us see whether they are similar or not.



I am observing that, the two polygons drawn by Palash are similar. I write reasons by understanding it.

Let us work-out 18.1

1. Let us write the right answer in

- (i) All squares are [congruent/similar]
- (ii) All circles are [congruent/similar]
- (iii) All [equilateral/isosceles] triangles are always similar.
- (iv) Two quadrilaterals will be similar if their corresponding angles are [equal/proportional] and corresponding sides are [unequal/proportional].

2. Let us write whether the following statements are true or false :

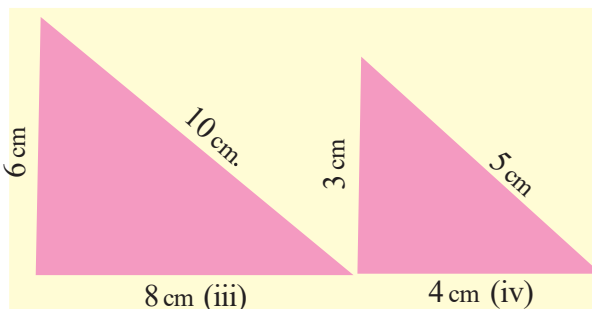
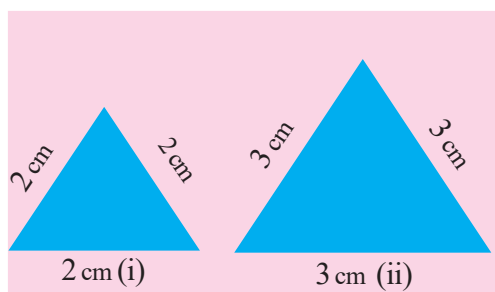
- (i) Any two congruent figures are similar.
- (ii) Any two similar figures are always congruent.
- (iii) The corresponding angles of any two similar polygonal figures are equal.
- (iv) The corresponding sides of any two similar polygonal figures are proportional.
- (v) The square and rhombus are always similar.

3. Let us write the examples of one pair of similar pictures/figures.

4. Let us draw a pair of figures which are not similar.

My friend Shakil has made triangles of different colours and of different types.

I keep the similar triangles in separate groups by measuring the angles and the lengths of the sides of these triangles.



The two figures (i) & (ii) are similar and the two figures (iii) & (iv) are similar. Because their corresponding sides are and their corresponding angles are equal.

9 What the corresponding angles of two triangles are called when they are equal ?



If the corresponding angles of two triangles are equal, then they are called similar triangle.

I have understood that, if two triangles are similar, then (i) their corresponding sides are proportional and (ii) the angles of two triangles are equal.

Greek Mathematician Thales told a truth about similar triangles. The truth is — "the corresponding sides of similar triangles are proportional." Many persons believe that he used a result and it is called Basic Proportionality Theorem or Thales theorem.

The Basic proportionality Theorem or Thales Theorem is —

If a line is drawn parallel to one side of a triangle intersecting the others two sides, then the other two sides or their extended sides are divided in the same ratio.

10 But what shall we understand by the division of a straight line in a fixed ratio ?

P is any point on a line segment AB. Here, the point P divides internally the line-segment AB in the ratio of AP : PB.



If P is the mid-point of the line-segment AB, then $\frac{AP}{PB} = 1$.



If the point P is near to A from the mid-point but far from the point B, then the value of $\frac{AP}{PB}$ will be less than 1.



Again if the point P is near to B from the mid-point but far from the point A, then the value of $\frac{AP}{PB}$ will be greater than 1.



11 But what will happen when the point P will lie on the extended line-segment AB ?

Here, the point P divides the line-segment AB externally in the ratio AP : PB. As the length of AP is greater than the length of PB, the value of $\frac{AP}{PB}$ will be greater than 1.

12 But what will happen, when the point P will lie on the extended line-segment BA ?

Here also, the point P divides externally the line-segment AB in the ratio AP : PB. As the length of PB is greater than the length of AP, the value of $\frac{AP}{PB}$ is less than 1.

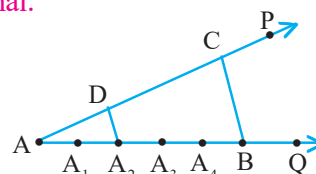
I have understood that, **if a line parallel to any side of a triangle divides other two sides (or extended two sides) in the same ratio, then they will be proportional.**

By hands on trial, I justify Thales Theorem.

- (1) I drew $\angle PAQ$ on my copy. On the ray AQ I took five points A_1, A_2, A_3, A_4 and B in such a way that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4B$

- (2) I draw a straight line from the point B which intersects the ray AP at the point C.

- (3) I draw a line parallel to BC through A_2 which intersects the ray AP at the point D
 $\therefore$ I have got, $\frac{AA_2}{A_2B} = \frac{2}{3}$



Measuring with a scale, I am observing that, $\frac{AD}{DC} = \frac{2}{3}$ [Let me justify it by drawing]

$\therefore$ I have got that if $BC \parallel A_2D$ in $\triangle ABC$, then $\frac{AA_2}{A_2B} = \frac{AD}{DC}$

$\therefore$ **By hands on trial, I have got that a line parallel to any side of a triangle divides proportionally other two sides (or, the extended two sides).**



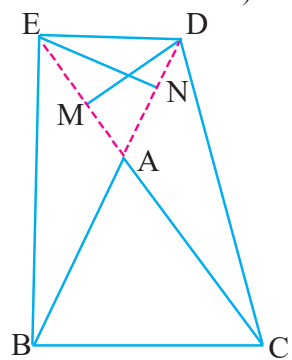
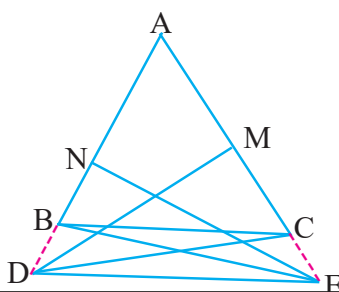
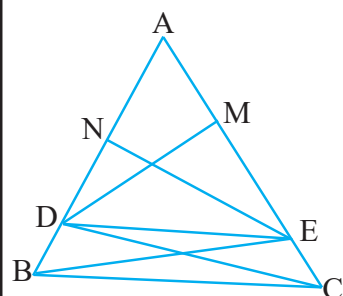
Let us prove with reason

Theorem : 43. "A straight line parallel to any side of any triangle divides other two sides (or the extended two sides) proportionally" (Proof is not included in Evaluation)

Given : A line parallels to the side BC of $\triangle ABC$ intersects the sides AB and AC or the extended sides of AB and AC (the extended sides of BA and CA) at the points D and E respectively.

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Construction : I joined B, E and C, D and I drew $DM \perp AC$ (or the extended side of CA) and $EN \perp AB$ (or the extended side of BA).



Proof: Area of $\triangle ADE = \frac{1}{2} \times \text{base} \times \text{height}$
 $= \frac{1}{2} \times AD \times EN$

Similarly, Area of $\triangle BDE = \frac{1}{2} \times \text{base} \times \text{height}$
 $= \frac{1}{2} \times DB \times EN$



$$\therefore \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \text{———— (I)}$$

Again, area of $\triangle ADE = \frac{1}{2} \times AE \times DM$

And area of $\triangle DEC = \frac{1}{2} \times EC \times DM$

$$\therefore \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \text{———— (II)}$$

Again, $\triangle BDE$ and $\triangle DEC$ lie on the same base DE and they are situated within the same pair of parallel lines DE and BC .

$\therefore \text{Area of } \triangle BDE = \text{Area of } \triangle DEC$

$\therefore$ From (I) and (II) we get, $\frac{AD}{DB} = \frac{AE}{EC}$ **[proved]**

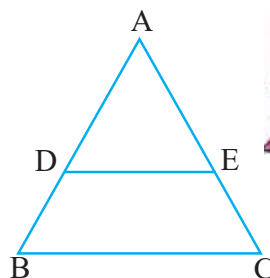
Corollary : 1 If a straight line parallel to BC of $\triangle ABC$ intersects AB and AC at the points D and E respectively, then let us prove that,

(i) $\frac{AB}{DB} = \frac{AC}{EC}$ (ii) $\frac{AD}{AB} = \frac{AE}{AC}$

Proof: (i) From Thales Theorem we get, $\frac{AD}{DB} = \frac{AE}{EC}$

or, $\frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

or, $\frac{AD + DB}{DB} = \frac{AE + EC}{EC} \therefore \frac{AB}{DB} = \frac{AC}{EC}$ **[Proved]**



(ii) From Thales Theorem we get, $\frac{AD}{DB} = \frac{AE}{EC}$

or, $\frac{DB}{AD} = \frac{EC}{AE}$

or, $1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$ or, $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$

or, $\frac{AB}{AD} = \frac{AC}{AE} \therefore \frac{AD}{AB} = \frac{AE}{AC}$ **[Proved]**

Is the converse of Thales Theorem possible ? That is if a straight line divides any two sides (or their extended sides) of a triangle in the same ratio, will the straight line be parallel to the third side of the triangle ?



Hands on trial

- (1) I drew $\angle PAQ$ on my copy. I took the points $A_1, A_2, A_3, A_4, A_5, A_6$ and B on the ray AP such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6B$

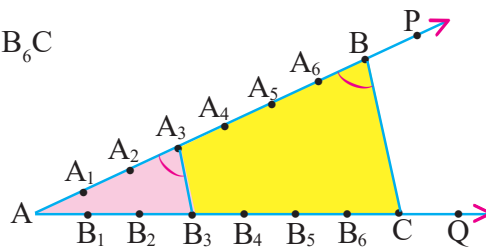
→ Again I took the points $B_1, B_2, B_3, B_4, B_5, B_6$ and C on the ray AQ such that

$$AB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6C$$

- (2) Now I got $\triangle ABC$ by joining B and C .

By joining A_3 and B_3 , I am observing that,

$$\frac{AA_3}{A_3B} = \frac{AB_3}{B_3C} = \frac{3}{4}$$



- (3) Now by cutting off $\angle AA_3B_3$ and putting it on $\angle ABC$, I am observing that the two angles are overlapping.

∴ I have got $\angle AA_3B_3 = \angle ABC$, which are corresponding angles.

∴ $A_3B_3 \parallel BC$

Similarly by joining A_1, B_1 , we shall get $\frac{AA_1}{A_1B} = \frac{AB_1}{B_1C} = \frac{\square}{\square}$ and $A_1B_1 \parallel BC$

By joining A_2, B_2 , we shall get $\frac{AA_2}{A_2B} = \frac{AB_2}{B_2C} = \frac{\square}{\square}$ and $A_2B_2 \parallel BC$ [putting $=$ /]]

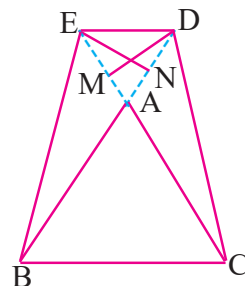
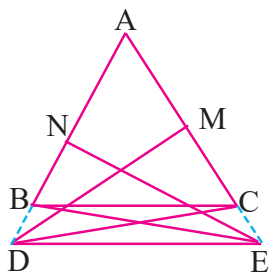
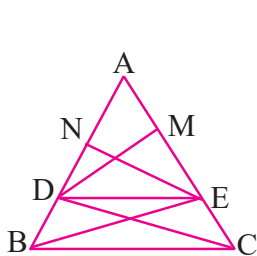
What we shall get by joining A_4, B_4 or A_5, B_5 or A_6, B_6 , Let me justify it and write it myself.



In a similar way, I have drawn another triangle and a straight line which has divided any other two sides (or their extended sides) in the same ratio and I observe that the straight line is parallel to third side. **[Let me do it myself]**

∴ By hands on trial, we have got, **the straight line which divides any two sides (or their extended sides) of a triangle in the same ratio, will be parallel to the third side.**

Theorem : 44. I prove with reason that "if a straight line divides any two sides (or their extended sides) in the same ratio, it will be parallel to third side." (The proof is not included in the evaluation)



Given : ABC is a triangle. A line-segment intersects the two sides AB and AC or their extended sides (or the extended sides of BA and CA) at the points D and E in such a way that,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

To prove : $DE \parallel BC$

Construction : I join B, E and C, D. I draw $DM \perp AC$ (or the extended side of CA) and $EN \perp AB$ (or the extended side of BA).

Proof: Area of $\triangle ADE = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times AD \times EN$

Area of $\triangle BDE = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times DB \times EN$

$$\therefore \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB}$$

Again, Area of $\triangle ADE = \frac{1}{2} \times AE \times DM$

Area of $\triangle DEC = \frac{1}{2} \times EC \times DM$

$$\therefore \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$$

Since, $\frac{AD}{DB} = \frac{AE}{EC}$ (Given)

$$\text{So, } \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle BDE} = \frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle DEC}$$

$\therefore$ Area of $\triangle BDE = \text{Area of } \triangle DEC$

As $\triangle BDE$ and $\triangle DEC$ lie on the same base DE and situated on the same side of DE and their areas are equal, so they lie within the same pair of parallel lines.

$\therefore DE \parallel BC$ **[Proved]**

Application : 1. In adjoining figure, $DE \parallel BC$ of $\triangle ABC$; if $AD = 5$ cm, $DB = 6$ cm and $AE = 7.5$ cm, then let us write by calculating, the length of AC .

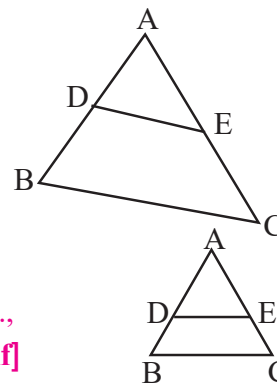
In $\triangle ABC$, $DE \parallel BC$, $\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [From Thales theorem]



$$\therefore \frac{5}{6} = \frac{7.5}{EC}$$

$$\therefore EC = 7.5 \times \frac{6}{5} \text{ cm} = 9 \text{ cm}$$

$$\therefore AC = AE + EC = \boxed{} + \boxed{} = \boxed{}$$



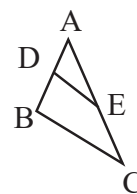
Application : 2. If $BC \parallel DE$ in $\triangle ABC$, $\frac{AD}{DB} = \frac{2}{5}$ and $AC = 21$ cm., then let us write by calculating, the value of AE [Let me do it myself]

Application : 3. The line parallel to the side BC of the triangle ABC intersects AB and AC at the points D and E respectively. If $AE = 2 AD$, then let us calculate the value of $DB : EC$.

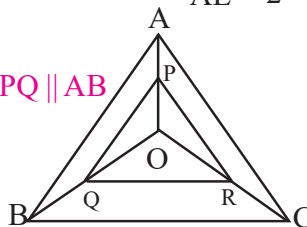
In $\triangle ABC$, $DE \parallel BC$, $\therefore \frac{AD}{DB} = \frac{AE}{EC}$

$$\therefore \frac{DB}{EC} = \frac{AD}{AE} = \frac{1}{2} \quad [\because AE = 2AD \quad \therefore \frac{AD}{AE} = \frac{1}{2}]$$

$$\therefore DB : EC = 1 : 2$$



Application : 4. If in the adjoining figure, $PQ \parallel AB$ and $PR \parallel AC$, then let us prove that $QR \parallel BC$.



In $\triangle OAB$, $PQ \parallel AB$, $\therefore \frac{OP}{PA} = \frac{OQ}{QB}$ [From Thales theorem] _____ (i)

Again, in $\triangle AOC$, $PR \parallel AC$, $\therefore \frac{OP}{PA} = \frac{OR}{RC}$ [From Thales theorem] _____ (ii)

From (i) and (ii), we get, $\frac{OQ}{QB} = \frac{OR}{RC}$

$\therefore$ I have got two points Q and R on OB and OC of $\triangle OBC$ respectively so that $\frac{OQ}{QB} = \frac{OR}{RC}$

$\therefore$ From the converse of Thales theorem, we have got $QR \parallel BC$.

Application : 5. A straight line intersects AB and AC of $\triangle ABC$ at the points D and E respectively in such a way that $\frac{AD}{DB} = \frac{AE}{EC}$. If $\angle ADE = \angle ACB$, then let us prove that $\triangle ABC$ is an isosceles triangle.

Given : The line segment DE of $\triangle ABC$ intersects the side AB and AC at the points D and E respectively in such a way that $\frac{AD}{DB} = \frac{AE}{EC}$

To prove : $\triangle ABC$ is an isosceles triangle

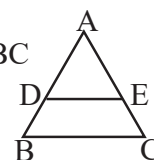
Proof : Since, $\frac{AD}{DB} = \frac{AE}{EC}$, so from converse of Thales theorem, we get $DE \parallel BC$

$$\therefore \angle ADE = \angle ABC \text{ [Corresponding angles]} \text{ _____ (i)}$$

$$\text{Again, } \angle ADE = \angle ACB \text{ [Given]} \text{ _____ (ii)}$$

$$\therefore \text{From (i) \& (ii), we get, } \angle ACB = \angle ABC$$

$$\therefore AB = AC \quad \therefore \triangle ABC \text{ is an isosceles triangle.}$$



Application : 6 I have drawn a trapezium ABCD whose $AB \parallel DC$; I have drawn a straight line parallel to AB which has intersected AD and BC at the points E and F respectively. I prove that $AE : ED = BF : FC$

Given : $AB \parallel DC$ in the trapezium ABCD; the straight line parallel to AB has intersected AD and BC at the points E and F respectively.

To prove : $AE : ED = BF : FC$

Construction : I joined A, C which has intersected EF at the point G.

Proof : In $\triangle ADC$, $DC \parallel EG$

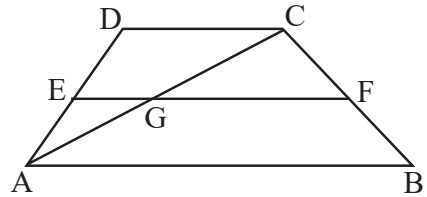
So, from Thales theorem, we get, $\frac{AE}{ED} = \frac{AG}{GC}$ ————— (i)

Again, in $\triangle ACB$, $AB \parallel GF$

$\therefore$ From Thales theorem, we get, $\frac{AG}{GC} = \frac{BF}{FC}$ ————— (ii)

So, from (i) and (ii), we get $\frac{AE}{ED} = \frac{BF}{FC}$

$\therefore AE : ED = BF : FC$



Application : 7. With the help of the converse of Thales theorem, let us prove that the line segment joining two mid-points of a triangle is parallel to its third side. **[Let me do it myself]**

Application : 8. I draw a trapezium whose $AB \parallel DC$; I take two points P and Q on AD and BC respectively so that $AP : PD = BQ : QC$. Let us prove that $PQ \parallel DC$.

Given : In the trapezium, $AB \parallel DC$; the two points P and Q are on the two sides AD and BC respectively in such a way that $AP : PD = BQ : QC$

To prove : $PQ \parallel DC$

Construction : Let $AB < DC$; DA and CB are extended. The extended sides of DA and CB meet at O. P, Q are joined.

Proof : $AB \parallel DC$ in $\triangle ODC$

$\therefore \frac{OA}{AD} = \frac{OB}{BC}$ [From Thales theorem] ————— (i)

Again $\frac{AP}{PD} = \frac{BQ}{QC}$ [given]

i.e., $\frac{PD}{AP} = \frac{QC}{BQ}$ or, $1 + \frac{PD}{AP} = 1 + \frac{QC}{BQ}$ or, $\frac{AP + PD}{AP} = \frac{BQ + QC}{BQ}$

$\therefore \frac{AD}{AP} = \frac{BC}{BQ}$ ————— (ii)

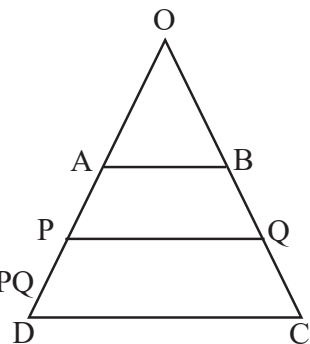
From (i) and (ii), we get, $\frac{OA}{AD} \times \frac{AD}{AP} = \frac{OB}{BC} \times \frac{BC}{BQ}$

$\therefore \frac{OA}{AP} = \frac{OB}{BQ}$

$\therefore$ I have got, in $\triangle OPQ$ $\frac{OA}{AP} = \frac{OB}{BQ}$

$\therefore$ From the converse of Thales theorem, we get $AB \parallel PQ$

Again, $AB \parallel DC$, $\therefore PQ \parallel DC$ **[Proved]**



Application : 9 I prove with reason that an internal or external bisector of any angle of a triangle divides its opposite side in the ratio of the lengths of the adjacent sides of the angle internally or externally. (The proof is not included in the evaluation)

Given: The internal bisector AD (fig-i) or external bisector (fig-ii) of $\angle BAC$ of $\triangle ABC$ intersects BC or extended BC at the point D.

To prove : $BD : DC = AB : AC$

Construction : I draw a line parallel to DA through the point C which intersects the extended side of BA or BA at the point E.

Proof: $DA \parallel CE \therefore \angle DAC = \text{alternate } \angle ACE$

$DA \parallel CE, \therefore \angle BAD$ (or $\angle FAD$ in the fig-ii) = Corresponding $\angle AEC$

But $\angle BAD$ (or $\angle FAD$ in fig-ii) = $\angle DAC$

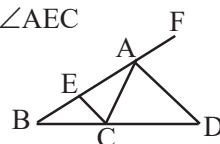
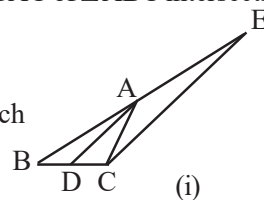
$\therefore \angle ACE = \angle AEC$

So, $AC = AE$

In $\triangle BEC$ (fig-i) or $\triangle BDA$ (fig-ii) $DA \parallel CE; \therefore \frac{BD}{DC} = \frac{BA}{AE}$ (By Thales theorem)

i.e, $BD : DC = AB : AE$

So, $BD : DC = AB : AC$ ($\because AE = AC$) **[proved]**



Application : 10 I prove with reason, if a line drawn from any angle of a triangle divides its opposite side internally or externally in the ratio of the lengths of its adjacent sides, then the line will be an internal or external bisector of the angle. (The proof is not included in the evaluation)

Given: In $\triangle ABC$ a line drawn through A intersects the side BC (fig-i) or extended BC at the point D (fig-ii) in such a way that, $BD : DC = AB : AC$

To prove : AD is an internal (fig-i) or an external (fig-ii) bisector of $\angle BAC$.

Construction : I draw a line parallel to the side DA through C which intersects the extended BA (fig-i) or BA (fig-ii) at the point E.

Proof: In $\triangle BCE$ (fig-i) or in $\triangle ABD$ (fig-ii), $DA \parallel CE$

$\therefore \frac{BD}{DC} = \frac{AB}{AE}$ (By Thales theorem)

But, $\frac{BD}{DC} = \frac{AB}{AC}$ (Given)

$\therefore \frac{AB}{AE} = \frac{AB}{AC}$

$\therefore AE = AC$

$\therefore \angle AEC = \angle ACE$

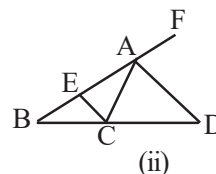
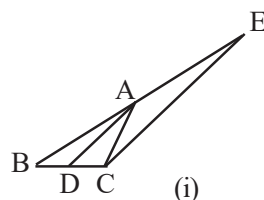
Again, $DA \parallel CE$;

$\therefore \angle DAC = \text{alternate } \angle ACE$ and $\angle BAD$ (fig-i) or $\angle FAD$ (fig-ii) = Corresponding $\angle AEC$.

Since, $\angle AEC = \angle ACE$,

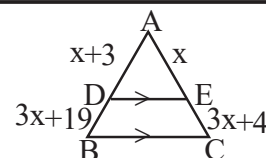
So $\angle BAD$ (fig-i) or $\angle FAD$ (fig-ii) = $\angle DAC$.

So AD is an internal (fig-i) or an external (fig-ii) bisector of $\angle BAC$. **[proved]**

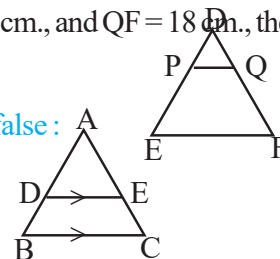


1. A line parallel to the side BC of $\triangle ABC$ intersects the sides AB and AC at the points P and Q respectively.
 - (i) If $PB = AQ$, $AP = 9$ units, $QC = 4$ units, then let us calculate the length of PB.
 - (ii) The length of PB is twice of AB and the length of QC is 3 units more than the length of AQ, then let us write, by calculating the length of AC.
 - (iii) If $AP = QC$, the length of AB is 12 units and the length of AQ is 2 units, then let us calculate the length of CQ.
2. I took two points X and Y on the sides PQ and PR respectively of $\triangle PQR$.
 - (i) If $PX = 2$ units, $XQ = 3.5$ units, $YR = 7$ units and $PY = 4.25$ units, then let us write with reason, whether XY and QR will be parallel to each other or not.
 - (ii) If $PQ = 8$ units, $YR = 12$ units, $PY = 4$ units and the length of PY is less by 2 units than the length of XQ, then let us write with reason, whether XY and QR will be parallel or not.
3. Let us prove that the line drawn through mid-point of one side of a triangle parallel to another side bisects the third side. [Let us prove it with the help of Thales theorem.]
4. P is a point in the median AD of $\triangle ABC$. The extended BP and CP intersects AC and AB at the points Q and R respectively. Let us prove that, $RQ \parallel BC$.
5. The two medians BE and CF of $\triangle ABC$ intersect each other at the point G and if the line segment FE intersects the line segment AG at the point O, then let us prove that $AO = 3 OG$.
6. Let us prove that, the line segment joining the mid-points of two transverse sides of a trapezium is parallel to its parallel sides.
7. D is any point on the side BC of $\triangle ABC$. P and Q are centroids of $\triangle ABD$ and $\triangle ADC$ respectively. Let us prove that $PQ \parallel BC$.
8. I have drawn two triangles $\triangle PQR$ and $\triangle SQR$ on same base QR and on same side of QR and their areas are equal. If F and G are two centroids of two triangles, then let us prove that, $FG \parallel QR$.
9. Let us prove that the two adjacent angles of any parallel side of a trapezium are equal.
10. $\triangle ABC$ and $\triangle DBC$ are situated on the same base BC and on the same side of BC. E is any point on the side BC. A line through the point E and parallel to AB and BD intersects the sides AC and DC at the points F and G respectively. Let us prove that, $AD \parallel FG$.
11. **V.S.A.**
(A) **M.C.Q.**
 - (i) A line parallel to the side BC of $\triangle ABC$ intersects the sides AB and AC at the points X and Y respectively. If $AX = 2.4$ cm., $AY = 3.2$ cm. and $YC = 4.8$ cm., then the length of AB is
(a) 3.6 cm. (b) 6 cm. (c) 6.4 cm. (d) 7.2 cm.
 - (ii) The point D and E are situated on the sides AB and AC of $\triangle ABC$ in such a way that $DE \parallel BC$ and $AD : DB = 3 : 1$; if $EA = 3.3$ cm., then the length of AC is

- (a) 1.1 cm. (b) 4 cm. (c) 4.4 cm. (d) 5.5 cm.
- (iii) In the adjoining figure if $DE \parallel BC$, then the value of x is
(a) 4 (b) 1 (c) 3 (d) 2

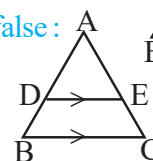


- (iv) In the trapezium ABCD, $AB \parallel DC$ and the two points P and Q are situated on the sides AD and BC in such a way that $PQ \parallel DC$; if $PD = 18$ cm., $BQ = 35$ cm., $QC = 15$ cm., then the length of AD is
(a) 60 cm. (b) 30 cm. (c) 12 cm. (d) 15 cm.
- (v) In the adjoining figure, if $DP = 5$ cm., $DE = 15$ cm., $DQ = 6$ cm., and $QF = 18$ cm., then
(a) $PQ = EF$ (b) $PQ \parallel EF$ (c) $PQ \neq EF$ (d) $PQ \nparallel EF$



(B) Let us write whether the following statements are true or false :

- (i) Two similar triangles are always congruent.
- (ii) In the adjoining figure, if $DE \parallel BC$, then $\frac{AB}{BD} = \frac{AC}{CE}$

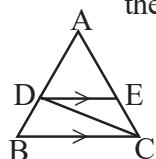


(C) Let us fill up the blanks :

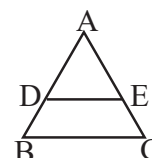
- (i) The line segment parallel to any side of a triangle divides other two sides or the extended two sides _____.
- (ii) If the bases of two triangles are situated on a same line and the other vertex of the two triangles are common, then the ratio of the areas of two triangles are _____ to the ratio of their bases.
- (iii) The straight line parallel to the parallel sides of a trapezium divides _____ other two sides.

12. **S.A.**

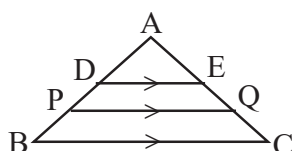
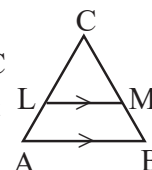
- (i) In the adjoining figure, if in $\triangle ABC$, $\frac{AD}{DB} = \frac{AE}{EC}$ and $\angle ADE = \angle ACB$, then let us write the type of the triangle according to side.



- (ii) In the adjoining figure if $DE \parallel BC$ and $AD : BD = 3 : 5$, then let us write area of $\triangle ADE$: area of $\triangle CDE$.

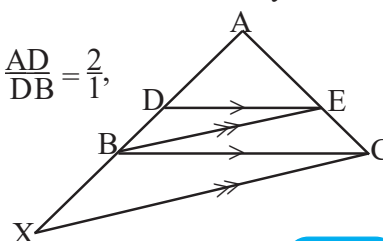


- (iii) In the adjoining figure, if $LM \parallel AB$ and $AL = (x-3)$ unit, $AC = 2x$ unit, $BM = (x-2)$ unit and $BC = (2x+3)$ unit, then let us determine the value of x .



- (iv) In the adjoining figure, if in $\triangle ABC$, $DE \parallel PQ \parallel BC$ and $AD = 3$ cm., $DP = x$ cm., $PB = 4$ cm., $AE = 4$ cm., $EQ = 5$ cm., $QC = y$ cm., then let us determine the value of x and y .

- (v) In the adjoining figure, if $DE \parallel BC$, $BE \parallel XC$ and $\frac{AD}{DB} = \frac{2}{1}$, then let us determine the value of $\frac{AX}{XB}$



Today we have decided that we will hung the similar triangles among the triangular regions of the card board made by Shakil in our classroom by fixing them on an artpaper.



But are the corresponding sides of all these similar triangles proportional? That is, will the two triangles be similar? Let us justify it by hands on trial.

Hand on trial

- At first, I took two coloured triangular regions ABC and PQR whose corresponding angles are same i.e., $\angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$ and $PQ > AB$, $PR > AC$, $QR > BC$.

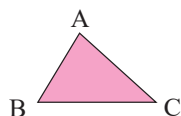


fig (i)

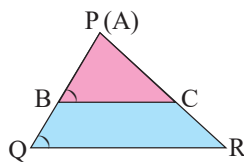


fig (ii)

- I put the triangular region ABC on the triangular region PQR in such a way that the vertex A overlaps on the vertex P and the side AB remains on the side PQ. [Like the fig (ii)]

I am observing that, (i) The side AC of $\triangle ABC$ overlaps on the side PR [Let us justify it myself]

$$(ii) \frac{PB}{PQ} = \frac{PC}{PR} \therefore \frac{AB}{PQ} = \frac{AC}{PR} \text{ [Let us justify it myself] } \text{————— (I)}$$

Explanation : In the figure (ii) $\angle B = \angle Q$

$\therefore BC \parallel QR$ [$\because$ the corresponding angles are equal]

$\therefore \frac{PB}{BQ} = \frac{PC}{CR}$ [By Thales theorem]

$$\frac{AB}{BQ} = \frac{AC}{CR}$$

$$\text{or, } \frac{BQ}{AB} = \frac{CR}{AC}$$

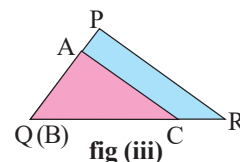
$$\text{or, } 1 + \frac{BQ}{AB} = 1 + \frac{CR}{AC}$$

$$\text{or, } \frac{AB + BQ}{AB} = \frac{AC + CR}{AC}$$

$$\text{or, } \frac{AQ}{AB} = \frac{AR}{AC} \therefore \frac{PQ}{AB} = \frac{PR}{AC} \text{ so, } \frac{AB}{PQ} = \frac{AC}{PR}$$



3. In the same way I put the triangular region ABC on the triangular region PQR in such a way like the adjoining figure that the vertex 'B' and the vertex Q overlaps to each other and the side AB remains on the side PQ.



I am observing that, (i) The side BC overlaps with side QR. [Let us justify it by hands on trial]

and (ii) $\frac{AQ}{PQ} = \frac{QC}{QR}$, $\therefore \frac{AB}{PQ} = \frac{BC}{QR}$ [Let me justify it myself] _____ (II)

I write the explanation myself as above. [Let me do it myself]

$\therefore$ From (I) & (II) we have got, $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

$\therefore$ By hands on trial, we have got, the corresponding sides of two similar triangles are proportional, i.e. two triangles are similar.

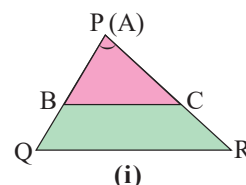
But can its converse be possible? i.e. if the sides of two triangles are proportional, will their corresponding angles be equal? Let us justify by hands on trial, whether two triangles are similar or not if their corresponding sides are in proportional.



Hands on trial

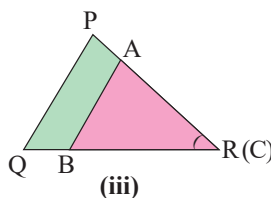
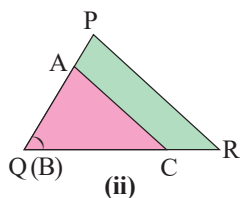
1. At first by cutting out a coloured art paper I made two triangular regions ABC and PQR whose sides are in proportional. i.e. $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

2. Now as in the adjoining figure, I put the triangular region ABC on the triangular region PQR in such a way that the vertex A and the vertex P are overlapped with each other and the side AB remains on the side PQ.



I am observing that, the side AC has overlapped with the side PR. i.e. I have got $\angle A = \angle P$

3. In the same way, like the following fig (ii) and fig (iii), by putting $\triangle ABC$ on $\triangle PQR$, I am observing that, $\angle B = \square$ and $\angle C = \square$ [Let us write by justifying it]



$\therefore$ By hands on trial, I have got, if the corresponding sides of two triangles are proportional, then their corresponding angles are equal i.e. two triangles are similar.

By taking any other two triangles in the same way justifying hands on trial we are observing that,

- (i) if two triangles are similar, their corresponding sides are proportional,
again (ii) if the sides of the triangles are proportional then they are similar. [Let me do it myself]

Let us prove with reason

Theorem : 45. If two triangles are similar then their corresponding sides are in same ratio i.e. their corresponding sides are proportional [Proof is not included in Evaluation]

Given : ABC and DEF are two similar triangles. i.e. $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

To prove : $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

Construction : From $\triangle DEF$, I cut off DP and DQ of equal length with AB & AC respectively from DE or extended DF. I joined P and Q.

Proof : In $\triangle ABC$ and $\triangle DPQ$

$AB = DP$, $\angle A = \angle D$ and $AC = DQ$

$\therefore \triangle ABC \cong \triangle DPQ$ (By S-A-S criterion of congruency)

$\therefore \angle B = \angle P$

Again, $\angle B = \angle E$ [Given]

$\therefore \angle P = \angle E$

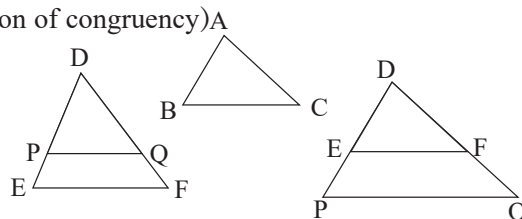
So, $PQ \parallel EF$ [$\because$ Corresponding angles are equal]

$\therefore \frac{DP}{DE} = \frac{DQ}{DF}$ (From Thales theorem)

$\therefore \frac{AB}{DE} = \frac{AC}{DF}$ (I) [$\because DP = AB$ and $DQ = AC$]

Similarly, by cutting off from ED or extended ED the length equal to BA and from EF or extended EF the length equal to BC, I can prove that, $\frac{AB}{DE} = \frac{BC}{EF}$ (II)

$\therefore$ From (I) & (II), we have got, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ [Proved]



We have understood that, if two angles of one triangle are equal to two angles of another triangle, then their corresponding sides are proportional, let me write by understanding the reason. [Let me do it myself]

Let us prove with reason

Theorem : 46. If the sides of two triangles are in same ratio, then their corresponding angles are equal. i.e. two triangles are similar. [Proof does not belong to evaluation]

Given : In $\triangle ABC$ and $\triangle DEF$, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

To prove : $\triangle ABC$ and $\triangle DEF$ are similar i.e. $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$

Construction : From DE or extended DE. DP equals with AB and from DF or extended DF DQ equals with AC are cut off. P and Q are joined.

Proof : $\frac{AB}{DE} = \frac{AC}{DF}$; So, $\frac{DP}{DE} = \frac{DQ}{DF}$ [By construction $AB = DP$ and $AC = DQ$]

$\therefore$ With the help of converse of Thales theorem, we get, $PQ \parallel EF$

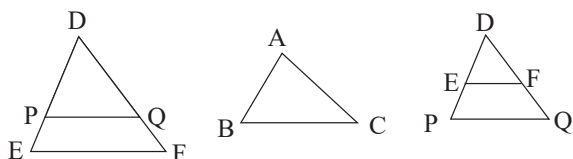
$PQ \parallel EF$ and DE is transversal

$\therefore \angle P = \angle E$

$PQ \parallel EF$ and DF is transversal

$\therefore \angle Q = \angle F$

$\therefore \triangle DPQ$ and $\triangle DEF$ are similar



$$\therefore \frac{DP}{DE} = \frac{PQ}{EF}$$

$$\text{or, } \frac{AB}{DE} = \frac{PQ}{EF} \quad [\because \text{By construction, } DP = AB] \quad \text{————— (i)}$$

$$\text{But } \frac{AB}{DE} = \frac{BC}{EF} \quad [\text{Given}] \quad \text{————— (ii)}$$

$$\therefore \text{ From (i) and (ii), I get, } \frac{PQ}{EF} = \frac{BC}{EF}$$

$$\therefore PQ = BC.$$

$\therefore$ In $\triangle ABC$ and $\triangle DPQ$, $AB = DP$, $BC = PQ$ and $AC = DQ$

$\therefore \triangle ABC \cong \triangle DPQ$ (By S-S-S criterion of congruency)

$$\therefore \angle A = \angle D, \angle B = \angle P = \angle E \text{ and } \angle C = \angle Q = \angle F$$

$$\therefore \angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F$$

$\therefore \triangle ABC$ and $\triangle DEF$ are similar

13 Two polygons will be similar when

(i) The sides of the polygons are proportional and (ii) Corresponding angles of the polygons are equal.

14 But when will two triangles be similar?

We are observing from the above proof that, two triangles will be similar if, the sides of the triangles are proportional or the corresponding angles of the triangles are equal.

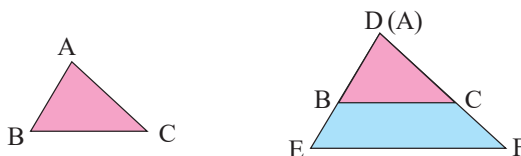


But in two triangles, if an angle of one is equal to an angle of another and if their adjacent sides are proportional, will then the two triangles be similar? Let us justify it by hands on trial.



Hands on trial

- By cutting out a paper I made two coloured triangular regions ABC and DEF whose $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$



- Like the above figure, I put $\triangle ABC$ on $\triangle DEF$ in such a way that the vertex A and the vertex D overlap with each other and the side DE remains on the side AB.

I am observing that, (i) The side AC overlaps with the side DF [$\because \angle A = \angle D$]

$$\text{and (ii) } \frac{BC}{EF} = \frac{\square}{\square} \quad [\text{Let me write by measuring it myself}] \quad \frac{AB}{DE} = \frac{AC}{DF}$$

By hands on trial, I am observing that, in two triangles if an angle of one triangle is equal to an angle of another triangle and the adjacent sides of the angles are proportional then two triangles will be similar.

Let us prove with reason

Theorem : 47. If in two triangles, an angle of one triangle is equal to an angle of another triangle and the adjacent sides of the angle are proportional, then two triangles are similar. [Proof is not included in the evaluation]

Given : In $\triangle ABC$ and $\triangle DEF$, $\angle A = \angle D$ and $\frac{AB}{DE} = \frac{AC}{DF}$

To prove : $\triangle ABC$ and $\triangle DEF$ are similar

proof : $\frac{AB}{DE} = \frac{AC}{DF}$ or, $\frac{DB}{DE} = \frac{DC}{DF}$ or, $\frac{DB}{BE} = \frac{DC}{CF}$ $\therefore BC \parallel EF$ [We get the converse of Thales theorem]

So, $\angle B = \text{corresponding } \angle E$ and $\angle C = \text{Corresponding } \angle F$

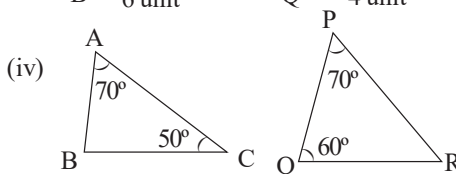
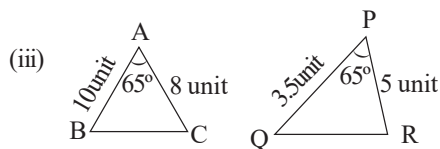
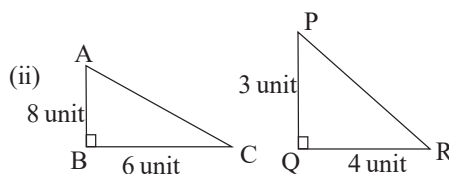
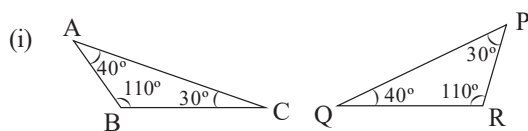
i.e. $\triangle ABC$ and $\triangle DEF$ are similar. $\therefore \triangle ABC$ and $\triangle DEF$ are similar.



If two triangles are congruent, then we use the congruent sign $\cong$ but in case of two similar triangles, can we use any sign for it?

If in $\triangle ABC$ and $\triangle DEF$, $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$ then the two triangles are similar, so we write now $\triangle ABC \sim \triangle DEF$

Application : 11. Let us write by calculating, which pairs of the following triangles are similar.



(i) $\angle A = \angle Q$, $\angle B = \angle R$ and $\angle C = \angle P$

$\therefore \triangle ABC$ and $\triangle QRP$ are similar or $\triangle ABC \sim \triangle QRP$

(ii) $\frac{AB}{RQ} = \frac{BC}{QP} = \frac{CA}{PR}$

$\therefore$ The sides of $\triangle ABC$ and $\triangle RQP$ are proportional.

$\therefore \triangle ABC$ and $\triangle RQP$ are similar or $\triangle ABC \sim \triangle QRP$



In the same way, let us write by calculating, the sides and angles of the figures (iii) and (iv)

[Let me do it myself]

Application : 12. Let us see the adjoining figure and write the value of $\angle P$ by calculating it.

Solution : In $\triangle ABC$ and $\triangle PQR$

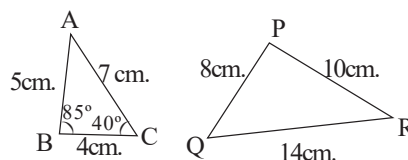
$$\frac{AB}{PR} = \frac{5}{10} = \frac{1}{2}, \quad \frac{BC}{PQ} = \frac{4}{8} = \frac{1}{2} \quad \text{and} \quad \frac{AC}{QR} = \frac{\boxed{}}{\boxed{}} = \frac{1}{2}$$

$$\therefore \frac{AB}{RP} = \frac{BC}{PQ} = \frac{CA}{QR} = \frac{1}{2}$$

$\therefore \triangle ABC$ and $\triangle RPQ$ are similar

$\therefore \angle A = \angle R$, $\angle B = \angle P$ and $\angle C = \angle Q$

$\therefore \angle P = \angle B = 85^\circ$



Application 13. Let us prove that the perimeters of two similar triangles are proportional to the corresponding sides of two triangles.

Given : ABC and PQR are two similar triangles.

To prove : $\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$

Proof : Let, in $\triangle ABC$, $BC = a$ unit

$CA = b$ unit and $AB = c$ unit

Let, in $\triangle PQR$, $QR = p$ unit, $RP = q$ unit and $PQ = r$ unit

$\triangle ABC$ and $\triangle PQR$ are similar

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$$

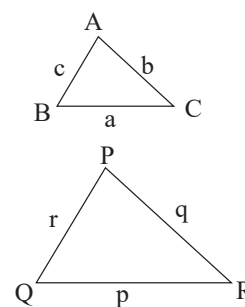
$$\text{or, } \frac{a}{p} = \frac{b}{q} = \frac{c}{r} = k \ (\neq 0) \text{ (suppose) } \text{--- (i)}$$

$$\therefore a = pk, b = qk, c = rk.$$

$$\therefore \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} = \frac{a+b+c}{p+q+r} = \frac{pk+qk+rk}{p+q+r} = \frac{k(p+q+r)}{(p+q+r)} = k \text{ --- (ii)}$$

$\therefore$ From (i) and (ii) we get

$$\frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle PQR} = \frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} \quad \text{[Proved]}$$



Application 14. The perimeters of two similar triangles are 20 cm and 16 cm. respectively, if the length of one side of first triangle is 9 cm., then let us write by calculating, the length of the corresponding side of second triangle. [Let me do it myself]

Hints : $\frac{\text{Perimeter of first triangle}}{\text{Perimeter of second triangle}} = \frac{9 \text{ cm.}}{\text{Length of the corresponding side of second triangle}}$

Application 15. Let us prove that the line drawn through the mid-point of one side of a triangle parallel to other side bisects the third side and intercepted part of the parallel line by two sides is half of the second side.

Given : In $\triangle ABC$, the line through mid-point P of AB parallel to BC intersects AC at the point Q.

To prove : (i) Q is the mid-point of AC, (ii) $PQ = \frac{1}{2} BC$

Proof : In $\triangle APQ$ and $\triangle ABC$

$$\angle PAQ = \angle BAC \text{ [Common angle]}$$

$$\angle APQ = \angle ABC \text{ [} \because PQ \parallel BC \text{ and AB is transversal]}$$

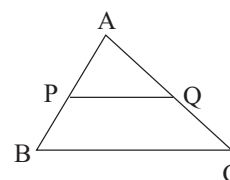
$\therefore \triangle APQ$ and $\triangle ABC$ are similar.

$$\therefore \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$$

$$\text{But } \frac{AP}{AB} = \frac{1}{2} \text{ [} \because P \text{ is the mid-point of AB]}$$

$$\therefore \frac{AQ}{AC} = \frac{1}{2}, \text{ or, } AQ = \frac{1}{2} AC, \therefore Q \text{ is the mid-point of AC} \quad \text{[(i) is proved]}$$

$$\text{Again, } \frac{PQ}{BC} = \frac{1}{2}, \therefore PQ = \frac{1}{2} BC \quad \text{[(ii) is proved]}$$



Application : 16. In $\triangle ABC$, $\angle B = \angle C$, the points D and E are situated on BA and CA in such a way that $BD = CE$, let us prove that, $DE \parallel BC$ [Let me do it myself]

Application : 17. I have drawn a median AD in a triangle $\triangle ABC$. If a line parallel to BC intersects the two sides AB and AC at the points P and Q respectively, then let us prove that the line segment PQ is bisected by AD.

Given : AD is a median of ABC. The line parallel to the side BC intersects AB, AD and AC at the points P, R and Q respectively.

To prove : $PR = RQ$

Proof : In $\triangle APR$ and $\triangle ABD$, $\angle PAR = \angle BAD$ [same angle]

and $\angle APR = \text{Corresponding } \angle ABD$ [$\because PR \parallel BD$ and AB is transversal]

$\therefore \triangle APR$ and $\triangle ABD$ are similar

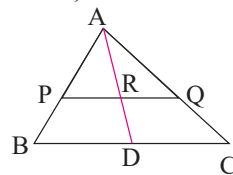
$$\therefore \frac{PR}{BD} = \frac{AR}{AD} \text{ ————— (i)}$$

From $\triangle ARQ$ and $\triangle ADC$, it can be proved similarly that, $\frac{RQ}{DC} = \frac{AR}{AD}$ ————— (ii)

From (i) & (ii), we get, $\frac{PR}{BD} = \frac{RQ}{DC}$

But, $BD = DC$ [$\because$ AD is median]

$\therefore PR = RQ$ [Proved]



Application : 18. I have drawn a cyclic quadrilateral ABCD. The extended two sides of AB and DC meet at the point P, let me prove that, $PA.PB = PC.PD$

Given : In cyclic quadrilateral ABCD, the extended two sides of AB and DC meet at the point P.

To prove : $PA.PB = PC.PD$

Proof : ABCD is a cyclic quadrilateral.

$$\therefore \angle DAB + \angle DCB = 180^\circ$$

$$\text{Again, } \angle DCB + \angle BCP = 180^\circ$$

$$\therefore \angle DAB + \angle DCB = \angle DCB + \angle BCP$$

$$\therefore \angle DAB = \angle BCP \text{ ————— (i)}$$

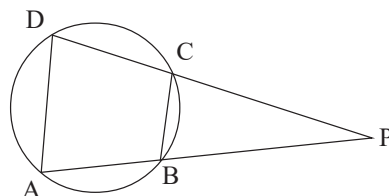
In $\triangle APD$ and $\triangle CPB$, $\angle APD = \angle CPB$ [Same angle]

and $\angle PAD = \angle BCP$ [from (i)]

$\therefore \triangle APD$ and $\triangle CPB$ are similar.

$$\therefore \frac{PA}{PC} = \frac{PD}{PB}$$

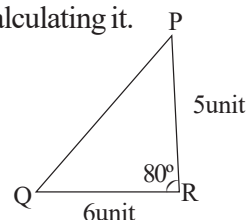
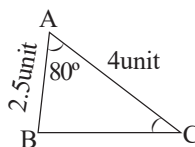
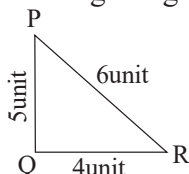
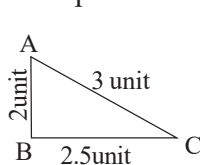
So, $PA.PB = PC.PD$ (Proved)



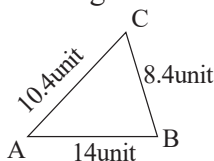
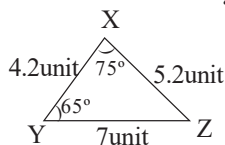
I am observing in above proof that in $\triangle APD$ and $\triangle CPB$, PA and PC are corresponding sides and PD and PB are corresponding sides.

Let us work out 18.3

1. Which pairs of the following triangles are similar-let us write by calculating it.



2. I look at the following pair of triangles and I write by calculating, the value of $\angle A$.



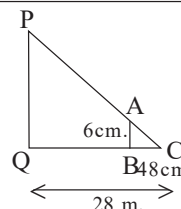
3. In our field, a shadow of 4 cm length of a stick with the length of 6 cm. has fallen on the ground. In the same time, If the length of the shadow of a high tower is 28m., then let us write by calculating, the height of the tower.

Hints : Let, PQ be tower and AB be stick

$\therefore BC = 4 \text{ cm.}, QC = 28 \text{ m.}$

$\triangle PQC$ and $\triangle ABC$ are similar.

$$\therefore \frac{PQ}{AB} = \frac{QC}{BC} \quad \text{[Let me do it myself]}$$



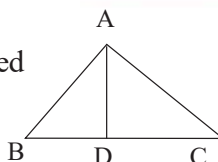
4. Let us prove that the line segment joining mid-points of any two sides of a triangle is parallel to third side and is equal to half of it.
5. Two parallel lines intersect three concurrent straight lines at the points A, B, C and X, Y, Z respectively. Let us prove that, $AB : BC = XY : YZ$
6. I have drawn a trapezium PQRS whose $PQ \parallel SR$; if two diagonals PR and QS intersect each other at the point O, then let me prove that $OP : OR = OQ : OS$; if $SR = 2PQ$, then let me prove that the point O will be a trisector of each of two diagonals.
7. PQRS is a parallelogram. If a line passing through the point S intersects PQ and extended RQ at the points X and Y respectively, then let us prove that $PS : PX = QY : QX = RY : RS$.
8. The two acute-angled triangles $\triangle ABC$ and $\triangle PQR$ are similar. Their circum-centres are X and Y respectively; if BC and QR are corresponding sides, then let us prove that, $BX : QY = BC : QR$.
9. The two chords PQ and RS of a circle intersects each other at the point X within the circle. By joining P, S and R, Q, let us prove that, $\triangle PXS$ and $\triangle RSQ$ are similar. From this, let us prove that $PX : XQ = RX : XS$.
or, if two chords of a circle intersects internally, then the rectangle of two parts of one is equal to the rectangle of two parts of other.
10. The two points P and Q are on a straight line. At the points P and Q, PR and QS are perpendiculars on the straight line. PS and QR intersect each other at the point O. OT is perpendicular on PQ. Let us prove that, $\frac{1}{OT} = \frac{1}{PR} + \frac{1}{QS}$
11. $\triangle ABC$ is inscribed in a circle; AD is a diameter of the circle and AE is perpendicular on the side BC, which intersects the side BC at the point E. Let us prove that, $\triangle AEB$ and $\triangle ACD$ are similar. From this, let us prove that, $AB \cdot AC = AE \cdot AD$.

Debmalya has separated the right angled triangular regions from the triangular regions which we have made in the cardboard.

Among these, by taking two right angled triangles Nusarat has made a perpendicular on hypotenuse from the right-angular point of the triangle by folding the paper.



From the right-angular point of the right-angled triangle, Nusarat has drawn $AD \perp BC$.

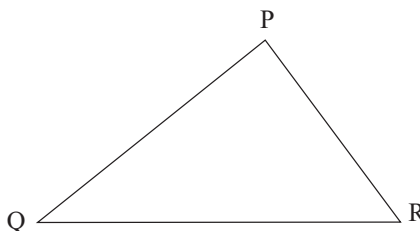
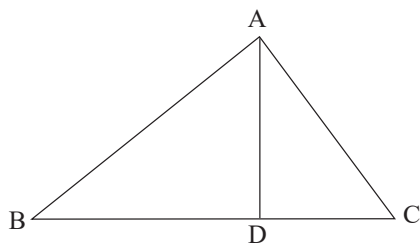


Unfolding the paper, I got three triangles ABD, CAD and ABC. But are the three triangles similar to each other? Let us justify it by hands on trial.



Hands on trial

1. I have made two triangles ABC and PQR of same measures whose $\angle A = \angle P = 90^\circ$, $\angle B = \angle Q$ and $\angle C = \angle R$ and I have cut out the two triangular regions.
2. Now by folding the paper I have drawn the perpendicular AD on the hypotenuse BC of $\triangle ABC$.



3. Now I cut out $\triangle ABD$ and $\triangle CAD$ and by putting on $\triangle PQR$ I am observing that

$$\angle A = \angle P = \angle CDA = \angle ADB$$

$$\angle B = \angle Q = \angle ABD = \angle CAD$$

$$\angle C = \angle R = \angle DAB = \angle ACD \quad \text{[Let me do it myself]}$$

$\therefore$ I have got, $\triangle ABD$, $\triangle CAD$ and $\triangle ABC$ are similar.

$\therefore$ By hands on trial, I have got, $\triangle ABD$, $\triangle CAD$ and $\triangle ABC$ are similar.

- $\therefore$ By hands on trial, I have got, from the right angular point of a right angled triangle, a perpendicular drawn on hypotenuse divides the triangle into two triangles which are similar to each other and each of them is similar to original triangle.



Let us prove with reason

Theorem : 48. In any right angled triangle if a perpendicular is drawn from right angular point on hypotenuse then the two triangles on both sides of this perpendicular are similar and each of them is similar to original triangle.

Given : ABC is a right angled triangle whose $\angle A$ is right angle and from right angular point A, AD is perpendicular on the hypotenuse BC.

To prove : (i) $\triangle DBA$ and $\triangle ABC$ are similar to each other. (ii) $\triangle DAC$ and $\triangle ABC$ are similar to each other. (iii) $\triangle DBA$ and $\triangle DAC$ are similar to each other.

Proof : In $\triangle DBA$ and $\triangle ABC$

$$\angle BDA = \angle BAC = 90^\circ$$

and $\angle ABD = \angle CBA$. So remaining $\angle BAD = \angle BCA$

$\therefore \triangle DBA$ and $\triangle ABC$ are similar to each other. **[(i) is proved]**

Again, in $\triangle DAC$ and $\triangle ABC$

$$\angle ADC = \angle BAC = 90^\circ$$

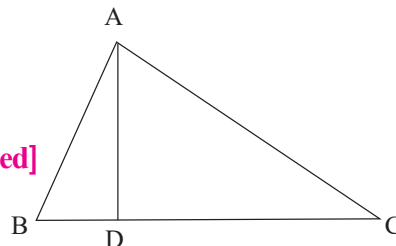
$\angle ACD = \angle BCA$. So remaining $\angle CAD = \angle CBA$

$\therefore \triangle DAC$ and $\triangle ABC$ are similar to each other. **[(ii) is proved]**

$\triangle DBA$ and $\triangle ABC$ are similar to each other.

Again, $\triangle DAC$ and $\triangle ABC$ are similar to each other.

So $\triangle DBA$ and $\triangle DAC$ are similar to each other. **[(iii) is proved]**



If the acute angles of each of two right-angled triangles are equal then the two right-angled triangles are **[Let me do it myself]**

Application : 19. In right angled triangle ABC, $\angle A$ is right angle. I draw a perpendicular AD on the hypotenuse BC from the right-angular point A. Let me prove that, (i) $AB^2 = BC \cdot BD$, (ii) $AD^2 = BD \cdot CD$ and (iii) $AC^2 = BC \cdot CD$

Given : In ABC, $\angle BAC = 90^\circ$; $AD \perp BC$.

To prove : (i) $AB^2 = BC \cdot BD$, (ii) $AD^2 = BD \cdot CD$ and (iii) $AC^2 = BC \cdot CD$

Proof : (i) $\triangle DBA$ and $\triangle ABC$ are similar ($\because$ In ABC, AD is perpendicular on the hypotenuse BC from the right-angular point A)

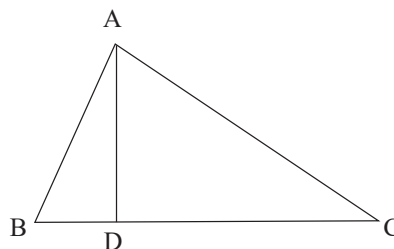
$$\therefore \frac{AB}{BC} = \frac{BD}{AB}, \text{ so, } AB^2 = BC \cdot BD \text{ [(i) is Proved]}$$

(ii) $\triangle DBA$ and $\triangle DAC$ are similar.

$$\therefore \frac{AD}{CD} = \frac{BD}{AD}, \text{ so, } AD^2 = BD \cdot CD \text{ [(ii) is Proved]}$$

(iii) $\triangle DAC$ and $\triangle ABC$ are similar.

$$\therefore \frac{AC}{BC} = \frac{CD}{AC}, \text{ so, } AC^2 = BC \cdot CD \text{ [(iii) is Proved]}$$

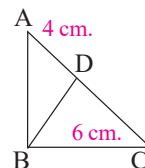


In case of (ii) of the above proof, I am observing that, in right angled triangle ABC ($\angle A$ is right-angle) if the perpendicular AD is drawn on the hypotenuse BC from the right-angular point A, then the length of the line segment AD is mean proportion of the lengths of two parts of the hypotenuse into which it is divided.

Application 20. In $\triangle ABC$, $\angle ABC = 90^\circ$ and $BD \perp AC$; if $BD = 6$ cm. and $AD = 4$ cm. then let us write by calculating, the length of CD .

$\triangle DAB$ and $\triangle DBC$ are similar.

$$\therefore BD^2 = AD \cdot CD \quad \text{or, } 6^2 = 4 \times CD \quad \therefore CD = \boxed{} \text{ [Let me write it myself]}$$



Application : 21. In $\triangle ABC$, $\angle ABC = 90^\circ$ and $BD \perp AC$; if $AB = 6$ cm. and $BD = 3$ cm. and $CD = 5.4$ cm., then let us write by calculating the length of BC . [Let me write it myself]

Application : 22. I drew a perpendicular AD on the side BC from the vertex A of $\triangle ABC$.

If $\frac{BD}{DA} = \frac{DA}{DC}$, then let us prove that, $\triangle ABC$ is a right-angled triangle.

Proof : In $\triangle BDA$ and $\triangle ADC$, $\angle BDA = \angle ADC = 90^\circ$ [$\because AD \perp BC$] and $\frac{BD}{DA} = \frac{DA}{DC}$

$\therefore \triangle BDA$ and $\triangle ADC$ are similar. [If an angle of one of two triangles is equal to an angle of the other and the adjacent sides of the angles are proportional then these two triangles are similar]

So, $\angle ABD = \angle CAD$ and $\angle BAD = \angle ACD$

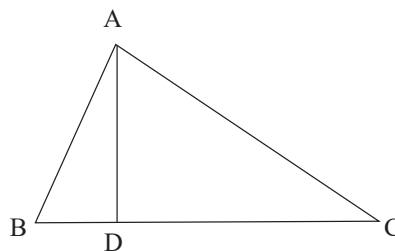
$$\therefore \angle ABD + \angle ACD = \angle CAD + \angle BAD$$

$$\text{or, } \angle B + \angle C = \angle A$$

$$\text{or, } \angle A + \angle B + \angle C = 2\angle A$$

$$\text{or, } 2\angle A = 180^\circ \quad \therefore \angle A = 90^\circ$$

$\therefore \triangle ABC$ is a right-angled triangle.



Application : 23. If I draw a perpendicular on the hypotenuse from the right-angular point A of a right angled triangle ABC and if AC , AB and BC are in continued proportion, then let me prove that the greatest part of the hypotenuse is equal to the smallest side of the triangle.

Given : In right angled triangle ABC , $\angle A$ is right angle; I drew the perpendicular AD on the hypotenuse BC from A . Let, AC be the smallest side. $AC : AB = AB : BC$

To prove : The greatest part of the hypotenuse BC is equal to the side AC . Since DC of the right angled triangle ADC can not be equal to the hypotenuse AC , so I have to prove $BD = AC$.

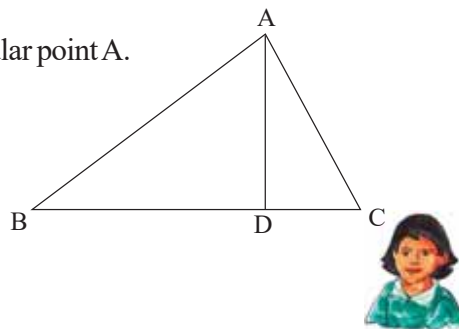
Proof : AD is perpendicular on BC from right-angular point A .

$\therefore \triangle ABD$ and $\triangle ABC$ are similar.

$$\therefore \frac{BD}{AB} = \frac{AB}{BC}$$

$$\text{But, } \frac{AC}{AB} = \frac{AB}{BC} \text{ [given]. So, } \frac{BD}{AB} = \frac{AC}{AB}$$

$\therefore BD = AC$ [Proved]



Application : 24. I have drawn a circle with its diameter AB and centre O; I draw a perpendicular on the diameter AB from a point P on the circle which intersects AB at the point N. Let me prove that, $PB^2 = AB \cdot BN$

Given : AB is a diameter of a circle with its centre O. P is any point on the circle and $PN \perp AB$

To prove : $PB^2 = AB \cdot BN$

Proof: AB is a diameter of the circle. So $\angle APB$ is an angle in a semicircle.

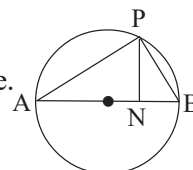
$\therefore \angle APB = 1$ right angle.

In right angled triangle APB, PN is a perpendicular drawn on the hypotenuse from the right angular point P.

$\therefore \triangle ABP$ and $\triangle PBN$ are similar to each other.

$$\text{So, } \frac{PB}{BN} = \frac{AB}{PB}$$

$\therefore PB^2 = AB \cdot BN$ **[Proved]**



Application : 25. Two circles touch each other externally at the point A. PQ is a direct common tangent of two circles. If the lengths of radii of two circles are r and r' respectively, then let us prove that, $PQ^2 = 4rr'$

Given : Two circles with centres at R and S and radii of lengths r and r' respectively touch each other externally at the point A. PQ is a direct common tangent of those two circles and it touches the two circles at the points P and Q respectively.

To prove : $PQ^2 = 4rr'$

Construction: R, A and A, S are joined, one common tangent of two circles are drawn at the point A which intersects PQ at the point B. R, B and S, B are joined.

Proof: The two tangents of the circle with centre R from the point B are BP and BA.

$\therefore BP = BA$ and RB is the bisector of $\angle ABP$

$$\text{So, } \angle RBA = \frac{1}{2} \angle PBA$$

Again, the two tangents of the circle with centre S from the point B are BQ and BA.

$\therefore BQ = BA$ and BS, is the bisector of $\angle ABQ$.

$$\therefore \angle SBA = \frac{1}{2} \angle QBA$$

$$\angle RBA + \angle SBA = \frac{1}{2} (\angle PBA + \angle QBA)$$

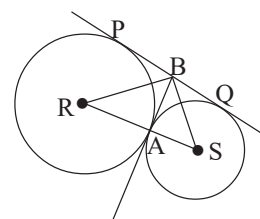
$$\text{or, } \angle RBS = \frac{1}{2} \angle PBQ = \frac{1}{2} \times 180^\circ = 90^\circ$$

$\therefore \angle RBS = 1$ right angle.

R, S are centres of two circles and A is point of contact.

$\therefore$ The three points R, A, S are collinear and $AB \perp RS$ [$\because$ The tangent of a circle and radius passing through point of contact are perpendicular to each other]

BA is perpendicular on the hypotenuse RS drawn from the right-angular point B of the right angled triangle RBS.



$\therefore \triangle ABR$ and $\triangle ASB$ are similar to each other.

$$\text{So, } \frac{AB}{AS} = \frac{AR}{AB}$$

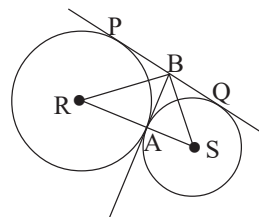
$$\text{or, } AB^2 = AR \cdot AS$$

$$= r \cdot r' \quad [\because AR = r \text{ and } AS = r']$$

$$\therefore 4AB^2 = 4r \cdot r'$$

$$\text{or, } (2AB)^2 = 4r r'$$

$$\therefore PQ^2 = 4r r' \quad [\because PQ = PB + BQ = 2AB; \because PB = BA \text{ and } QB = BA] \text{ (Proved)}$$



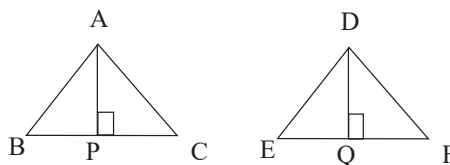
Application : 26. Let us prove with reason that the ratio of the areas of two similar triangles is equal to the ratio of the square of the corresponding sides.

[The Proof is not included in the evaluation]

Given : $\triangle ABC \sim \triangle DEF$;

$$\therefore \angle A = \angle D, \angle B = \angle E \text{ and } \angle C = \angle F$$

To Prove :
$$\frac{\triangle ABC}{\triangle DEF} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2}$$



Construction : In $\triangle ABC$, $AP \perp BC$ and in $\triangle DEF$, $DQ \perp EF$ are drawn.

Proof:
$$\triangle ABC = \frac{1}{2} BC \cdot AP$$

$$\text{and } \triangle DEF = \frac{1}{2} EF \cdot DQ$$

$$\frac{\triangle ABC}{\triangle DEF} = \frac{\frac{1}{2} BC \cdot AP}{\frac{1}{2} EF \cdot DQ} = \frac{BC}{EF} \cdot \frac{AP}{DQ}$$

In $\triangle ABP$ and $\triangle DEQ$, $\angle ABP = \angle DEQ$ ($\because \angle B = \angle E$)

$\angle APB = \angle DQE$ (each is right angle)

So, remaining $\angle PAB = \angle QDE$.

$\therefore \triangle ABP$ and $\triangle DEQ$ are similar.

$$\therefore \frac{AB}{DE} = \frac{AP}{DQ}$$

Again $\triangle ABC$ and $\triangle DEF$ are similar.

$$\text{So, } \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} \quad \therefore \frac{AP}{DQ} = \frac{BC}{EF}$$

$$\therefore \frac{\triangle ABC}{\triangle DEF} = \frac{BC}{EF} \cdot \frac{AP}{DQ} = \frac{BC}{EF} \cdot \frac{BC}{EF} = \frac{BC^2}{EF^2}$$

$$\text{Since, } \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$$

$$\text{So, } \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2} \quad \therefore \frac{\triangle ABC}{\triangle DEF} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2} \quad \text{[Proved]}$$



Let us work-out 18.4

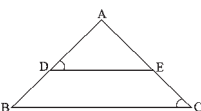
1. In $\triangle ABC$, $\angle ABC = 90^\circ$ and $BD \perp AC$; if $BD = 8$ cm. and $AD = 5$ cm., then let us calculate the length of CD .
2. ABC is a right angled triangle whose $\angle B$ is right angle and $BD \perp AC$; if $AD = 4$ cm. and $CD = 16$ cm., then let us calculate the lengths of BD and AB .
3. AB is a diameter of a circle with centre O , P is any point on the circle, the tangent drawn through the point P intersects the two tangent drawn through the points A and B , at the points Q and R respectively. If the radius of the circle be r , let us prove that, $PQ \cdot PR = r^2$
4. I have drawn a semicircle with a diameter AB . I have drawn a perpendicular on AB from any point ' C ' on AB which intersects the semicircle at the point D . Let us prove that CD is mean proportion of AC and BC .
5. In right angled triangle ABC , $\angle A$ is right angle. AD is perpendicular on the hypotenuse BC , let us prove that $\frac{\triangle ABC}{\triangle ACD} = \frac{BC^2}{AC^2}$
6. AB is a diameter of a circle with centre O . A line drawn through the point A intersects the circle at the point C and the tangent through B at the point D . Let us prove that
(i) $BD^2 = AD \cdot DC$ (ii) The area of the rectangle formed by AC and AD for any straight line is always equal.
7. **V.S.A.**

(A) M.C.Q :

- (i) In $\triangle ABC$ and $\triangle DEF$, if $\frac{AB}{DE} = \frac{BC}{FD} = \frac{AC}{EF}$ then
(a) $\angle B = \angle E$ (b) $\angle A = \angle D$ (c) $\angle B = \angle D$ (d) $\angle A = \angle F$
- (ii) In $\triangle DEF$ and $\triangle PQR$, if $\angle D = \angle Q$ and $\angle R = \angle E$, then let us write which of the following is not right :
(a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{QR}{PQ} = \frac{EF}{DF}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QR}$
- (iii) In $\triangle ABC$ and $\triangle DEF$, if $\angle A = \angle E = 40^\circ$, $AB : ED = AC : EF$ and $\angle F = 65^\circ$, then the value of $\angle B$ is
(a) 35° (b) 65° (c) 75° (d) 85°
- (iv) In $\triangle ABC$ and $\triangle PQR$, if $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then
(a) $\angle A = \angle Q$ (b) $\angle A = \angle P$ (c) $\angle A = \angle R$ (d) $\angle B = \angle Q$
- (v) In $\triangle ABC$, $AB = 9$ cm., $BC = 6$ cm., and $CA = 7.5$ cm. In $\triangle DEF$, the corresponding side of BC is EF ; $EF = 8$ cm. and if $\triangle DEF \sim \triangle ABC$, then the perimeter of $\triangle DEF$ will be
(a) 22.5 cm. (b) 25 cm. (c) 27 cm. (d) 30 cm.

(B) Let us write whether the following statements are true or false :

- (i) If the corresponding angles of two quadrilaterals are equal, then they are similar.

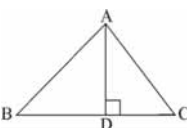
- (ii)  In the adjoining figure, if $\angle ADE = \angle ACB$ then $\triangle ADE \sim \triangle ACB$

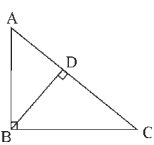
- (iii) In $\triangle PQR$, D is a point on the side QR so that $PD \perp QR$; so $\triangle PQD \sim \triangle RPD$

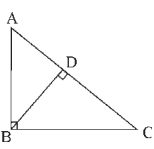
(C) Let the fill in the blank :

- (i) Two triangles are similar if their _____ sides are proportional.
- (ii) The perimeters of $\triangle ABC$ and $\triangle DEF$ are 30 cm. and 18 cm. respectively. $\triangle ABC \sim \triangle DEF$; BC and EF are corresponding sides. If $BC = 9$ cm., then $EF =$ _____ cm.

8. **S.A.**

- (i)  In the adjoining figure, if $\angle ACB = \angle BAD$ and $AD \perp BC$; $AC = 15$ cm., $AB = 20$ cm., and $BC = 25$ cm., then let us write the length of AD.

- (ii)  In the adjoining figure, $\angle ABC = 90^\circ$ and $BD \perp AC$; if $AB = 30$ cm., $BD = 24$ cm. and $AD = 18$ cm., then let us determine the length of BC.

- (iii)  In the adjoining figure, $\angle ABC = 90^\circ$ and $BD \perp AC$; if $BD = 8$ cm. and $AD = 4$ cm. then let us write the length of CD.

- (iv) In trapezium ABCD, $BC \parallel AD$ and $AD = 4$ cm. The two diagonals AC and BD intersect at the point O in such a way that, $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$. Let us calculate the length of BC.
- (v) $\triangle ABC \sim \triangle DEF$ and in $\triangle ABC$ and $\triangle DEF$, the corresponding sides of AB, BC and CA are DE, EF and DF respectively; if $\angle A = 47^\circ$ and $\angle E = 83^\circ$, then let us write the value of $\angle C$.

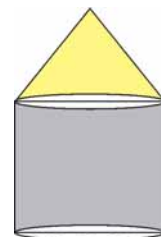
19

REAL LIFE PROBLEMS RELATED TO DIFFERENT SOLID OBJECTS

Last year, in December during winter vacation, we along with our aunty's family went for a tour to our village house at Atpur. At that time, paddy was being threshed in the fields and the granary was filled with paddy.



Application : 1 The lower portion of the granary of our house is right circular cylindrical shaped and the upper portion of it is cone shaped. The length of base diameter of this granary is 2.1 m, the height of the cylindrical part is 2m. and the height of conical part is 1.4 m. But in this granary $\frac{2}{3}$ part of it is filled with paddy. How I shall get the volume of paddy kept in the granary.



The length of base radius of the granary = $\frac{21}{2}$ m. = $\frac{21}{20}$ m.

The volume of cylindrical part = $\frac{22}{7} \times \frac{21}{20} \times \frac{21}{20} \times 2 \text{ m}^3 = 6.93 \text{ m}^3$

The volume of conical part = $\frac{1}{3} \times \frac{22}{7} \times \frac{21}{20} \times \frac{21}{20} \times \frac{14}{10} \text{ m}^3$
 $= \frac{77 \times 21}{1000} \text{ m}^3 = 1.617 \text{ m}^3$

$\therefore$ Volume of that granary = $(6.93 + 1.617) \text{ m}^3 = 8.547 \text{ m}^3$

$\therefore$ The volume of paddy kept in that granary = $\frac{2}{3} \times 8.547 \text{ m}^3 = 5.698 \text{ m}^3$



I have understood that, since some part of the granary is cylindrical and some part is conical shaped, so the two volumes are determined separately and taking their sums we have got the volume of granary.

Application : 2 But if the length of outer diameter of a hollow iron cylinder with the height of 25 cm. is 14 cm. and the length of inner diameter of it is 10 cm. and if by melting the cylinder, a solid right circular cone having half of its height is made, then let us determine the length of base diameter of the cone.

The length of outer radius of hollow cylinder (R) = $\frac{14}{2}$ cm. = 7cm., the length of inner radius (r) = $\frac{10}{2}$ cm. = 5cm., the height of the cylinder = 25cm.

$\therefore$ The quantity of iron in the cylinder = $\pi (R^2 - r^2) \times \text{height} = \pi (7^2 - 5^2) \times 25 \text{ cm}^3$
 $= \pi \times 24 \times 25 \text{ cm}^3$

The height of solid cone = $\frac{25}{2}$ cm.

Let the length of base radius of the cone = r cm.

According, to the condition, $\frac{1}{3} \pi \times r^2 \times \frac{25}{2} = \pi \times 24 \times 25$

$\therefore r = \pm \boxed{}$ [Let me write it by calculating myself]

But the length of radius can not be negative. $\therefore r \neq -\boxed{}$ so, $r = \boxed{}$

$\therefore$ The base diameter of the cone = $2 \times r \text{ cm.} = \boxed{} \text{ cm.}$



Application : 3 The length of external diameter of the face of a hemispherical bowl made of silver sheet is 8 cm. and the length of internal diameter of it is 4 cm. By melting the bowl a solid cone is made whose diameter is 8 cm. in length. Let us calculate the height of the cone. **[Let me do it myself]**

Application : 4 There is some water in a right circular drum. A conical iron piece having the diameter of 2.8 dcm. and of the height of 3 dcm. is immersed completely in to this water. For this, water level of the drum is raised by 0.64 dcm. Let us determine the base diameter of the drum.

The length of the base radius of the cone = $\frac{2.8}{2}$ dcm. = 1.4 dcm. and height = 3 dcm.

$$\therefore \text{Volume of the cone} = \frac{1}{3} \pi \times (1.4)^2 \times 3 \text{ dcm}^3$$

Let the length of base radius of drum be r dcm.

For immersing conical iron piece into the water of the drum, water-level is raised by 0.64 dcm.

$\therefore$ The volume of increased water in the drum = Volume of conical iron piece.

$$\therefore \pi r^2 \times 0.64 = \frac{1}{3} \pi \times (1.4)^2 \times 3 \quad \text{or, } r^2 = \boxed{} \quad \text{or, } r = \pm \boxed{} \quad \text{[Let me do it myself]}$$

But the length of radius can not be negative. $\therefore r \neq -\boxed{}$, So, $r = \boxed{}$

$\therefore$ The length of base diameter of the drum = $\boxed{}$ dcm.



Application : 5 There is some water in a right circular cylinder with the diameter of 28 cm. Three solid iron spheres can be immersed completely into it. After immersing the spheres, the height of the water level is increased by 7cm. than it was before the immersion of the spheres into water. Let us determine the length of the diameter of the spheres. **[Let me do it myself]**

Hints : Let the length of the radius of the spheres be r cm.

According to the condition, $3 \times \frac{4}{3} \pi r^3 = \pi \times 14^2 \times 7 \quad \therefore r = \boxed{}$

Application : 6 I took a right circular drum with the radius of 21 cm. and the height of 21 cm. and a solid iron sphere with the diameter of 21 cm. Let us write, by calculating the ratio of volumes of that drum and that of the solid iron sphere. (I shall not consider the thickness of the drum). Now by filling the drum completely with water, the sphere is taken out after immersing it into the drum completely. As a result, let us determine the depth of water in the drum now.

The length of radius of the drum is 21 cm. and its height is 21 cm.

$$\therefore \text{The volume of the drum} = \pi \times 21^2 \times 21 \text{ cm}^3.$$

The length of diameter of the sphere is 21 cm., $\therefore$ The length of radius is $\frac{21}{2}$ cm.

$$\therefore \text{The Volume of the sphere} = \frac{4}{3} \times \pi \times \left(\frac{21}{2}\right)^3 \text{ cm}^3$$

$$\therefore \text{The Volume of the drum : The Volume of the sphere} = \pi \times 21 \times 21 \times 21 : \frac{4}{3} \pi \times \frac{21 \times 21 \times 21}{2 \times 2 \times 2} = 6 : 1$$

Immersing the sphere and taking it out, the sphere removed water of its equal volume.

Let the height of water removed by the sphere be h cm.

$$\text{According to the condition, } \pi \times 21^2 \times h = \frac{4}{3} \pi \times \frac{21 \times 21 \times 21}{2 \times 2 \times 2} \quad \text{or, } \pi \times 21 \times 21 \times h = \frac{4}{3} \pi \times \frac{21 \times 21 \times 21}{8}, \quad \therefore h = 3.5$$

$\therefore$ Now, the depth of water in the drum $21 \text{ cm.} - 3.5 \text{ cm.} = 17.5 \text{ cm.}$



Application : 7 If the lengths of base diameter and the heights of a solid hemisphere and a solid cone are equal, then let us write by calculating, the ratio of their volumes and the ratio of their curved surface areas.

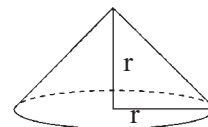
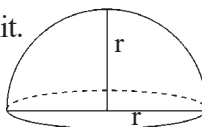
The lengths of base diameters of hemisphere and cone are equal. So, the lengths of their radii are equal.

Let, the length of the radius of the hemisphere and cone be r unit.

The height of the hemisphere = the length of the radius of the hemisphere.

According to the given condition, height of cone = r unit.

$\therefore$ Slant height of cone = $\sqrt{r^2 + r^2}$ unit = $\sqrt{2} r$ unit.



Hence, $\frac{\text{Volume of sphere}}{\text{Volume of cone}} = \frac{\frac{2}{3} \pi r^3}{\frac{1}{3} \pi r^2 \cdot r} = \frac{2}{1}$, $\therefore$ The ratio of the volumes of hemisphere and cone is 2:1

and the ratio of the curved surface areas of hemisphere and cone = $\frac{2\pi r^2}{\pi r \cdot \sqrt{2} r} = \frac{\sqrt{2}}{1} = \sqrt{2} : 1$

Application : 8 If recasting a solid metallic sphere of radius 2.1 cm. into a right circular rod, then let us write by calculating, the length of diameter of the rod.

Let us determine the ratio of total surface areas of the rod and sphere.

Let the length of the right circular rod be r cm.

According to the condition, $\pi r^2 \times 7 = \frac{4}{3} \pi \times (2.1)^3$ [$\because 2.1 \text{ dcm.} = 2.1 \text{ cm.}$]

$\therefore r^2 = \frac{4}{3} \times \frac{21 \times 21 \times 21}{7}$ or, $r = \pm \boxed{}$ [Let me do it myself]

But radius can not be negative.

so, $r \neq -\boxed{}$ $\therefore r = \boxed{}$

$\therefore$ The length of the radius of the rod is 42 cm. $\therefore$ The length of the diameter of the rod is 84cm.

The ratio of total surface areas of the rod and the sphere is = $\boxed{} : \boxed{}$ [Let me write it by calculating myself]

Application : 9 A hemisphere can with internal radius of 9 cm. is completely filled with water. I shall fill this water in a cylindrical bottle with a diameter of 3 cm. and height of 4 cm. Let me calculate the number of bottles to be required to make the bottle empty.

Let the number of bottle be n .

Water can be put in 1 bottle = $\pi \left(\frac{3}{2}\right)^2 \times 4 \text{ cm}^3$ [$\because$ length of radius is $\frac{3}{2}$ cm.]

$\therefore$ water can be put in n bottles = $\pi \left(\frac{3}{2}\right)^2 \times 4 \times n \text{ cm}^3$

According to the condition, $\pi \left(\frac{3}{2}\right)^2 \times 4 \times n = \frac{2}{3} \pi (9)^3$, $\therefore n = \frac{\frac{2}{3} \times 9 \times 9 \times 9}{4 \times \frac{3}{2} \times \frac{3}{2}} = \frac{6 \times 9 \times 9}{3 \times 3} = 54$

$\therefore$ 54 bottles are required to make the water filled can empty.



Application : 10 If a wooden log with a shape of rectangular parallelopiped with square cross-section is made by wasting minimum wood from a right circular wooden log having the diameter of $12\sqrt{2}$ cm. and length of 21 m. Let us calculate the quantity of remaining wood and the quantity of wasted wood.

The length of radius of right circular cylindrical wooden log $= \frac{12\sqrt{2}}{2}$ cm. $= 6\sqrt{2}$ cm.

The length of wooden log $= 21$ m. $= 2100$ cm.

Volume of wooden log $=$ base area $\times$ height

$$= \frac{22}{7} \times 6\sqrt{2} \times 6\sqrt{2} \times 2100 \text{ cm}^3.$$

$$= 475200 \text{ cm}^3. = 475.2 \text{ dcm}^3.$$



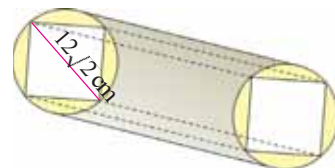
Wasting minimum quantity of wood, one wooden log with square cross-section is to be formed from a wooden log with circular cross-section.

Therefore, length of diagonal of the square $=$ length of diameter of the circle.

Let the length of the side of the square be a cm.

$$\text{So, } \sqrt{2} a = 12\sqrt{2}$$

$$\therefore a = 12$$



$$\therefore \text{Volume of rectangular parallelopiped shaped wooden log} = \text{base area} \times \text{height}$$

$$= 12 \times 12 \times 2100 \text{ cm}^3$$

$$= 302400 \text{ cm}^3$$

$$= 302.4 \text{ dcm}^3$$

$\therefore$ The quantity of remaining wood in that wooden log is 302.4 dcm^3
and the quantity of wasted wood will be $(475.2 - 302.4) \text{ dcm}^3 = 172.8 \text{ dcm}^3$

Application : 11 An out let pipe in the roof with length of 13m. and breadth of 11m. was closed at the time of rain fall. After rain fall, it was seen that water having logged in the roof with 7cm. depth. The length of diameter of outlet pipe is 7cm. and water comes out cylindrically with 200m. length/min. Let us calculate the time required to drain out whole amount of water after opening the pipe.

Volume of logged water in the roof $= 1300 \times 1100 \times 7 \text{ cm}^3$.

$$\text{Water exits from the pipe per minute } \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 200 \times 100 \text{ cm}^3$$

$$= 11 \times 7 \times 100 \times 100 \text{ cm}^3$$



[$\therefore$ The length of radius of cylindrical waterpillar $= \frac{7}{2}$ cm. and its length is 200m. $= 200 \times 100$ cm.]

Time required for draining out whole amount of water after opening the pipe

$$= \frac{1300 \times 1100 \times 7}{11 \times 7 \times 100 \times 100} \text{ minutes}$$

$$= 13 \text{ minutes}$$

Let us work out

19

1. There is a solid iron pillar in front of Anowar's house whose lower portion is right circular cylinder shaped and upper portion is cone shaped. The length of their base diameter is 20cm. and, the height of cylindrical portion is 2.8m. and the height of conical portion is 42cm. If the weight of 1 cm^3 iron is 7.5 gm., then let us calculate the weight of iron pillar.
2. The height of a solid right circular cone is 20cm. and its slant height is 25cm. If the height of a solid right circular cylinder, having as much volume as that of the cone, is 15 cm., then let us calculate the base diameter of the cylinder.
3. There is some water in a right circular cylindrical can with the diameter of 24 cm. If 60 solid conical iron pieces with base diameter of 6 cm. and height of 4 cm. are immersed completely into water, then let us write by calculating, the increased height of water level.
4. If the ratio of the curved surface areas of a solid cone and a solid right circular cylinder having same base radii and same height is 5:8, then let us determine the ratio of their base radii and heights.
5. By melting a solid iron sphere with 8 cm. radius, how many marble balls of 1 cm. diameter can be obtained? Let us write by calculating it.
6. The base radius of a solid right circular iron rod is 32 cm. and its length is 35 cm. Let us calculate the number of solid cones of 8 cm. radius and 28 cm. height can be made by melting this rod.
7. Let us determine the volume of a solid right circular cone which can be made from a solid wooden cube of 4.2 dcm. edge length by wasting minimum quantity of wood.
8. If the radii and the volumes of a solid sphere and a solid right circular cylinder are equal then let us calculate the ratio of the radius and height of the cylinder.
9. Let us determine the number of solid spheres with diameter of 2.1 dcm. can be made by melting a solid copper rectangular parallelepiped piece with length of 6.6 dcm., breadth of 4.2dcm. and thickness of 1.4dcm. and also calculate the quantity of metal in dcm^3 in each sphere.
10. Let us determine the length of a right circular rod of 2.8cm. radius made by recasting a solid gold sphere of 4.2cm. radius.
11. If a solid silver sphere of a diameter of 6 dcm. is melted and recasted into a solid right circular rod, then let us determine the length of diameter of the rod.
12. The length of the radius of cross-section of a solid right circular rod is 3.2dcm. By melting the rod 21 solid spheres are made. If the radius of the sphere is 8cm., then let us determine the length of the rod.

13. Half of a tank of 21 dcm. length, 11 dcm. breadth and 6 dcm. depth is full of water. Now if 100 iron spheres of 21 cm. diameter is immersed completely into the water of this tank, then let us calculate the rise of water level in dcm.
14. Let us determine the ratio of volumes of a solid cone, a solid hemisphere and a solid cylinder of same base diameter and same height.
15. The external radius of a hollow sphere made of lead sheet of 1 cm. thickness is 6 cm. If melting the sphere, a solid right circular rod of 2 cm. radius is made, then let us calculate the length of the rod.
16. The cross-section of a rectangular parallelepiped wooden log of 2 m. length is a square and each of its side is 14 dcm. in length. If this log can be converted into a right circular log by wasting minimum amount of wood, then let us calculate what amount of wood (in m^3) will be remained in it and what amount of wood (in m^3) will be wasted.

[**Hints :** The circumcircle inscribed in a rectangular figure, the length of the diameter of the circle is equal to the length of the side of the square.]

17. **V.S.A.**

(A) **M.C.Q.**

- (i) A solid sphere of r unit radius is melted and from it a solid right circular cone is made. The base radius of the cone is
 (a) $2r$ unit (b) $3r$ unit (c) r unit (d) $4r$ unit
- (ii) By melting a solid right circular cone, a solid right circular cylinder of same radius is made whose height is 5 cm. The height of the cone is
 (a) 10 cm. (b) 15 cm. (c) 18 cm. (d) 24 cm.
- (iii) The length of the radius of a right circular cylinder is r unit and height is $2r$ unit. The length of the diameter of the largest sphere that can be kept in the cylinder is
 (a) r unit (b) $2r$ unit (c) $\frac{r}{2}$ unit (d) $4r$ unit
- (iv) The volume of the largest solid that can be cut out from a solid hemisphere of r unit radius is
 (a) $4\pi r^3 \text{ unit}^3$ (b) $3\pi r^3 \text{ unit}^3$ (c) $\frac{\pi r^3}{4} \text{ unit}^3$ (d) $\frac{\pi r^3}{3} \text{ unit}^3$
- (v) If from a solid cube with the edge of x unit length, the largest solid sphere is cut out, then the length of the diameter of the sphere is
 (a) x unit (b) $2x$ unit (c) $\frac{x}{2}$ unit (d) $4x$ unit

(B) **Let us write whether the following statements are true or false :**

- (i) If two solid hemispheres of same type whose base radii are r unit each and if they are connected along base, then the total surface area of the connected solid is $6\pi r^2 \text{ sq. unit}$.

- (ii) The base radius of a solid right circular cone is r unit, height is h unit and slant height is l unit. The base of the cone is joined along a base of a right circular cylinder. If the base radii and heights of the cylinder and cone are same, then the total surface area of the connected solid is $(\pi r l + 2\pi r h + 2\pi r^2)$ sq. unit.

(C) Let us fill in the blank :

- (i) The base radii of a solid right circular cylinder and two hemispheres are same. If two hemispheres are fixed with two plane surfaces of the cylinder, then the total surface area of the new solid = curved surface area of one hemisphere + curved surface area of _____ + curved surface area of other hemisphere.
- (ii) The shape of a pencil cutting one face is a combination of cone and _____.
- (iii) By melting a solid sphere, a solid right circular cylinder is made. The volume of the sphere and cylinder _____.

18. **S.A.**

- (i) A solid right circular cylinder is made by melting a solid right circular cone. The radii of both are equal. If the height of the cone is 15 cm., then let us determine the height of the solid cylinder.
- (ii) The radii and volumes of a solid circular cone and the solid sphere are equal. Let us determine the ratio of the diameter of the sphere and the height of the cone.
- (iii) Let us determine the ratio of the volumes of a solid right circular cylinder, a solid right circular cone and a solid sphere of equal diameter and equal height.
- (iv) The shape of the lower portion of a solid is hemisphere and the shape of upper portion of it is right circular cone. If the surface areas of two parts are equal, then let us write by calculating, the ratio of the radius and height of the cone.
- (v) The base radius of a solid right circular cone is equal to the length of the radius of a solid sphere. If the volume of the sphere is twice of that of the cone, then let us write by calculating the ratio of the height and base radius of the cone.

20

TRIGONOMETRY : CONCEPT OF MEASUREMENT OF ANGLE

Every afternoon we, the friends, play different types of games in a big field near the pond. We have decided that today we will fly kites in the field. So we all assemble in the field at 4 pm. with some kites and spool.



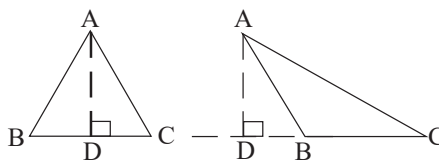
Suva and I can not fly the kite well. So we are looking at the kites being flown by our friends sitting near the field. At first, Rina flied kites with the help of other friends. I am observing that Rina's kite is being risen well from the ground.

1 But how shall we get the height of Rina's kite from the ground ?

The measurement of this type of height, the height of a pillar [without rising on the pillar], the breadth of the streaming river etc. with the help of a simple method which is discussed in a special branch of mathematics. This special branch of mathematics is known as **Trigonometry**.

The term trigonometry has come out from the Greek word "Tri" [which means three], "gon" [which means angle] and "metron" [which means measurement].

"Trigonometry" is the discussion about the relation of angle and side of a right angled triangle. Besides this, we shall discuss about acute angled triangles and obtused angled triangles in trigonometry. But in those cases, the triangles are converted into right-angled triangles. As for example :



Trigonometry is used for determining the distances of the planets and stars from the Earth in earliar times. But now, the higher technique is used in engineering and in physical science also, the trigonometrical view is used.

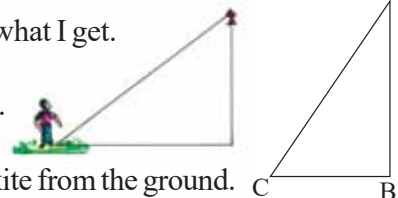
I draw the position of Rina's kite in my copy and see what I get.

Let, in the adjoining figure,

The point C is the position of Rina on the ground.

The point A is the position of Rina's kite,

and AB is the perpendicular distance or height of the kite from the ground.

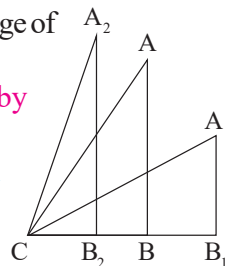


I am observing that, I have got a right-angled triangle whose $\angle ACB$ is an angle [acute/ obtuse]

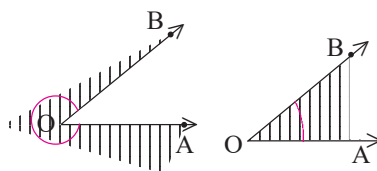
Again, in right angled triangle ABC, the height AB is changing with the change of values of $\angle ACB$

2 Can the value of an angle be greater than 360° ? I try to know it by drawing the picture.

If two rays are coming out off a point, then the two rays divide a plane in which it lies into two regions. Each of two regions produced by those two rays is called the produced angle at that point.



In the adjoining figure, for coming out two rays OA and OB from the point 'O', $\angle AOB$ and $\angle BOA$ are formed, these are called "**geometric angle**". In case of geometric angle, measures of angle is mainly considered without its direction.



I have understood, that geometric angle may be of any measures lying between 0° and 360° .

Can a conception be formed about an angle by the rotation of a ray ?

Let us see what we shall get if the ray rotates clockwise or anti-clockwise direction.

If we rotate a ray in anticlockwise direction by keeping its end point fixed from which the ray comes out in the same plane, then after rotation of the ray, each of next position makes an angle with the first position at that end point.

In the adjoining figure, keeping the end point O of the ray OA fixed, in the same plane by rotating the ray in anti clockwise direction, I have got the position OA_1 , OA_2 , OA_3 etc. of the ray, which make different angles at the point O, $\angle AOA_1$, $\angle AOA_2$, $\angle AOA_3$ etc. respectively. These angles are called **trigonometrical angles**.

In case of trigonometrical angles, both the direction of rotating ray and the measurement of angle formed by it are considered. If the rotating ray rotates anti clockwise direction, the angle formed is called **positive angle** according to the rule, and if the ray rotates clockwise direction, the angle formed is called **negative angle**.

In the adjoining figure, the ray OA by rotating anticlockwise direction keeping the centre at O, has produced $\angle AOA_1 = +\theta^\circ$ (Theta) and $\angle AOA_2 = +\alpha^\circ$ (Alpha). But $\angle AOA_1 = \theta^\circ$ and $\angle AOA_2 = \alpha^\circ$ are written [(+) sign is not used].

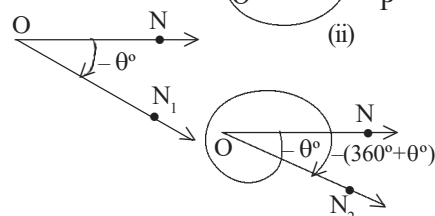
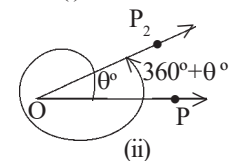
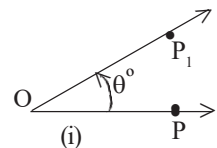
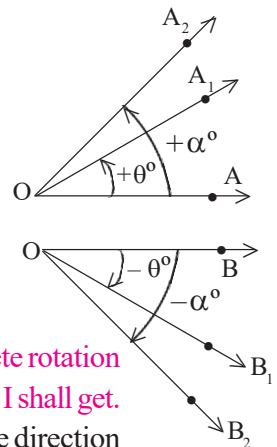
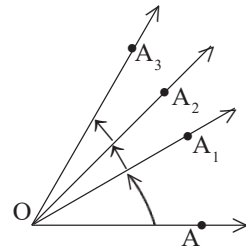
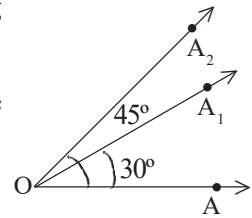
But in the adjoining figure, the ray OB by rotating clockwise direction keeping the centre at O has produced $\angle BOB_1 = -\theta^\circ$ and $\angle BOB_2 = -\alpha^\circ$ ($\theta > 0$, $\alpha > 0$)

But if a ray of an angle comes back to its first position after one complete rotation in anticlockwise direction, then let us see the measurement of angle that I shall get.

As many times as a ray of an angle completes rotation in anticlockwise direction so as to the measurement of the angle will be increased by 360° accordingly. Again as many times as a ray completes rotation in clockwise direction, so as to the angle will be decreased by 360° accordingly.

I have understood, in the adjoining figure, the ray OP_1 with the centre at O, after one complete anticlockwise rotation [ie. after rotation of 360° more] has come back again in the position OP_2 from the initial position OP_1 and in that case, I have got, $\angle POP_2 = 360^\circ + \theta^\circ$ where $\angle POP_1 = \theta^\circ$ [Here in the figure (ii), the two rays OP_2 and OP_1 has overlapped]

In the same way, from the adjoining figure, we are observing that the ray ON_1 with centre at O has come back in the position ON_2 after one complete clockwise rotation. (overlapped on the ray ON_1) and in this case $\angle NON_2 = -(360^\circ + \theta^\circ)$ where, $\angle NON_1 = -\theta^\circ$



3 If any rotating ray rotates an angle of 30° more after two successive complete anticlockwise rotations from its initial position centering at its end, then in that case, let us calculate the measurement of angle what it will be.

The required measurement of angle = $2 \times 360^\circ + 30^\circ = \boxed{}$



If any rotating ray rotates an angle of 45° more after three successive complete anticlockwise rotations, then in that case, let us calculate what will be the measurement of the angle. **[Let me do it myself]**

The lowest and highest measurements of geometric angles are 0° and 360° respectively; but except 0° and 360° as limits, the trigonometric angle may be of any measurement of below 0° and any measurement of above 360° .

4 But let us see the methods required to measure the trigonometrical angles.

The two general systems to measure trigonometrical angles are (i) **Sexagesimal System** and (ii) **Circular System**

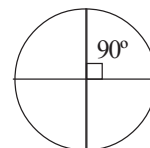
Sexagesimal System : The angle formed by two perpendicular intersecting lines is called right angle.

In this system, one right angle is divided into 90 equal parts and each part is called one degree (1°) and for this reason, one right angle = 90° ; again one degree is divided into 60 equal sexagesimal minutes and each minute is divided into 60 sexagesimal seconds.

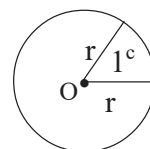
$\therefore$ We have got, 1 right angle = 90° (degree)

1° (degree) = 60' (minute)

$1'$ (minute) = 60" (second)



Circular System : The measure of an angle subtended at the centre by an arc having equal length with radius is called one radian and it is written as 1^c ; the ratio of circumference and diameter of any circle is constant.

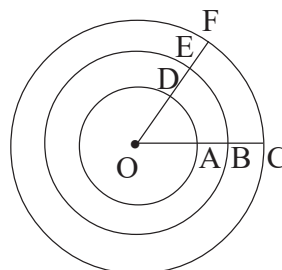


On the basis of this relation, the unit of this has been established.

But if radian is defined with the help a circle with any radius, will it always be a constant angle? Let us try to justify it by hands on trial.

Hands on trial

- (1) I drew three concentric circles on an art paper.
- (2) From the smallest circle I cut off an arc AD having equal length with radius along the circle by putting a string and by joining O, A and O, D I have got $\angle AOD$ which is an angle subtended at the centre by an arc having equal length with its radius of that smallest circle.
- (3) OA and OD are extended (produced) which intersect other two circles at the points B, C and E, F respectively.



By measuring, I am observing that, the length of the arc BE ie, the length of the string putting along BE = OB = the radius of the corresponding circle

Again the length of the arc CF = $\square$ [Let me justify it by hands on trial]
= the length of radius of the corresponding circle.

$\therefore$ I have got, $\angle BOE$ and $\angle COF$ are the angles subtended at the centre by the arcs having lengths equal to the radii of the corresponding circles.

$\therefore$ By hands on trial, I have got that the arcs having lengths equal with the radii of the corresponding circles make fixed angle at the centre. ie, radian is a constant angle

By drawing any circle, I am observing that radian is a constant angle by justifying it with hands on trial. **[Let me do it myself]**

5 I prove with reason that radian is a constant angle.

Proof: Let, the centre of a circle be O and the radius be OA

Let, OA = r unit

The arc AB having length equal with the radius OA subtended an angle $\angle AOB$ at the centre O. According to the definition $\angle AOB = 1$ radian.

We know that the circle subtends 4 right angle at the centre.

$$\therefore \frac{\text{The length of the arc AB}}{\text{Circumference of the circle}} = \frac{r}{2\pi r} = \frac{1}{2\pi}$$

$$\text{Again, } \frac{\angle AOB}{4 \text{ right angle}} = \frac{1 \text{ radian}}{4 \text{ right angle}}$$

From geometry, it can be found that the angle subtended at the centre by arcs of different lengths is equal to the ratio of the lengths of their arcs.

$$\text{So, } \frac{\angle AOB}{4 \text{ right angle}} = \frac{\text{The length of the arc AB}}{\text{circumference of the circle}}$$

$$\text{or, } \frac{1 \text{ radian}}{4 \text{ right angle}} = \frac{1}{2\pi}$$

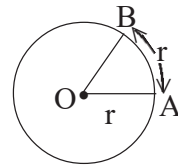
$$\text{or, } 1 \text{ radian} = \frac{4 \text{ right angle}}{2\pi} = \frac{2 \text{ right angle}}{\pi} \quad \text{and this is constant as both 2 right angle and } \pi \text{ are constants}$$

$$\therefore \text{ we have got, } 1 \text{ radian} = \frac{2 \text{ right angle}}{\pi}$$

$$\text{or, } 1^{\circ} = \frac{2 \text{ right angle}}{\pi}$$

$$\text{so, } \pi \text{ radian} = 2 \text{ right angle or, } 180^{\circ}$$

$$\text{or, } \pi^{\circ} = 2 \text{ right angle or, } 180^{\circ}$$



6 I write the value of 1 radian in sexagesimal system by calculating it.

π radian = 2 right angle = 180°

$$\begin{aligned}\therefore 1 \text{ radian} &= \frac{180^\circ}{\pi} = \frac{180^\circ}{\frac{22}{7}} \quad [\text{for calculation the approximate value of } \pi \\ &\quad \text{is taken as } \frac{22}{7}] \\ &= \frac{180^\circ \times 7}{22} = 57^\circ 16' 22'' \quad (\text{approximately}) \therefore 1 \text{ radian} = 57^\circ 16' 22'' \quad (\text{approx})\end{aligned}$$



$\frac{630^\circ}{11} \rightarrow$	$\begin{array}{r} 57 \\ 11 \overline{) 630} \\ \underline{- 55} \\ 80 \\ \underline{- 77} \\ 3 \end{array}$	$3^\circ = 3 \times 60' = 180'$	$\begin{array}{r} 16 \\ 11 \overline{) 180} \\ \underline{- 11} \\ 70 \\ \underline{- 66} \\ 4 \end{array}$	$4' = 4 \times 60'' = 240''$	$\begin{array}{r} 21.8 \\ 11 \overline{) 240} \\ \underline{- 22} \\ 20 \\ \underline{- 11} \\ 90 \\ \underline{- 88} \\ 2 \end{array}$
$\frac{630^\circ}{11} = 57^\circ 16' 22'' \quad (\text{approx})$				$21.8'' \approx 22''$	

7 I see by calculating the value of 1° in circular system

$$180^\circ = \pi^\circ$$

$$\text{or, } 1^\circ = \left(\frac{\pi}{180}\right)^\circ = \left(\frac{22}{7 \times 180}\right)^\circ, (\text{approximately}) \therefore \text{I am observing that, } 1^\circ < 1^\circ \left[\because \frac{22}{7 \times 180} < 1\right]$$

We have got a relation between the units of two systems.

[Let me fill in the box myself]

Sexagesimal System	Circular System
360°	2π radian = $2\pi^\circ$
180°	= π°
90°	$\frac{\pi}{2}$ radian =
60°	= $\frac{\pi^\circ}{3}$
	$\frac{\pi}{4}$ radian = $\frac{\pi^\circ}{4}$
30°	$\frac{\pi}{6}$ radian =

We shall remember : (1) For understanding angle in Sexagesimal system the sign “ $^\circ$ ” is used above the measure of an angle, as 60 degree = 60° ; again in circular system, for denoting angle, the sign “ $^\circ$ ” is used above the measure of an angle, example : 1 radian = 1°
(2) In circular system, we express the value of the angle by π and its parts. We do not use always the sign “ $^\circ$ ”. Example, $\frac{\pi}{3}$ is written for 60° .

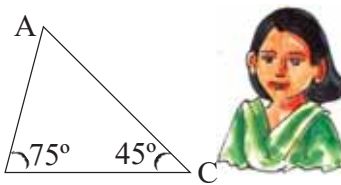
Application : 1. The values of two angles of a triangle in Sexagesimal system are 75° and 45° respectively; let us determine the value of the third angle in circular system.

Let, in $\triangle ABC$, $\angle ABC = 75^\circ$ and $\angle ACB = 45^\circ$

$$\therefore \angle BAC = 180^\circ - (75^\circ + 45^\circ) = \text{$$

$$\text{Again, } 180^\circ = \pi, \therefore 60^\circ = \frac{\pi}{3}$$

$\therefore$ The required value of the third angle in circular system is $\frac{\pi}{3}$



Application : 2. If the Sexagesimal values of two angles of a triangle are 65° and 85° respectively, let us determine the Circular value of third angle. [Let us do it myself]

Application : 3. A rotating ray rotates 30° more after two complete rotations in anticlockwise direction; let us determine the Sexagesimal and Circular value of the angle.

Since, the ray is rotating anticlockwise direction,

$\therefore$ The angle will be [positive/negative]

For one complete rotation of the rotating ray degree angle is formed.

$\therefore$ For 2 complete rotations, the angle will be $2 \times 360^\circ = 720^\circ$

Since after 2 complete rotations it rotates 30° more angle,

So, the sexagesimal value of the angle is $720^\circ + 30^\circ = 750^\circ$

Again, $180^\circ = \pi$, $\therefore 750^\circ = \left(\frac{750}{180}\pi\right) = 4\frac{1}{6}\pi$



Application : 4. I express $3750''$ into degrees, minutes and seconds.

$$60'' = 1' \quad \therefore 60 \overline{) \begin{array}{r} 62 \\ 3750 \\ - 360 \\ \hline 150 \\ - 120 \\ \hline 30 \end{array}}$$

$$60' = 1^\circ \quad \therefore 60 \overline{) \begin{array}{r} 1 \\ 62 \\ - 60 \\ \hline 2 \end{array}}$$

$$\therefore 3750'' = 1^\circ 2' 30''$$

Application : 5. I express 85.12° into degrees, minutes and seconds.

$$\begin{aligned} 85.12^\circ &= 85^\circ + (0.12)^\circ \\ &= 85^\circ + (0.12 \times 60') \quad [\because 1^\circ = 60'] \\ &= 85^\circ + 7.2' \\ &= 85^\circ + 7' + 0.2' = 85^\circ + 7' + (0.2 \times 60'') \quad [\because 1' = 60''] \\ &= 85^\circ + 7' + 12'' = 85^\circ 7' 12'' \end{aligned}$$

Application : 6. I express $40^\circ 16' 24''$ into radian.

$$\begin{aligned} 40^\circ 16' 24'' &= 40^\circ + 16' + 24'' \\ &= 40^\circ + 16' + \left(\frac{24}{60}\right)' \quad [\because 60'' = 1'] \\ &= 40^\circ + 16' + \frac{2}{5}' = 40^\circ + \left(16 + \frac{2}{5}\right)' \\ &= 40^\circ + \frac{82}{5}' = 40^\circ + \left(\frac{82}{5 \times 60}\right)^\circ \quad [\because 60' = 1^\circ] \\ &= 40^\circ + \left(\frac{41}{150}\right)^\circ = \boxed{}^\circ \end{aligned}$$



Since, $180^\circ = \pi$

[Let me do it myself]

$$\therefore \frac{6041}{150}^\circ = \frac{\pi}{180} \times \frac{6041}{150} = \frac{6041}{27000}\pi$$

Application : 7. Let me express $22^\circ 30'$ into radian. [Let me do it myself]

Application : 8. The difference of two acute angles in a triangle is $\frac{2\pi}{5}$ radian. Let me express the values of two angles into radian and degree.

Let the values of two acute angles be x° and y° and $x > y$

According to the condition, $x + y = \frac{\pi}{2}$ and $x - y = \frac{2\pi}{5}$

$$x + y = \frac{\pi}{2}$$

$$x - y = \frac{2\pi}{5}$$

$$\hline 2x = \frac{\pi}{2} + \frac{2\pi}{5} = \frac{9\pi}{10}$$

$$\therefore x = \frac{9\pi}{20}$$

$$\therefore y = \frac{\pi}{2} - \frac{9\pi}{20} = \frac{\pi}{20}$$

Again, $\pi = 180^\circ$

$$\therefore x = \frac{9\pi}{20} = \frac{9 \times 180^\circ}{20} = 81^\circ$$

and $y = \frac{\pi}{20} = \frac{180^\circ}{20} = 9^\circ$ $\therefore$ The values of two angles are $\frac{9\pi}{20}$ or 81° and $\frac{\pi}{20}$ or 9°

Application : 9. The ratio of the angles of a triangle is $2 : 5 : 3$; let us determine the circular value of the smallest angle of the triangle.

Let the values of the angles be $2x$, $5x$ and $3x$ radian, where x is common multiple and $x > 0$

$$\therefore 2x + 5x + 3x = \pi$$

$$\text{or, } 10x = \pi$$

$$\therefore x = \frac{\pi}{10}$$

$$\therefore \text{The value of the smallest angle will be } 2x = \frac{2\pi}{10} = \frac{\pi}{5}$$

Application : 10. I have drawn an equilateral triangle ABC. The line joining mid-point D of BC and the vertex A is AD; let us write by calculating, the circular value of $\angle BAD$. [Let me do it myself]

Hints : In equilateral triangle ABC, $\angle BAC = 60^\circ$ and the median of equilateral triangle is the bisector of corresponding angle. $\therefore \angle BAD = 30^\circ$

My friend Suva has drawn a circle in his copy and he has drawn an arc XKY of any length in that circle.

8 That arc XKY of any length will make an angle at the centre, let us see, how shall we get its circular value. I draw a circle with centre O and radius r unit length and let us determine what radian angle will be produced by an arc of s unit length at the centre.

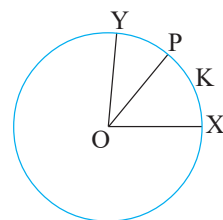
In the adjoining figure, in the circle with centre O, $OX = r$ unit,

Let the length of the arc XKY be s unit.

The arc XKY of s unit length subtends $\angle XOY$ at the centre.

Let, the arc XKP of r unit length subtends $\angle XOP$ at the centre.

$\therefore \angle XOP = 1$ radian [According to the definition]



In a circle the ratio of the angles at the centre subtended by different arcs is equal to the ratio of the lengths of their arcs.

$$\text{So, } \frac{\angle XOY}{\angle XOP} = \frac{\text{Length of the arc XKY}}{\text{Length of the arc XKP}} = \frac{s}{r}$$

$$\text{or, } \frac{\angle XOY}{1 \text{ radian}} = \frac{s}{r}$$

$$\text{or, } \theta = \frac{s}{r} \quad [\text{let, } \angle XOY = \theta \text{ radian}]$$

$$\therefore s = r\theta$$

$\therefore$ We have got, that in a circle of radius r unit length, if the circular value of an angle subtended by an arc of s unit length at the centre is θ then, $s = r\theta$

Application : 11. If the length of a radius in a circle drawn by Suva is 7 cm, then let us determine the circular value of an angle subtended by an arc of 5.5 cm length at the centre of that circle.

Here, $r = 7$ cm and $s = 5.5$ cm.

Let, the circular value of the angle subtended by 5.5 cm. arc be θ

$$s = r\theta$$

$$\therefore 5.5 = 7 \times \theta$$

$$\text{or, } \theta = \frac{5.5}{7} = \frac{55}{70} = \frac{11}{14}$$

$\therefore$ The required circular value of the angle is $\frac{11}{14}$ radian or $\frac{11^\circ}{14}$ or $\frac{\pi^c}{4}$ ($\because \frac{22}{7} \approx \pi$)

Application : 12. If the length of the radius of a circle is 6 cm, then let us write by calculating, the circular value of an angle which is subtended by arc of 15 cm. length at the centre.

[Let me do it myself]

Application : 13. In a circle, the ratio of two angles subtended by two arcs of unequal lengths is 5 : 3 and the sexagesimal value of the second angle is 45° ; let us determine the sexagesimal value and circular value of the first angle.

Let, the sexagesimal value of the first angle be θ°

$$\text{According to the condition, } \frac{\theta^\circ}{45^\circ} = \frac{5}{3} \quad \text{or, } \theta^\circ = \frac{5 \times 45^\circ}{3} = 75^\circ$$

$$\text{Since, } 180^\circ = \pi^c \quad \text{so, } 75^\circ = \frac{75}{180} \times \pi^c = \frac{5}{12} \pi^c$$

$\therefore$ The sexagesimal value of the first angle is 75° and its circular value is $\frac{5}{12} \pi^c$

Application : 14. If the measures of two angles of a triangle are $35^\circ 57' 4''$ and $39^\circ 2' 56''$, then let us determine the circular value of third angle.

$$\begin{aligned} \text{The sum of two angles of the triangle} &= 35^\circ 57' 4'' + 39^\circ 2' 56'' \\ &= 74^\circ 59' 60'' \\ &= 74^\circ 60' \\ &= 75^\circ \end{aligned}$$

$$\therefore \text{The value of the third angle} = 180^\circ - 75^\circ = 105^\circ = 105 \times \frac{\pi^c}{180} = \frac{7}{12} \pi^c$$

Therefore, the circular value of the third angle is $\frac{7}{12} \pi$



Application : 15. Let us write the value of complementary angle of $65^{\circ}35'25''$ in sexagesimal system.

$$\begin{aligned} 90^{\circ} &= 89^{\circ}60' \\ &= 89^{\circ}59'60'' \\ &\quad 89^{\circ}59'60'' \\ &\quad - 65^{\circ}35'25'' \\ \hline &\quad 24^{\circ}24'35'' \end{aligned}$$

$\therefore$ The complementary angle of $65^{\circ}35'25''$ is $24^{\circ}24'35''$



Application : 16. Let us write the value of complementary angle of $27^{\circ}27'27''$ in sexagesimal system. [Let me do it myself]

Application : 17. Let us write the supplementary angle of $75^{\circ}36'24''$ in sexagesimal system.

$$\begin{aligned} 180^{\circ} &= 179^{\circ}60' \\ &= 179^{\circ}59'60'' \\ &\quad 179^{\circ}59'60'' \\ &\quad - 75^{\circ}36'24'' \\ \hline &\quad 104^{\circ}23'36'' \end{aligned}$$

$\therefore$ The supplementary angle of $75^{\circ}36'24''$ is $104^{\circ}23'36''$



Application : 18. Let us write the supplementary angle of $85^{\circ}32'36''$ in sexagesimal system. [Let me do it myself]

Let us work out 20

- Let us express the following into degrees, minutes and seconds.
(i) $832'$ (ii) $6312''$ (iii) $375''$ (iv) $27\frac{1}{12}^{\circ}$ (v) 72.04°
- Let us determine the circular values of the followings
(i) 60° (ii) 135° (iii) -150° (iv) 72° (v) $22^{\circ}30'$ (vi) $-62^{\circ}30'$ (vii) $52^{\circ}52'30''$ (viii) $40^{\circ}16'24''$
- In $\triangle ABC$, $AC = BC$ and BC is extended upto the point D . If $\angle ACD = 144^{\circ}$, then let us determine the circular value of each of the angles of $\triangle ABC$.
- If the difference of two acute angles of a right angled triangle is $\frac{2\pi}{5}$; let us write the sexagesimal values of two angles.
- The measure of one angle of a triangle is 65° and other angle is $\frac{\pi}{12}$; let us write the sexagesimal value and circular value of third angle.
- If the sum of two angles is 135° and their difference is $\frac{\pi}{12}$, then let us determine the sexagesimal value and circular value of two angles.
- If the ratio of three angles of a triangle is $2:3:4$, then let us determine the circular value of the greatest angle.
- The length of a radius of a circle is 28 cm. Let us determine the circular value of angle subtended by an arc of 5.5 cm length at the centre of this circle.
- The ratio of two angles subtended by two arcs of unequal lengths at the centre is $5:2$ and if the sexagesimal value of the second angle is 30° , then let us determine the sexagesimal value and the circular value of the first angle.

10. A rotating ray makes an angle $-5\frac{1}{12}\pi$. Let us write by calculating, in which direction and how many times the ray has completely rotated and there after what more angle it has produced
11. I have drawn an isosceles triangle ABC whose included angle of two equal sides is $\angle ABC = 45^\circ$; the bisector of $\angle ABC$ intersects the side AC at the point D. Let us determine the circular values of $\angle ABD$, $\angle BAD$, $\angle CBD$ and $\angle BCD$.
12. The base BC of the equilateral triangle ABC is extended upto the point E so that $CE = BC$. By joining A, E, let us determine the circular values of the angles of $\triangle ACE$.
13. If the measures of three angles of quadrilateral are $\frac{\pi}{3}$, $\frac{5\pi}{6}$ and 90° respectively, then let us determine and write the sexagesimal and circular values of fourth angle.
14. **(V.S.A.)**
(A) (M.C.Q) :
 - (i) The end point of the minute hand of a clock rotates in 1 hour
 (a) $\frac{\pi}{4}$ radian (b) $\frac{\pi}{2}$ radian (c) π radian (d) 2π radian
 - (ii) $\frac{\pi}{6}$ radian equals to (a) 60° (b) 45° (c) 90° (d) 30°
 - (iii) The circular value of each internal angle of a regular hexagon is
 (a) $\frac{\pi}{3}$ (b) $\frac{2\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$
 - (iv) The measurement of θ in the relation to $s = r\theta$ is determined by (a) sexagesimal system (b) circular system (c) those two methods (d) none of these two methods.
 - (v) In cyclic quadrilateral ABCD, if $\angle A = 120^\circ$, then the circular value of $\angle C$ is
 (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$**(B) Let us write whether the following statements are true or false :**
 - (i) The angle, formed by rotating a ray centering its end point in anticlockwise direction is positive.
 - (ii) The angle, formed for completely rotating a ray twice by centering its end point is 720°**(C) Let us fill in the blanks :**
 - (i) π radian is a _____ angle.
 - (ii) In sexagesimal system 1 radian equals to _____ (approx)
 - (iii) The circular value of the supplymentary angle of the measure $\frac{3\pi}{8}$ is _____.
15. **(V.S.A.)**
 - (i) If the value of an angle in degree is D and in radian is R ; then let us determine the value of $\frac{R}{D}$
 - (ii) Let us write the value of complementary angle of the measure $63^\circ 35' 15''$
 - (iii) If the measures of two angles of a triangle are $65^\circ 56' 55''$ and $64^\circ 3' 5''$, then let us determine the circular value of third angle.
 - (iv) In a circle, if an arc of 220 cm. length subtends an angle of measure 63° at the centre, then let us determine the radius of the circle.
 - (v) Let us write the circular value of an angle formed by the end point of hour hand of a clock in 1 hour rotation.



There is a music school at the ground floor of our house. Many children of our locality come to learn the art of singing. I also learn here. The mat spread on the floor of music school is spoiled. My friend Sumit ordered to make a rectangular mat of 4m. length and 3m. breadth which he has brought it. The room of the music school is square shaped. Hence the mat cannot be spread well on the floor.

But if this rectangular mat would be square-shaped, could this square-shaped mat be laid on the floor? Let us write the length of one side of the square-shaped mat whose area is equal to the area of the rectangular mat.

The length of one side of the square-shaped mat, whose area is equal to the area of rectangular mat of 4m. length and 3m. breadth, is $\sqrt{4 \times 3}$ m.

The mean proportional of 4 and 3 is to be determined. $\therefore$ The square root of 12 is to be determined.

But is it possible to determine the mean proportional of 4 and 3 or the value of $\sqrt{12}$ in geometric method?

Construction : 1. Determination of the mean proportional of a and b or the value of $\sqrt{ab}$ in geometric method.

First method I have drawn two line segments of lengths 'a' unit and 'b' unit. Let us draw a line segment whose length is the mean proportional of the lengths of these two line segments.

Construction Procedure :

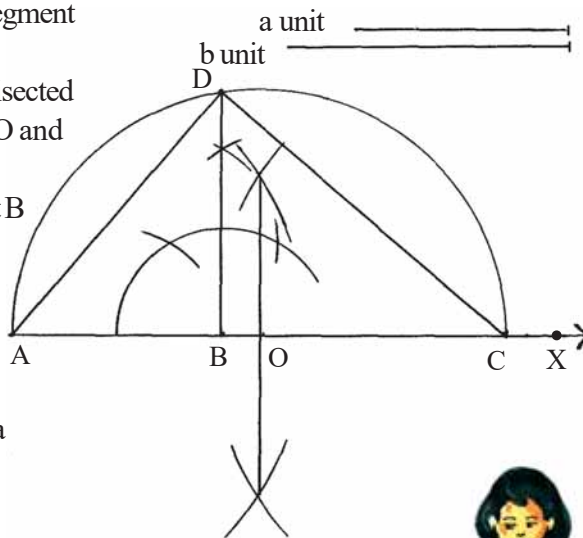
- (i) I draw any ray AX.
- (ii) From AX, the part AB having length equals to the line segment of a unit length and from BX, the part BC having length equals to the line segment of b unit length are cut off.
- (iii) I bisect the line segment AC. Let AC is bisected at the point O. A semicircle with centre at O and radius of OA or OC is drawn.

- (iv) I draw a perpendicular on BC at the point B which intersects the semicircle at the point D.

Then the length of the line segment BD is the mean proportional of two lengths AB and BC. i.e, the value of the length of the line segment BD

= (The value of the mean proportional of a unit and b unit)

= $\sqrt{ab}$ unit



Proof : I join A, D and C, D. $\angle ADC$ is an angle in a semicircle.

$\therefore \angle ADC = 1$ right angle

In right angled triangle ADC, from right angular point D, DB is perpendicular on AC.

$\therefore \triangle ABD$ and $\triangle CBD$ are similar.



$$\therefore \frac{AB}{BD} = \frac{BD}{BC} \quad \text{or, } BD^2 = AB \cdot BC = a \cdot b$$

$$\therefore BD = \sqrt{ab} \text{ unit}$$



In another method I draw a mean proportional of the lengths of two line segments.

Second method I draw two line segments of lengths 'a' unit and 'b' unit. I draw a line segment whose length is the mean proportional of the lengths of these two line segments.

Construction Procedure :

(i) From a ray AX, the line segment AB of a unit length and from BA, the line segment BC of b unit length are cut off.

(ii) I bisect the line segment AB. Let AB is bisected at the point O. I draw a semicircle by taking centre at O and radius of OA or OB length.

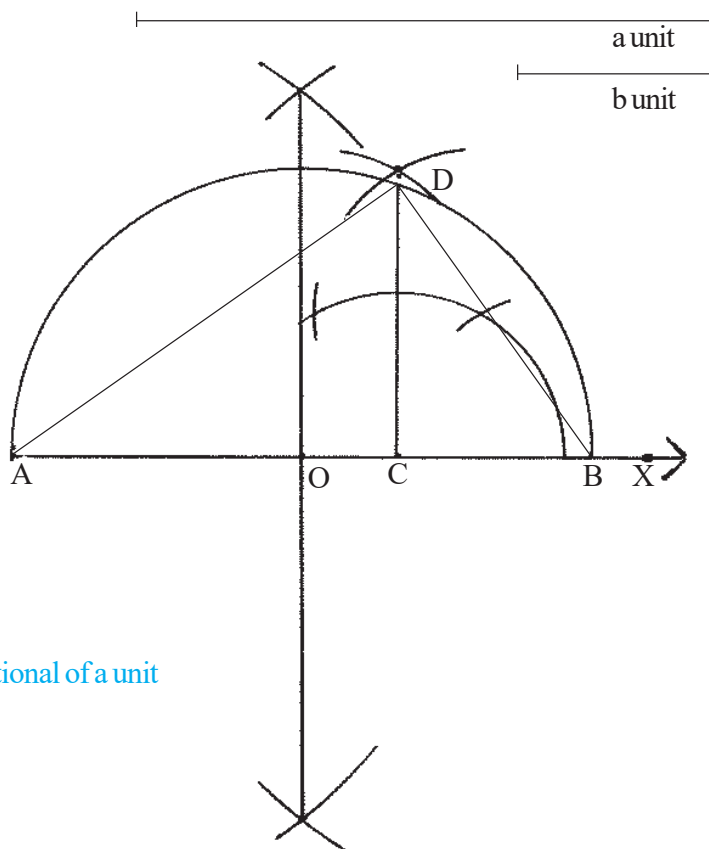
(iii) I draw a perpendicular on the line segment AB at the point C which intersects the semicircle at the point D.

$\therefore$ The length of BD is mean proportional of the lengths of two line segments AB and BC.

i.e. The Value of the length of the line segment BD

= (The value of the mean proportional of a unit and b unit)

$$= \sqrt{ab} \text{ unit}$$



Proof : I join A and D.

$\angle ADB$ is an angle in a semicircle

$\therefore \angle ADB = 1 \text{ rt. angle.}$

$\therefore$ In the rt. angled triangle ADB, from the right-angular point D, DC is perpendicular on AB.

$\therefore \triangle ABD$ and $\triangle DBC$ are similar

$$\therefore \frac{AB}{BD} = \frac{BD}{BC} \quad \text{or, } BD^2 = AB \cdot BC = a \cdot b$$

$$\therefore BD = \sqrt{ab} \text{ unit}$$



Application : 1. Now I determine the value of $\sqrt{4 \times 3} = \sqrt{12}$ in geometric method.

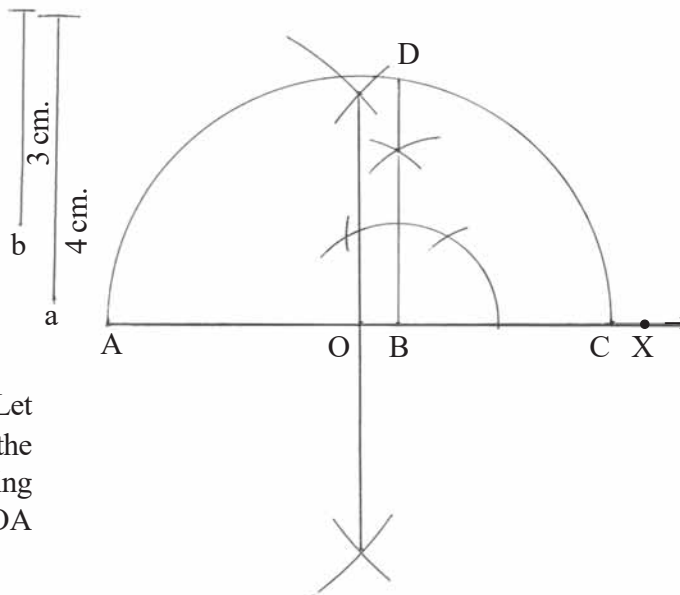


Construction Procedure :

(i) I take two line segments a and b whose lengths are 4 cm. and 3 cm. respectively.

(ii) From the ray AX , the line segment AB of 4 cm. length and from the ray BX , the line segment BC of 3 cm. length are cut off.

(iii) I bisect the line segment AC . Let the line segment AC is bisected at the point O . I draw a semicircle by taking the centre at O and the radius of OA or OC length.



(iv) At the point B , I draw a perpendicular on BC which intersects the semicircle at the point D .

$\therefore$ BD is mean proportional of the lengths of two line segments AB and BC .

$\therefore$ The length of the line segment BD is $\sqrt{12}$ cm.

By measuring with a scale, I am observing that

$BD = 3.5$ cm. (approx)

$\therefore \sqrt{12} = 3.5$ (approx)

Proof : BD is mean proportional of AB and BC .

$\therefore BD^2 = AB \cdot BC = a \cdot b$ sq cm. $= 4 \cdot 3$ sq cm. $= 12$ sq cm.

$\therefore BD = \sqrt{12}$ cm.

Application : 2. I determine the values of $\sqrt{21}$ and $\sqrt{15}$ in geometric method or in geometric method, I determine the square roots of 21 and 15. [Let me do it myself]



Hints : $21 = 7 \times 3$. $\therefore$ In this case, I take two line segments a and b whose lengths are 7 units and 3 units respectively. and in the same method I shall determine mean proportional of a and b . Again $15 = \square \times \square$, I draw it by taking the lengths of a and b properly.

Application : 3. I determine the value of $\sqrt{12}$ in another constructional method based on Pythagoras theorem.

Another construction method based on Pythagoras Theorem.



Construction Procedure :

I draw a right angled triangle ABC, whose side AB = 2cm. and hypotenuse BC = 4cm.

∴ The length of AC is $\sqrt{12}$ cm.

Measuring with a scale, I am observing that, AC = 3.5 cm. (approx)

$$\therefore \sqrt{12} = 3.5 \text{ (approx)}$$

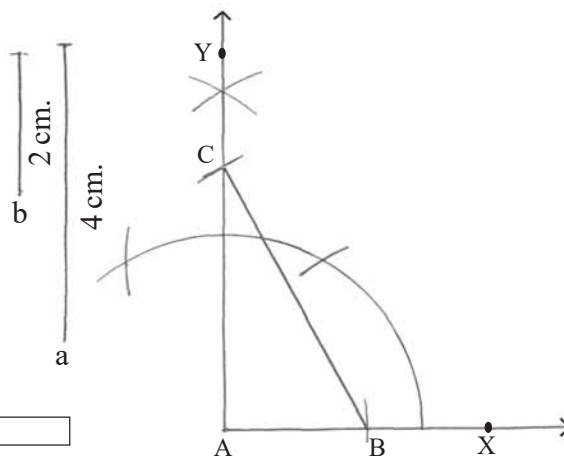
Proof:

ΔABC is a right angled triangle.

$$\therefore AC^2 = BC^2 - AB^2 = 4^2 - 2^2 = \boxed{}$$

[Let me write it myself]

$$\therefore AC = \sqrt{12} \text{ cm.}$$



Application : 4. I determine the value of $\sqrt{23}$ in geometric method.

$$\boxed{23 = 5 \times 4.6}$$

Construction Procedure :

(i) I take two line segments a and b whose lengths are 5 cm. and 4.6 cm. respectively.

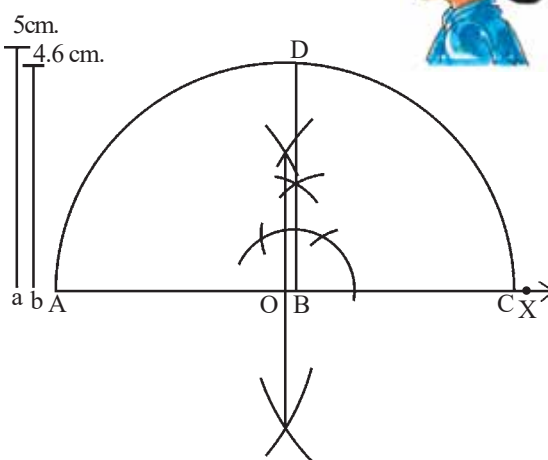
Measuring with a scale, I am observing that, BD = $\boxed{}$ cm. (approx)

$$\therefore \sqrt{23} = \boxed{} \text{ [approx]}$$

[I have understood i.e., if there is a two digit prime number, as for example, 17, 19, 29, 37 etc, then these numbers will be divided by 5,

example : $17 = 5 \times 3.4$,

$19 = 5 \times 3.8$, $29 = 5 \times 5.8$, $37 = 5 \times 7.4$ etc.]



Application : 5. How shall we get a squared mat whose area is equal to a rectangular mat of 4m. length and 3m. breadth? I try to draw a square with an area equals to a rectangle.

I draw a square whose area is equal to a rectangle of 6 cm. length and 3 cm. breadth.



Construction Procedure :

(i) I draw a rectangular figure ABCD of 6 cm. length and 3 cm. breadth.

(ii) I extended DC and cut off the part CE which is equal to CB from the extended part.

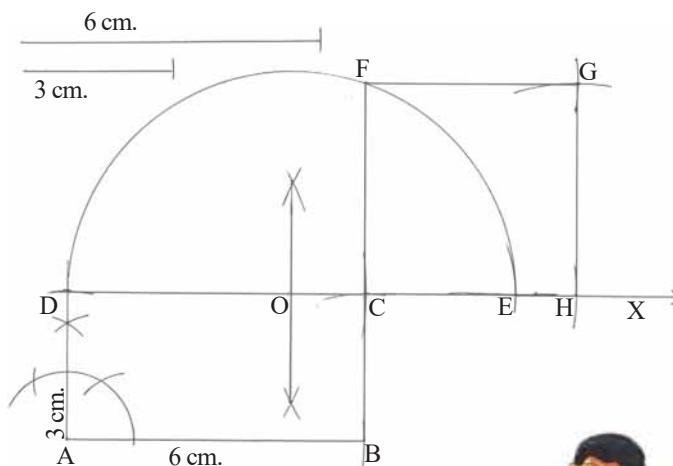
(iii) I bisect the line segment DE, Let the line segment is bisected at the point O. I draw a semicircle on DE by taking the centre at O and radius of OD or OE length.

(iv) I extend BC which intersects the semicircle at the point F.

(v) I drew the squared figure CFGH by taking the length equal to the side CF.

$\therefore$ CFGH is the required square whose area is equal to the area of the rectangle ABCD.

Therefore, area of the square CFGH = area of the rectangle ABCD.



Proof : ABCD is a rectangle.

$\therefore \angle BCD = 1$ rectangle. Hence CF is perpendicular on DE.

$\therefore$ According to the construction, the length of CF is mean proportional of the lengths of DC and CE.

$\therefore CF^2 = DC \cdot CE = AB \cdot CD = \text{area of the rectangle ABCD}.$

Application : 6. Let us draw a square whose area is equal to the area of a rectangle of 7cm. length and 4cm. breadth. **[Let me do it myself]**

Application : 7. I draw a triangle whose three sides are 7cm., 6cm. and 3 cm. respectively. I draw a square whose area is equal to the area of that triangle.



Construction Procedure :

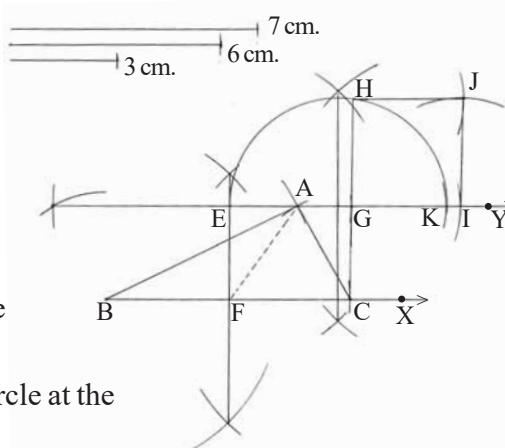
(i) I drew a triangle ABC of which AB, BC and CA are 7 cm., 6 cm. and 3 cm. respectively.

(ii) I drew a rectangle EFCG whose area is equal to the area of $\triangle ABC$.

(iii) Now from the extended EG, I cut off GK which is equal to GC.

(iv) Now I drew a semicircle by taking the line segment EK as diameter.

(v) I extended CG which intersects the semicircle at the point H.



(vi) I drew a squared figure HGIJ by taking the side GH.

∴ HGIJ is the required square whose area is equal to the area of $\triangle ABC$.

Proof: I joined AF.

-AF is a median of $\triangle ABC$. ∴ Area of $\triangle AFC = \frac{1}{2} \triangle ABC$ (i)

Again $\triangle AFC$ and the rectangle EFCG lie on the same base FC and within the pair of parallel lines FC and EG.

∴ Area of $\triangle AFC = \frac{1}{2}$ area of rectangle EFCG(ii)

From (i) and (ii), I have got, Area of $\triangle ABC$ = area of rectangle EFCG.

In the rectangle EFCG, $\angle CGE = 1$ rt. angle. Hence, $HG \perp EK$.

According to the construction, the length of HG is mean proportional of the length EG and GK.

∴ $HG^2 = EG \cdot GK = EG \cdot GC$ = area of the rectangle EFCG.

∴ Area of the square HGIJ = area of the rectangle EFCG.

Again, area of $\triangle ABC$ = area of the rectangle EFCG.

∴ Area of $\triangle ABC$ = area of the square HGIJ.

Application : 8. I draw a square whose area is equal to the area of an equilateral triangle having the side 7 cm. in length. [Let me do it myself]

Let us work out

21

- Let us draw the mean proportional of the following line segments and let us measure the values of the mean proportionals in each case with the help of a scale :

(i) 5 cm., 2.5 cm.	(ii) 4 cm., 3 cm.	(iii) 7.5 cm., 4 cm.
(iv) 10 cm., 4 cm.	(v) 9 cm., 5 cm.	(vi) 12 cm., 3 cm.
- Let us determine the square roots of the following numbers by geometric method :
 (i) 7 (ii) 8 (iii) 24 (iv) 28 (v) 13 (vi) 29
- Let us draw the squared figures by taking the following lengths as sides.
 (i) $\sqrt{14}$ cm. (ii) $\sqrt{22}$ cm. (iii) $\sqrt{31}$ cm. (iv) $\sqrt{33}$ cm.
- Let us draw the squares whose areas are equal to the areas of the rectangles by taking the following lengths as its sides :

(i) 8 cm., 6 cm.	(ii) 6 cm., 4 cm.
(iii) 4.2 cm., 3.5 cm.	(iv) 7.9 cm., 4.1 cm.
- Let us draw the squares whose areas are equal to the areas of the following triangles :
 - The lengths of three sides of a triangles are 10 cm., 7 cm. and 5 cm. respectively.
 - An isosceles triangle whose base is 7 cm. length and the length of each of two equal sides is 5 cm.
 - An equilateral triangle whose side is 6 cm. in length.

Last week, a science exhibition was held in our school. We had wandered about different sections of exhibition on different subjects through out whole day. We had seen some mathematical funs in the exhibition room of mathematics. But among them, the topic "funny games with match sticks" was very interesting. In this topic, the perimeter of area of different plane figures framed out putting match sticks together were determined or plane figures with fixed areas were framed.

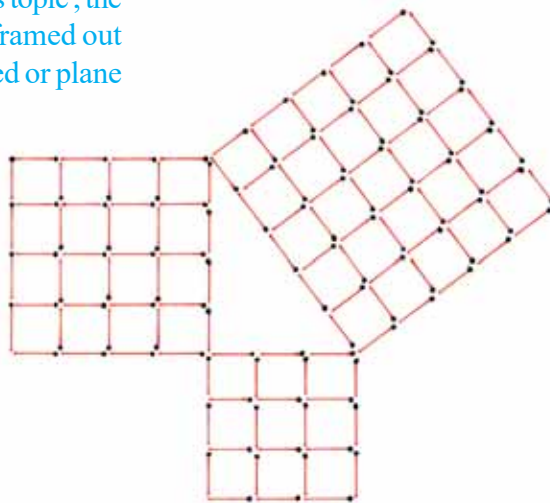


[Where  = 1 unit² was considered]

I observed there a funny figure of match sticks in a chart.



That was →



On the chart I am observing a right angled triangle and there are 5 sticks in hypotenuse, 4 sticks in perpendicular and  sticks in base.

Also I am observing that,

the area of the square on the hypotenuse = 25 sq.unit = 5^2 sq.unit

the area of the square on the base = 9 sq.unit = 3^2 sq.unit

the area of the square on the perpendicular = 16 sq.unit = 4^2 sq.unit

I am observing that, $5^2 = 3^2 + 4^2$

$\therefore (\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

that is, area of the square drawn on the hypotenuse

= area of the square drawn on the base + area of the square drawn on the perpendicular

1 From where have I got the relation among the hypotenuse, base and perpendicular of a right angled triangle? Is this relation possible in any right angled triangle?

I have got it from Pythagoras Theorem.



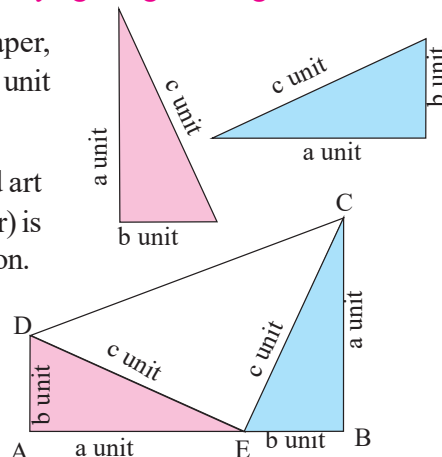
Pythagoras Theorem : In any right angled triangle, the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on other two sides.

Hands on trial Let us justify Pythagoras Theorem in any right angled triangle.

(1) I draw a right angled triangle on a coloured art paper, whose base is 'b' unit length, height (perpendicular) is 'a' unit length and I cut off the right angled triangular region.

(2) I draw another right angled triangle on a coloured art paper whose base is 'a' unit length, height (perpendicular) is 'b' unit length and cut off the right-angled triangular region.

(3) Let the length of the hypotenuse of two triangles be 'c' unit. Now in a white art paper as the adjoining figure fixing two right angled triangles I have got the region of the trapezium ABCD.



(4) Area of the region of the trapezium ABCD = $\frac{1}{2} (a + b) \times (a + b)$ sq.unit

Again area of the region of the trapezium ABCD

= area of $\triangle DAE$ + area of $\triangle CBE$ + area of $\triangle DEC$

$$\therefore \frac{1}{2} (a + b) (a + b) = \frac{1}{2} a \times b + \frac{1}{2} a \times b + \frac{1}{2} c \times c \quad [\because \angle DEC = 90^\circ]$$

$$\text{or, } (a + b)^2 = ab + ab + c^2$$

$$\text{or, } a^2 + 2ab + b^2 = 2ab + c^2$$

$$\therefore a^2 + b^2 = \boxed{}$$

$$\therefore \text{I got, (base)}^2 + \text{(perpendicular)}^2 = \text{(hypotenuse)}^2$$

$\therefore$ By hands on trial I have got, in any right angled triangle, the area of square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on other two sides.



I prove with reason

Theorem : 49. Pythagoras Theorem : In any right angled triangle the area of the square drawn on the hypotenuse is equal to the sum of the areas of the squares drawn on other two sides.

Given : ABC is a right angled triangle whose $\angle A$ is right angle.

To prove : $BC^2 = AB^2 + AC^2$



Construction : I draw a perpendicular AD on the hypotenuse BC from the right angular point A which intersects the side BC at the point D.

Proof : In right angled triangle ABC, AD is perpendicular on the hypotenuse BC

$\therefore \triangle ABD$ and $\triangle CBA$ are similar

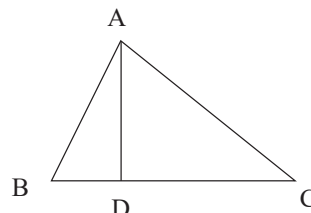
$$\text{Hence, } \frac{AB}{BC} = \frac{BD}{AB}, \therefore AB^2 = BC \cdot BD \dots\dots\dots(I)$$

Again, $\triangle CAD$ and $\triangle CBA$ are similar.

$$\text{Hence, } \frac{AC}{BC} = \frac{DC}{AC}, \therefore AC^2 = BC \cdot DC \dots\dots\dots(II)$$

$$\begin{aligned} \text{So, by adding (I) and (II), I get, } AB^2 + AC^2 &= BC \cdot BD + BC \cdot DC \\ &= BC (BD + DC) = BC \cdot BC = BC^2 \end{aligned}$$

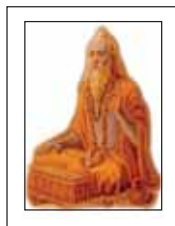
$$\therefore BC^2 = AB^2 + AC^2 \quad \text{[proved]}$$



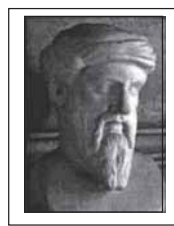
Long ago from today (nearly 800 B.C.) an ancient Indian Mathematician Baudhayan described Pythagoras Theorem in the following way, He said

The diagonal of a rectangle produces by itself the same area as produced by its both sides (i.e. length and breadth)

For this reason, sometimes this is called as Bandhayana Theorem.



Bandhayan



Pythagoras

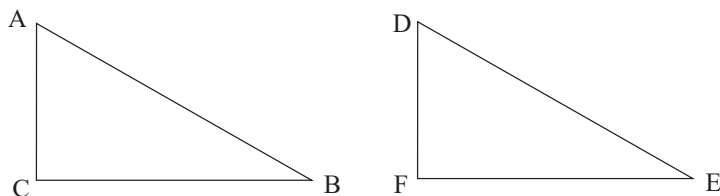


Is the converse of Pythagoras theorem possible? i.e. if in a triangle, the area of a square drawn on one side is equal to the sum of the areas of squares drawn on other two sides, will then the angle opposite to the first side be right angle? I prove it with reason.

I prove with reason

Theorem : 50. Converse of Pythagoras Theorem : If in a triangle, the area of a square drawn on one side is equal to the sum of the areas of squares drawn on other two sides, then the angle opposite to the first side will be right angle.

Given : In $\triangle ABC$, the area of the square drawn on the side AB is equal to the sum of the areas of the squares drawn on BC and AC . That is, $AB^2 = AC^2 + BC^2$



To prove : $\angle ACB = 1$ rt. angle.

Construction : I draw the line segment FE which is equal to CB . I draw a perpendicular on the side FE at the point F and cut off FD from that perpendicular which is equal to the side CA and join the two points D and E .

Proof : $AB^2 = BC^2 + AC^2$ [Given]
 $= EF^2 + DF^2$ [$\because$ acc.to the construction, $EF = BC$ and $AC = DF$]
 $= DE^2$ [$\because \angle DFE = 1$ rt. angle]
 $\therefore AB = DE$

Now in $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $BC = EF$ and $AC = DF$
 $\therefore \triangle ABC \cong \triangle DEF$ (by S-S-S axiom of congruency)
 $\therefore \angle ACB = \angle DFE = 1$ rt. angle [$\because DF \perp EF$ by construction]
 $\therefore \angle ACB = 1$ rt. angle **[proved]**





I have understood, we can easily determine whether a triangle is right-angled or not from converse of Pythagoras theorem. Example : The lengths of three sides of a triangle are 5cm., 12cm., and 13 cm. respectively. This triangle will be a triangle. Since, $13^2 = 5^2 + 12^2$ and the length of its hypotenuse is 13 cm.

2 But what the type of the sides of a right angled triangle would be, can its formula be found out? since, $(m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2$

Hence, if the sides of a triangle are $(m^2 - n^2)$ units, $2mn$ units and $(m^2 + n^2)$ units, then it will be a right angled triangle and its hypotenuse will be $(m^2 + n^2)$ units, (where, $m > n$)

Application 1 : Taking different proper values of m and n , let us write the sides of two right angled triangles. [Let me do it myself]

Application 2 : In our garden, a ladder of 25m length is inclined to a guardwall at the height of 24m. above the ground. Let us write by calculating, the distance of the foot of the ladder from the guardwall. Let the length of the ladder $AC = 25\text{m.}$, $AB = 24\text{m.}$, from Pythagoras theorem in right angled triangle ABC, we get,

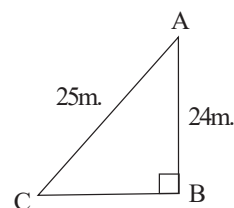
$$AB^2 + BC^2 = AC^2$$

$$\text{or, } (24 \text{ m.})^2 + (BC)^2 = (25 \text{ m.})^2$$

$$\text{or, } BC^2 = (25 \text{ m.})^2 - (24 \text{ m.})^2 = \boxed{}$$

$$\therefore BC = 7$$

$\therefore$ The foot of the ladder is in 7m. distance from the guardwall.

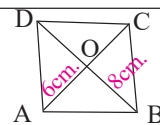


Application 3: If the lengths of two diagonals of a rhombus are 12cm. and 16cm. respectively, then let us write by calculating, the length of one side of this rhombus. [Let me do it myself]

Hints : The two diagonals of a rhombus bisect each other, perpendicularly.

$$\therefore OA = \frac{12}{2} \text{ cm.} = \boxed{} \text{ cm., } OB = \boxed{} \text{ cm.}$$

AOB is a right angled triangle, $\therefore$ from Pythagoras theorem, we get, $AB^2 = OA^2 + OB^2$



Application 4: I have drawn an isosceles triangle ABC whose $\angle B$ is right angle. I have drawn a bisector AD of $\angle BAC$ which intersects the side BC at the point D. Let us prove that, $CD^2 = 2BD^2$

Given : In isosceles triangle ABC, $\angle B = 90^\circ$. The bisector AD of $\angle BAC$ intersects the side BC at the point D.

To prove : $CD^2 = 2BD^2$

Construction : I draw the perpendicular DE on AC from the point D, which intersects AC at the point E.

Proof : ABC is an isosceles right angled triangle. $\therefore \angle ACB = 45^\circ$

$\therefore$ In right angled triangle DEC, $\angle DCE = 45^\circ$ ($\because \angle ACB = 45^\circ$);

Again, $\because \angle DEC = 90^\circ$, so, $\angle EDC = 45^\circ$; $\therefore$ in $\triangle DEC$, $DE = EC$

In $\triangle ABD$ and $\triangle AED$, $\angle BAD = \angle EAD$ [$\because$ AD is bisector of $\angle BAE$]

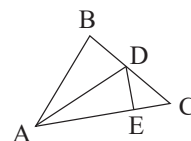
$\angle ABD = \angle AED$ (Each is 90°) and AD is common side.

$\therefore \triangle ABD \cong \triangle AED$ (by A-A-S criterion of congruency)

Hence, $BD = DE$ (corresponding sides of congruent triangles)

In the right angled triangle DEC, $DC^2 = DE^2 + CE^2 = 2DE^2$ ($\because CE = DE$)

$= 2BD^2$ ($\because DE = BD$) [proved]



Application 5: If in $\triangle ABC$, $AD \perp BC$, let us prove that, $AB^2 + CD^2 = AC^2 + BD^2$ [Let me do it myself]

Application 6: If in $\triangle ABC$, $\angle A$ is right angle and BP and CQ are two medians, then let us prove that, $5BC^2 = 4(BP^2 + CQ^2)$

Given : In $\triangle ABC$, $\angle BAC = 1\text{rt. angle}$. BP and CQ are two medians of the triangle

To prove : $5BC^2 = 4(BP^2 + CQ^2)$

Proof : In $\triangle ABC$, $\angle A$ is right angle

$$\therefore BC^2 = AB^2 + AC^2$$

$$= (2AQ)^2 + (2AP)^2 \quad [\because P \text{ and } Q \text{ are mid-points of the two sides } AC \text{ and } AB \text{ respectively.}]$$

$$\therefore BC^2 = 4(AQ^2 + AP^2) \dots\dots\dots (i)$$

Again, $\triangle BAP$ and $\triangle CAQ$ are right angled triangles.

$$\therefore BP^2 = AB^2 + AP^2 = (2AQ)^2 + AP^2 = 4AQ^2 + AP^2$$

$$CQ^2 = AC^2 + AQ^2 = (2AP)^2 + AQ^2 = 4AP^2 + AQ^2$$

$$\therefore BP^2 + CQ^2 = 4AQ^2 + AP^2 + 4AP^2 + AQ^2 = 5AQ^2 + 5AP^2 = 5(AQ^2 + AP^2) \dots\dots(ii)$$

$$5BC^2 = 5.4(AQ^2 + AP^2) \quad [\text{from (i), we get}]$$

$$= 4.5(AQ^2 + AP^2) = 4(BP^2 + CQ^2) \quad [\text{from (ii), we get}] \quad \text{[proved]}$$

Application 7: From the vertex A of $\triangle ABC$ I have drawn a perpendicular AD on the side BC which intersects the side BC at the point D and if $AD^2 = BD.CD$, then let us prove that, ABC is a right angled triangle and $\angle A = 90^\circ$

Given : In $\triangle ABC$, $AD \perp BC$ and $AD^2 = BD.CD$

To prove : $\angle BAC = 90^\circ$

Proof : In right angled triangle ADB , $\angle ADB = 90^\circ$

$$\therefore AB^2 = AD^2 + BD^2 \dots\dots\dots (i) \quad [\text{from Pythagoras theorem}]$$

Again, in right angled triangle ADC , $\angle ADC = 90^\circ$

$$\therefore AC^2 = AD^2 + CD^2 \dots\dots\dots (ii) \quad [\text{from Pythagoras theorem}]$$

By adding (i) and (ii), we get,

$$\begin{aligned} AB^2 + AC^2 &= BD^2 + CD^2 + 2AD^2 \\ &= BD^2 + CD^2 + 2BD.CD \quad [\because AD^2 = BD.CD] \\ &= (BD + CD)^2 = BC^2 \end{aligned}$$

$$\therefore AB^2 + AC^2 = BC^2$$

$\therefore$ From converse of Pythagoras theorem, we get, ABC is a right angled triangle whose $\angle BAC = 90^\circ$ [proved]

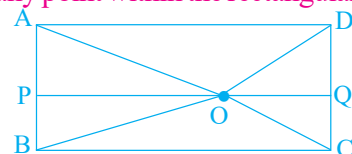
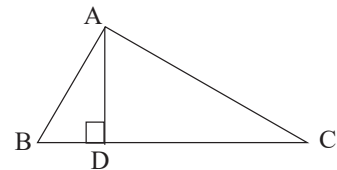
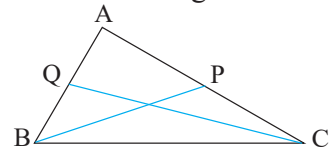
Application 8: Let us prove that the area of a square drawn on the diagonal of a square is twice of the area of that square. [Let me do it myself]

Application 9: I have drawn a rectangular figure $ABCD$. If O is any point within the rectangular figure, then let us prove that $OA^2 + OC^2 = OB^2 + OD^2$

Given : Within the rectangular figure $ABCD$, O is any point.

To prove : $OA^2 + OC^2 = OB^2 + OD^2$

Construction : I draw a line, parallel to BC through the point O , which intersects the two sides AB and DC at the points, P and Q respectively.



Proof : According to construction, $PQ \parallel BC$

$\therefore PQ \perp AB$ and $PQ \perp DC$ ($\because \angle B = 90^\circ$ and $\angle C = 90^\circ$)

$\therefore$ Each of $\triangle APO$, $\triangle BPO$, $\triangle CQO$ and $\triangle DQO$ is a right angled triangle

$$\therefore OA^2 = AP^2 + OP^2 \quad OC^2 = CQ^2 + OQ^2$$

$$OB^2 = BP^2 + OP^2 \quad OD^2 = DQ^2 + OQ^2$$

$$\therefore OA^2 + OC^2 = AP^2 + OP^2 + CQ^2 + OQ^2 \dots\dots\dots (i)$$

But according to the condition, each of $APQD$ and $BPQC$ is a rectangular figure

Hence, $AP = DQ$ and $CQ = BP$

$$\begin{aligned} \text{From (i) we get, } OA^2 + OC^2 &= DQ^2 + OP^2 + BP^2 + OQ^2 \\ &= (DQ^2 + OQ^2) + (BP^2 + OP^2) \\ &= OD^2 + OB^2 = OB^2 + OD^2 \quad \text{[proved]} \end{aligned}$$



Let us work out

22

- If the followings are the lengths of the three sides of a triangle, then let us write by calculating, the cases where the triangles are right-angled triangles : (i) 8cm., 15 cm. and 17 cm. (ii) 9 cm., 11 cm. and 6 cm.
- In the road of our locality there is a ladder of 15m. length kept in such a way that it has touched Millis' window at a height of 9m. above the ground. Now keeping the foot of the ladder at the same point of that road, the ladder is rotated in such a way that it touched our window situated on the other side of the road. If our window is 12m. above the ground, then let us determine the breadth of that road in our locality.
- If the length of one diagonal of a rhombus having the side 10cm. length is 12cm., then let us write, by calculating the length of other diagonal.
- I have drawn a triangle PQR whose $\angle Q$ is right angle. If S is any point on QR , let us prove that, $PS^2 + QR^2 = PR^2 + QS^2$.
- Let us prove that, the sum of squares drawn on the sides of a rhombus is equal to the sum of squares drawn on two diagonals.
- ABC is an equilateral triangle. AD is perpendicular on the side BC , let us prove that, $AB^2 + BC^2 + CA^2 = 4AD^2$.
- I have drawn a right angled triangle ABC whose $\angle A$ is right angle. I took two points P and Q on the sides AB and AC respectively. By Joining $P, Q; B, Q$ and C, P let us prove that, $BQ^2 + PC^2 = BC^2 + PQ^2$
- If two diagonals of a quadrilateral $ABCD$ intersect each other perpendicularly, then let us prove that, $AB^2 + CD^2 = BC^2 + DA^2$
- I have drawn a triangle ABC whose height is AD . If $AB > AC$, let us prove that, $AB^2 - AC^2 = BD^2 - CD^2$
- In $\triangle ABC$ I have drawn two perpendiculars from two vertices B and C on AC and AB ($AC > AB$) which are intersected each other at the point P . Let us prove that, $AC^2 + BP^2 = AB^2 + CP^2$
- ABC is an isosceles triangle whose $\angle C$ is right angle. If D is any point on AB , then let us prove that, $AD^2 + DB^2 = 2CD^2$
- In the triangle ABC , $\angle A$ is right angle, if CD is a median, let us prove that, $BC^2 = CD^2 + 3AD^2$
- From a point O within a triangle ABC , I have drawn the perpendiculars OX, OY and OZ on BC, CA and AB respectively. Let us prove that, $AZ^2 + BX^2 + CY^2 = AY^2 + CX^2 + BZ^2$.

14. In $\triangle RST$, $\angle S$ is right angle. The mid-points of two sides RS and ST are X and Y respectively; let us prove that, $RY^2 + XT^2 = 5XY^2$

15. **V.S.A.**

(A) M. C. Q. :

- A person goes 24m. west from a place and then he goes 10m. north. The distance of the person from starting point is (a) 34 m. (b) 17 m. (c) 26 m. (d) 25 m.
- If ABC is an equilateral triangle and $AD \perp BC$, then $AD^2 =$ (a) $\frac{3}{2} DC^2$ (b) $2DC^2$ (c) $3DC^2$ (d) $4DC^2$
- In an isosceles triangle ABC , if $AC = BC$ and $AB^2 = 2AC^2$, then the measure of $\angle C$ is (a) 30° (b) 90° (c) 45° (d) 60°
- Two rods of 13m. length and 7m. length are situated perpendicularly on the ground and the distance between their foots is 8m. The distance between the two vertices is (a) 9 m. (b) 10 m. (c) 11 m. (d) 12 m.
- If the lengths of two diagonals of a rhombus are 24cm. and 10cm., the perimeter of the rhombus is (a) 13cm. (b) 26 cm. (c) 52 cm. (d) 25 cm.

(B) Let us write whether the following statements are true or false :

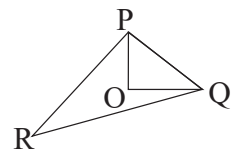
- If the ratio of the lengths of three sides of a triangle is 3:4:5, then the triangle will always be a right angled triangle.
- If in a circle of radius 10 cm. length, a chord subtends right angle at the centre, then the length of the chord will be 5 cm.

(C) Let us fill in blanks :

- In a right angled triangle, the area of a square drawn on the hypotenuse is equal to the _____ of the areas of the squares drawn on other two sides.
- In an isosceles right-angled triangle if the length of each of two equal sides is $4\sqrt{2}$ cm., then the length of the hypotenuse will be _____ cm.
- In a rectangular figure $ABCD$, the two diagonals AC and BD intersect each other at the point O , if $AB = 12$ cm., $AO = 6.5$ cm., then the length of BC is _____ cm.

16. **S.A.**

- In $\triangle ABC$, if $AB = (2a - 1)$ cm. $AC = 2\sqrt{2a}$ cm. and $BC = (2a+1)$ cm., then let us write the value of $\angle BAC$.
- In the adjoining figure, the point O is situated within the triangle PQR in such a way that, $\angle POQ = 90^\circ$, $OP = 6$ cm. and $OQ = 8$ cm. If $PR = 24$ cm. and $\angle QPR = 90^\circ$, then let us write the length of QR .
- The point O is situated within the rectangular figure $ABCD$ in such a way that $OB = 6$ cm., $OD = 8$ cm. and $OA = 5$ cm. Let us determine the length of OC .
- In the triangle ABC the perpendicular AD from the point A on the side BC meets the side BC at the point D . If $BD = 8$ cm., $DC = 2$ cm. and $AD = 4$ cm., then let us write the measure of $\angle BAC$.
- In a right angled triangle ABC , $\angle ABC = 90^\circ$, $AB = 3$ cm., $BC = 4$ cm. and the perpendicular BD on the side AC from the point B which meets the side AC at the point D . Let us determine the length of BD .



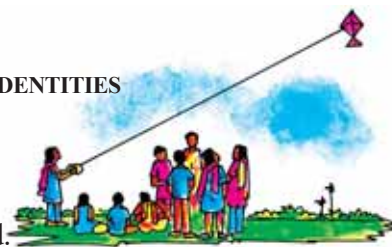
I have got a right angled triangle ABC in my copy by drawing the picture of Rina's flying kite.

Where, the point C is the position of Rina on the ground.

The point A is the position of Rina's kite.

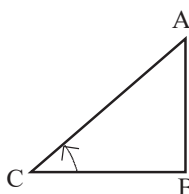
AB is the height of the kite's position from the ground.

and, $\angle BCA$ is an acute angle.



1 But what do we call AB and BC with respect to the acute angle $\angle BCA$?

In the right angled triangle ABC, the side AB is called the opposite side or perpendicular of $\angle BCA$ and the side BC is called the adjacent side or base of $\angle BCA$.



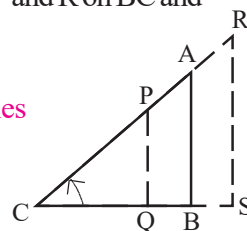
I have understood, in right angled triangle ABC, the opposite side of $\angle BAC$ is and the adjacent side of $\angle BAC$ is AB.

In the right angled triangle which I have drawn, Suva took a point P on the hypotenuse AC and a point R on the extended CA. He drew two perpendiculars from the points P and R on BC and on extended CB which intersect at the points Q and S respectively.

For this, I have got two more right angled triangles PQC and RSC.

2 We try to know the relations of the sides of the right angled triangles PQC, ABC and RSC and see what we will get.

I am observing that the right angled triangles PQC, ABC and RSC are similar [let me prove it myself]



$$(i) \frac{PQ}{CP} = \frac{AB}{CA} = \frac{RS}{CR} \quad (ii) \frac{CQ}{CP} = \frac{CB}{CA} = \frac{CS}{CR} \quad (iii) \frac{PQ}{CQ} = \frac{AB}{CB} = \frac{RS}{CS}$$

$$(iv) \frac{CP}{PQ} = \frac{CA}{AB} = \frac{CR}{RS} \quad (v) \frac{CP}{CQ} = \frac{CA}{CB} = \frac{CR}{CS} \quad (vi) \frac{CQ}{PQ} = \frac{CB}{AB} = \frac{CS}{RS}$$



I am observing that, in three right angled triangles, with respect to the acute angle $\angle BCA$

(i) The ratios $\frac{\text{Perpendicular}}{\text{hypotenuse}}$ are equal (ii) the ratios $\frac{\text{base}}{\text{hypotenuse}}$ are equal and

(iii) the ratios $\frac{\text{Perpendicular}}{\text{base}}$ are equal.

By drawing any right angled triangle and in the same way by drawing more than one similar right angled triangles with respect an acute angle, I am observing that the ratios (i), (ii) and (iii) are equal [Let me do it myself]

I have understood that, in a right angled triangle, the ratios of the sides with respect to an acute angle do not depend on the lengths of the sides of that triangle. The ratios totally depend on the measure of the acute angle.

3 But by what name the six types ratios which I have got by taking two sides of a right angled triangle, can be called separately — Let us see

By taking two sides out of three sides of a right angled triangle, the six types ratios which we find are called **trigonometric ratios**.



In the adjoining figure, the angle $\angle ABO$ of the right angled triangle ABO is 1 right angle.

$\therefore$ OA is hypotenuse and with respect to the acute angle $\angle BOA$, OB = base and AB = perpendicular; let $\angle AOB = \theta$

$$\text{sine of } \angle BOA = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{OA} = \sin \theta \text{ [writing in short]}$$

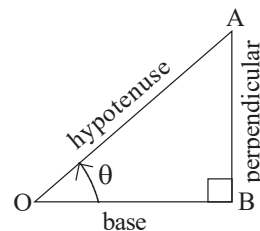
$$\text{cosine of } \angle BOA = \frac{\text{base}}{\text{hypotenuse}} = \frac{OB}{OA} = \cos \theta \text{ [writing in short]}$$

$$\text{tangent of } \angle BOA = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{OB} = \tan \theta \text{ [writing in short]}$$

$$\text{cosecant of } \angle BOA = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{OA}{AB} = \text{cosec } \theta \text{ [writing in short]}$$

$$\text{secant of } \angle BOA = \frac{\text{hypotenuse}}{\text{base}} = \frac{OA}{OB} = \sec \theta \text{ [writing in short]}$$

$$\text{cotangent of } \angle BOA = \frac{\text{base}}{\text{perpendicular}} = \frac{OB}{AB} = \cot \theta \text{ [writing in short]}$$



4 Let us see the above trigonometric ratios and try to write relations among them.

I am observing that cosec θ , sec θ and cot θ are reciprocals of sin θ , cos θ and tan θ respectively.

$$\text{i.e. } \text{cosec } \theta = \frac{1}{\sin \theta}, \sec \theta = \frac{1}{\cos \theta}, \cot \theta = \frac{1}{\tan \theta}$$

$$\text{Again, } \tan \theta = \frac{AB}{OB} = \frac{\frac{AB}{OA}}{\frac{OB}{OA}} = \frac{\sin \theta}{\cos \theta}$$

$$\therefore \text{ I have got, } \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\text{Again, } \cot \theta = \frac{OB}{AB} = \frac{\frac{OB}{OA}}{\frac{AB}{OA}} = \frac{\cos \theta}{\sin \theta} \quad \text{[Let me write it myself]}$$

$$\therefore \text{ I have got, } \cot \theta = \frac{\cos \theta}{\sin \theta}$$



I am observing that, in right angled triangle ABO, the trigonometric ratios of the acute angle $\angle BOA$ express the relation between the angle and sides of that triangle and the values of the trigonometric ratios with respect to acute angle are not changing with the lengths of the side of that right angled triangle.

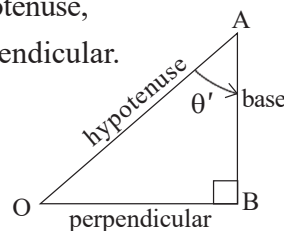
5 But in right angled triangle ABO, let us see what will be the trigonometric ratios of the acute angle $\angle OAB$.

In right angled triangle ABO, $\angle ABO = 1$ right angle. $\therefore$ OA = hypotenuse, with respect to acute angle $\angle OAB$, AB = base and OB = perpendicular.

Let, $\angle OAB = \theta'$

$$\therefore \sin \theta' = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{OB}{OA}$$

$$\cos \theta' = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{OA}$$

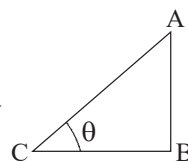


Let me write tan θ' , cosec θ' , sec θ' , cot θ'

I have understood, if Rina's kite is bound with a string having length AC and if the kite is flying in an angle of 60° with the horizontal line, then the kite will remain at a height of AB from the position of Rina.

$$\sin \theta = \frac{AB}{AC} \quad \therefore AB = AC \times \sin \theta$$

$\therefore$ Then Rina's kite will remain at a height of $AC \times \sin \theta$ from the position of Rina or from the base.



6 But is $\sin \theta$ is the product of sin and θ ?

" $\sin \theta$ " is the short form of sine of the angle θ . But **$\sin \theta$ is not a product of sin and θ** . In the same way, $\cos \theta$ is not a product of cos and θ and all other trigonometric ratios are of same type.

Aryabhata (500 A.D.) was the first to use the idea of sin. Later Aryabhata's work is translated into Arabic and Latin language. The word Sinus is used in Latin language. After then, the word 'Sine' is used instead of the word 'Sinus' in all cases of whole European Mathematics. English Astrophysicist professor Edmund Gunter (1581-1626) used first 'sin' in short form.



7 How shall I write the square of $\sin \theta$?

For convenience of writing $(\sin \theta)^2 = \sin^2 \theta$ is written, but $(\sin \theta)^2 \neq \sin \theta^2$

similarly $(\cos \theta)^2 = \cos^2 \theta$, $(\tan \theta)^3 = \tan^3 \theta$ etc.

$\operatorname{cosec} \theta = (\sin \theta)^{-1}$ can be written, but $\operatorname{cosec} \theta \neq \sin^{-1} \theta$ (it is called sin inverse θ)

$\sin \theta = \frac{\text{Perpendicular}}{\text{hypotenuse}}$, $\therefore$ can the value of $\sin \theta$ be greater than 1?



$\sin \theta = \frac{\text{Perpendicular}}{\text{hypotenuse}}$; since the perpendicular can not be greater than the hypotenuse,

hence, the value of $\sin \theta$ will not be greater than 1.

8 Can the value of $\cos \theta$ be greater than 1? [Let me do it myself]

Application : 1. α and β are such acute angles that $\sin \alpha = \sin \beta$. Let us prove that $\alpha = \beta$

Let, ABC and PQR be two right angled triangles. In $\triangle ABC$, $\angle ABC = 90^\circ$ and $\angle BCA = \alpha$; in $\triangle PQR$, $\angle PQR = 90^\circ$ and $\angle QRP = \beta$

In right angled triangle ABC, $\sin \alpha = \frac{AB}{AC}$;

In right angled triangle PQR, $\sin \beta = \frac{PQ}{PR}$

Since, $\sin \alpha = \sin \beta$, hence, $\frac{AB}{AC} = \frac{PQ}{PR}$ or, $\frac{AB}{PQ} = \frac{AC}{PR} = k$ (supposing) ($k > 0$)

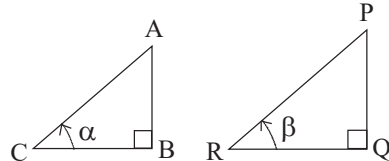
$\therefore AB = k.PQ$ and $AC = k.PR$

$\therefore BC = \sqrt{AC^2 - AB^2}$ and $QR = \sqrt{PR^2 - PQ^2}$

Hence, $\frac{BC}{QR} = \frac{\sqrt{AC^2 - AB^2}}{\sqrt{PR^2 - PQ^2}} = \frac{\sqrt{k^2 PR^2 - k^2 PQ^2}}{\sqrt{PR^2 - PQ^2}} = \frac{k \sqrt{PR^2 - PQ^2}}{\sqrt{PR^2 - PQ^2}} = k, \therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR}$

$\therefore \triangle ABC \sim \triangle PQR$, so, $\angle BCA = \angle QRP$, $\therefore \alpha = \beta$ [proved]

I have understood that, **if $\sin \alpha = \sin 45^\circ$, then $\alpha = 45^\circ$** [If $\sin \alpha = \sin \beta$, then we can not divide both sides by sin, since $\sin \alpha = \sin \times \alpha$. Hence, we cannot write it as $\alpha = \beta$ then, $\sin \alpha = \sin \beta$. It is wrong]



Application : 2. If $\sin(90^\circ - \theta) = \sin 2\theta$, then let us write the value of θ when θ is an acute angle.

$$\sin(90^\circ - \theta) = \sin 2\theta$$

$$\text{or, } 90^\circ - \theta = 2\theta$$

$$\text{or, } 3\theta = 90^\circ \quad \therefore \theta = 30^\circ$$



Application : 3. 5θ is an acute angle and if $\tan 5\theta = \tan(60^\circ + \theta)$, then let us determine the value of θ . [Let me do it myself]

we shall remember : Generally, (i) $\sin 2\theta \neq 2\sin \theta$

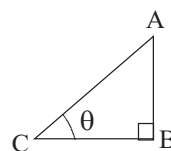
$$(ii) \frac{\sin \alpha}{\sin \beta} \neq \frac{\alpha}{\beta}$$

$$\text{and } (iii) \sin \alpha \pm \sin \beta \neq \sin(\alpha \pm \beta)$$

These are applicable also in the cases of cosine, tangent etc... of an angle.

Application : 4. In a right angled triangle, if $\sin \theta = \frac{12}{13}$ with respect to a positive acute angle θ , then let us determine the value of $\tan \theta$ and $\cos \theta$.

$$\sin \theta = \frac{12}{13} = \frac{\text{perpendicular}}{\text{hypotenuse}}$$



In right angled triangle ABC, $\angle ABC = 90^\circ$ and $\angle BCA = \theta$

Let, the perpendicular AB = 12k units and hypotenuse, AC = 13k units [where $k > 0$]

$$\therefore \text{Base BC} = \sqrt{AC^2 - AB^2} = \sqrt{(13k)^2 - (12k)^2} \text{ units} = \sqrt{169k^2 - 144k^2} \text{ units} = 5k \text{ units}$$

$$\therefore \tan \theta = \frac{\text{Perpendicular}}{\text{base}} = \frac{12k}{5k} = \frac{12}{5}$$

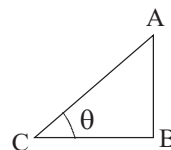
$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{5k}{13k} = \frac{5}{13}$$



Application : 5. If θ is a positive acute angle, and $\tan \theta = \frac{8}{15}$, then let us determine the values of $\sin \theta$ and $\cos \theta$ and prove that $\sin^2 \theta + \cos^2 \theta = 1$

ABC is a right angled triangle whose $\angle ABC = 90^\circ$ and $\angle ACB = \theta$

$$\therefore \tan \theta = \frac{AB}{BC} = \frac{8}{15}$$



Let, perpendicular AB = 8k units and base BC = 15k units [where $k > 0$]

$$\therefore \text{Hypotenuse AC} = \sqrt{AB^2 + BC^2} = \sqrt{(8k)^2 + (15k)^2} \text{ units} = \sqrt{64k^2 + 225k^2} \text{ units} = \sqrt{289k^2} \text{ units} = 17k \text{ units}$$

$$\therefore \sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{8k}{17k} = \frac{8}{17}$$

$$\cos \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{\boxed{}}{\boxed{}} = \frac{\boxed{}}{\boxed{}} \quad [\text{Let me write it myself}]$$

$$\therefore \sin^2 \theta + \cos^2 \theta = \left(\frac{8}{17}\right)^2 + \left(\frac{15}{17}\right)^2 = \frac{64}{289} + \frac{225}{289} = \boxed{} \quad [\text{Let me write it myself}]$$

$$\therefore \sin^2 \theta + \cos^2 \theta = 1 \quad [\text{proved}]$$



Application : 6. If $\tan \theta = \frac{4}{3}$, then let us show that, $\sin \theta + \cos \theta = \frac{7}{5}$ [Let me do it myself]

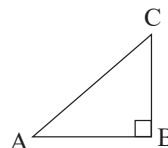
Application : 7. In $\triangle ABC$, $\angle B$ is right angle and the length of its hypotenuse is $\sqrt{13}$ units. If the sum of the lengths of other two sides is 5 units, then let us determine the value of $\sin C + \sin A$.

In right angled triangle ABC, AC is hypotenuse.

AB is perpendicular with respect to $\angle C$

and BC is perpendicular with respect to $\angle A$.

$$\therefore \sin C + \sin A = \frac{AB}{AC} + \frac{BC}{AC} = \frac{AB+BC}{AC} = \frac{5}{\sqrt{13}}$$



Alternative Proof : Let, AB = x unit, BC = (5-x) unit

According to Pythagoras theorem, in right angled triangle ABC,

$$x^2 + (5-x)^2 = (\sqrt{13})^2$$

$$\text{or, } x^2 + 25 + x^2 - 10x = 13$$

$$\text{or, } 2x^2 - 10x + 12 = 0$$

$$\text{or, } x^2 - 5x + 6 = 0$$

$$\text{or, } x^2 - 3x - 2x + 6 = 0$$

$$\text{or, } x(x-3) - 2(x-3) = 0$$

$$\text{or, } (x-3)(x-2) = 0$$

$$\text{Either, } x-3 = 0 \quad \therefore x = 3$$

$$\text{or, } x-2 = 0 \quad \therefore x = 2$$

If, AB = 3 units, then BC = (5-3) units = 2 units.

$$\text{Hence, } \sin C = \frac{AB}{AC} = \frac{3}{\sqrt{13}} \text{ and } \sin A = \frac{BC}{AC} = \frac{2}{\sqrt{13}}$$

$$\therefore \sin C + \sin A = \frac{3}{\sqrt{13}} + \frac{2}{\sqrt{13}} = \frac{5}{\sqrt{13}}$$

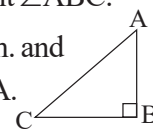


Again, if AB = 2 units, then BC = units and then $\sin C + \sin A =$ [in similar way, let me write it myself]

But if we want to determine the value of $\sin C - \sin A$, then whether we shall do it by first method or by alternative method, let me do it myself by thinking with reasons.

Let us work out 23.1

- I have drawn a right angled triangle ABC whose hypotenuse AB = 10 cm., base BC = 8 cm. and perpendicular AC = 6 cm. Let us determine the values of sine and tangent $\angle ABC$.
- Soma has drawn a right angled triangle ABC whose $\angle ABC = 90^\circ$, AB = 24 cm. and BC = 7 cm. By calculating, let us write the values of $\sin A$, $\cos A$, $\tan A$ and $\operatorname{cosec} A$.
- If in a right angled triangle ABC, $\angle C = 90^\circ$, BC = 21 units and AB = 29 units, then let us find the values of $\sin A$, $\cos A$, $\sin B$ and $\cos B$.
- If $\cos \theta = \frac{7}{25}$, then let us determine the values of all trigonometric ratios of the angle θ .



5. If $\cot\theta=2$, then let us determine the values of $\tan\theta$ and $\sec\theta$ and show that $1+\tan^2\theta=\sec^2\theta$.
6. If $\cos\theta=0.6$, then let us show that, $(5\sin\theta-3\tan\theta)=0$.
7. If $\cot A=\frac{4}{7.5}$, then let us determine the values of $\cos A$ and $\operatorname{cosec} A$ and show that $1+\cot^2 A=\operatorname{cosec}^2 A$
8. If $\sin C=\frac{2}{3}$, then let us write by calculating, the value of $\cos C \times \operatorname{cosec} C$.
9. Let us write with reason whether the following statements are true or false :
 - (i) The value of $\tan A$ is always greater than 1.
 - (ii) The value of $\cot A$ is always less than 1.
 - (iii) For an angle θ , it may be possible that, $\sin\theta=\frac{4}{3}$.
 - (iv) For an angle α , it may be possible that, $\sec\alpha=\frac{12}{5}$.
 - (v) For an angle β (Beta), it may be possible that, $\operatorname{cosec}\beta=\frac{5}{13}$.
 - (vi) For an angle θ , it may be possible that, $\cos\theta=\frac{3}{5}$.

Each of our friends has flown kites. All kites were flown at a long height. But Satish's kite was flown at the longest height.

Today, we have decided that after returning home we shall draw different right angled triangles (with pencil and compass), of which the measure of one angle is 30° or 45° or 60° and we shall try to determine the values of trigonometric ratios of that angle.



Asha has drawn a right angled triangle ABC in her copy whose $\angle ABC=90^\circ$ and the value of $\angle BCA$ is 45°

9 By calculating, let us write the values of the trigonometric ratios of $\angle BCA$ of the right angled triangle ABC drawn by Asha.

In right angled triangle ABC, $\angle ABC=90^\circ$ and $\angle BCA=45^\circ$

$$\therefore \angle CAB = 45^\circ$$

$\therefore \triangle ABC$ is an isosceles right angled triangle.

Hence, $BC=BA$

Let, $BA=BC=a$ units

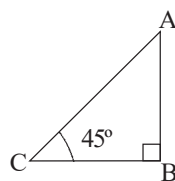
$$\therefore AC^2 = AB^2 + BC^2 = a^2 + a^2 = 2a^2$$

$$\therefore AC = a\sqrt{2} \text{ units}$$

$$\sin \angle BCA = \sin 45^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AB}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\cos \angle BCA = \cos 45^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{BC}{AC} = \frac{a}{a\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\tan \angle BCA = \tan 45^\circ = \frac{\text{perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{a}{a} = 1$$



I have understood, $\operatorname{cosec} 45^\circ = \frac{1}{\sin 45^\circ} = \sqrt{2}$, $\sec 45^\circ = \frac{1}{\cos 45^\circ} = \sqrt{2}$ and $\cot 45^\circ = \frac{1}{\tan 45^\circ} = \boxed{}$

Let me determine the values of trigonometric ratios of two angles 30° and 60° .

I have drawn an equilateral triangle ABC. Each angle of an equilateral triangle is [60°/45°]

$$\therefore \angle A = \angle B = \angle C = 60^\circ$$

From the vertex A, the perpendicular AD is drawn on the side BC which intersects the BC at the point D.

$\therefore$ In $\triangle ABD$ and $\triangle ACD$, $\angle ADB = \angle ADC = 90^\circ$, $\angle ABD = \angle ACD = 60^\circ$ and $AB = AC$.

$\therefore \triangle ABD \cong \triangle ACD$ (by A-A-S criterion of congruency)

$\therefore BD = DC$ and $\angle BAD = \angle CAD$

Again, ABD is a right angled triangle; $\angle ADB = 90^\circ$, $\angle DBA = 60^\circ$, $\therefore \angle BAD = 30^\circ$

Let, $AB = 2a$ unit, $\therefore BC = 2a$ unit

Hence, $BD = \frac{1}{2}BC = a$ unit

$\therefore$ From right angled triangle ABD, we get, $AD = \sqrt{AB^2 - BD^2} = \sqrt{(2a)^2 - a^2}$ units
 $= \sqrt{4a^2 - a^2}$ units $= a\sqrt{3}$ units

$$\sin 30^\circ = \frac{\text{Perpendicular}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{\text{Perpendicular}}{\text{base}} = \frac{BD}{AD} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

I have understood, $\operatorname{cosec} 30^\circ = \frac{1}{\sin 30^\circ} = \frac{1}{\frac{1}{2}} = 2$, $\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$ and $\cot 30^\circ = \frac{1}{\tan 30^\circ} = \sqrt{3}$ [Let me write it myself]

$$\sin 60^\circ = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{AD}{AB} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{\text{base}}{\text{hypotenuse}} = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$

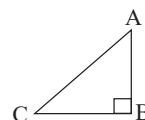
$$\tan 60^\circ = \frac{\text{Perpendicular}}{\text{base}} = \frac{AD}{BD} = \frac{a\sqrt{3}}{a} = \sqrt{3}$$

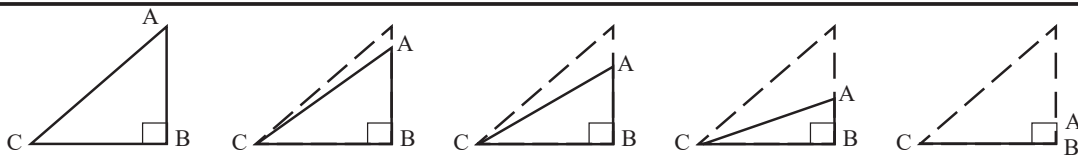
I have understood, $\operatorname{cosec} 60^\circ = \frac{1}{\sin 60^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$ [Let me write it myself]

$$\sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{\frac{1}{2}} = 2 \text{ and } \cot 60^\circ = \frac{1}{\tan 60^\circ} = \frac{1}{\sqrt{3}}$$

10 But in any right angled triangle ABC, if the value of its acute angle decreases continuously, then let us see by drawing what changes will be its trigonometric ratios.

ABC is right angled triangle whose $\angle B$ is right angle; the value of $\angle BCA$ decreases continuously until its value is nearly to 0° .





In the figure, we are observing that, as the value of $\angle BCA$ is decreasing, the point A is going or advancing towards the point B i.e. the length of the side AB is decreasing accordingly and if the point A comes nearly to the point B, the value of $\angle BCA$ becomes nearly to 0° , and the lengths of the sides AC and BC become nearly equal, i.e. if the value of $\angle BCA$ be nearly to 0° , the length of the side AB will be nearly to 0.

$\therefore$ The value of $\sin \angle BCA = \frac{AB}{AC}$ will be nearly to 0 when the value of $\angle BCA$ is nearly to 0° .

Again when the value of $\angle BCA$ is nearly to 0° , then the lengths of two sides AC and BC are nearly equal.

Hence the value of $\cos \angle BCA = \frac{BC}{AC}$ is nearly to 1 when the value of $\angle BCA$ is nearly to 0°

So, in this case the following trigonometric ratios are treated as definition.

$$\sin 0^\circ = 0 \text{ and } \cos 0^\circ = 1$$

$\therefore \tan 0^\circ = \frac{\sin 0^\circ}{\cos 0^\circ} = 0$ and $\cot 0^\circ = \frac{1}{\tan 0^\circ}$, which is undefined.

I have understood, $\operatorname{cosec} 0^\circ = \frac{1}{\sin 0^\circ}$, which is undefined and $\sec 0^\circ = \frac{1}{\cos 0^\circ} = 1$

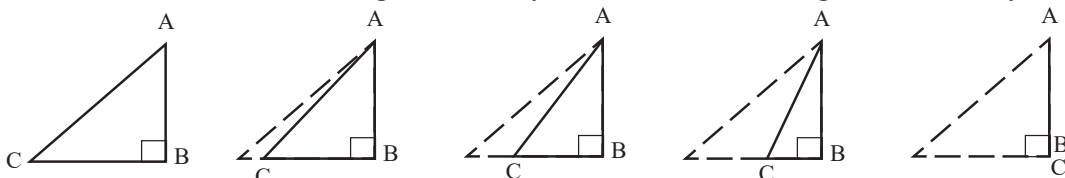


If in the above right angled triangle ABC, the value of the acute angle $\angle BCA$ is increasing continuously and reaches nearly to 90° , then let us see, what will be the values of trigonometric ratios.

ABC is a right angled triangle whose $\angle B$ is right angle.



The value of $\angle BCA$ is increasing continuously until the value of the angle reaches nearly to 90° .



In the figure, I am observing that, as the value of $\angle BCA$ is going nearly to 90° by increasing, the value of $\angle CAB$ is going nearly to 0° by decreasing and the point C is coming accordingly towards the point B and the length of the side CB is going nearly to 0 continuously.

Again, the length of the value of the side AC is becoming gradually nearly equal to the length of the side AB.

Hence, in that case the value of $\sin \angle ACB = \frac{AB}{AC}$ will be nearly to 1 when the value of $\angle BCA$ will be nearly to 90° .

Again, the value of $\cos \angle BCA = \frac{BC}{AC}$ will be nearly to 0, when the value of $\angle BCA$ will be nearly to 90° .

So, in this case, the following trigonometric ratios are treated as definition.

$$\sin 90^\circ = 1 \text{ and } \cos 90^\circ = 0$$

$\therefore \tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ}$, which is undefined and $\cot 90^\circ = \frac{\cos 90^\circ}{\sin 90^\circ} = 0$



By calculating, let us write the values of $\operatorname{cosec} 90^\circ$ and $\sec 90^\circ$ [Let me write it myself.]

I have got,

Values of θ Trigonometrical ratios of θ	0° or 0	30° or $\frac{\pi}{6}$	45° or $\frac{\pi}{4}$	60° or $\frac{\pi}{3}$	90° or $\frac{\pi}{2}$
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
$\operatorname{cosec} \theta$	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec \theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
$\cot \theta$	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

From the above table, we are observing that, as the value of θ is increasing from 0° to 90° , the value of $\sin \theta$ is increasing from 0 to 1 and the value of $\cos \theta$ is decreasing from 1 to 0.

We are also observing that, from 0° to 90° , the trigonometric ratios are zero, positive or undefined. But they are not negative.

Since, in the syllabus of class X, $0^\circ \leq \theta \leq 90^\circ$ is always positive acute angle, so trigonometric ratios can not be negative.

Application : 8. If Rina's kite is flown with a string of 150m. length and if the kite forms an angle 60° with the horizontal line, then let us calculate the height of the kite from Rina's position or from the ground.

In the following figure, ABC is a right angled triangle whose $\angle ABC = 90^\circ$, $AC = 150$ m length of the kite with string,

and $\angle BCA = 60^\circ$

AB = height of the kite from the ground.

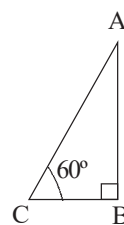
For determining AB in right angled triangle ABC, we shall take the trigonometric ratio having AB and AC , [$\because$ the length of AC is known]

In right angled triangle ABC, $\sin 60^\circ = \frac{AB}{AC}$

$$\text{or, } \frac{\sqrt{3}}{2} = \frac{AB}{150 \text{ m.}}$$

$$\text{or, } 2AB = 150\sqrt{3} \text{ m.}$$

$$\text{or, } AB = \frac{150\sqrt{3}}{2} \text{ m.} = 75\sqrt{3} \text{ m.}$$



$\therefore$ The kite is at a height of $75\sqrt{3}$ m from Rina's position or from the ground.

Application : 9. If the kite would be flown with a string of 120m. length and the kite is at an angle of 30° with the horizontal line, then let us calculate the height of the kite from Rina's position or from the ground. **[Let me do it myself]**

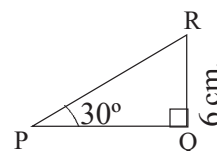
Application : 10. In the right angled triangle PQR, if $\angle Q$ is right angle and $\angle P = 30^\circ$; $RQ = 6$ cm., then let us write by calculating, the lengths of two sides PQ and PR.

In right angled triangle PQR, $\tan 30^\circ = \frac{RQ}{PQ}$

$$\text{or, } \frac{1}{\sqrt{3}} = \frac{6 \text{ cm.}}{PQ} \quad \therefore PQ = 6\sqrt{3} \text{ cm.}$$

In right angled triangle PQR, $\sin 30^\circ = \frac{RQ}{PR}$

$$\text{or, } \frac{1}{2} = \frac{6 \text{ cm.}}{PR} \quad \therefore PR = 12 \text{ cm.}$$



In other method, from Pythagoras theorem, we get $PR^2 = PQ^2 + RQ^2$
 $= (6\sqrt{3} \text{ cm.})^2 + (6 \text{ cm.})^2$
 $= 108 \text{ cm.}^2 + 36 \text{ cm.}^2$
 $= 144 \text{ cm.}^2$
 $\therefore PR = 12 \text{ cm.}$



Application : 11. In a right angled triangle ABC, $\angle B$ is right angle. If $AB = 5$ cm. and $AC = 10$ cm., then let us determine the values of $\angle BCA$ and $\angle CAB$.

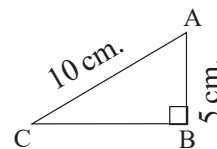
In right angled triangle ABC, $\angle B$ is right angle,

$AB = 5$ cm. and $AC = 10$ cm.

$\therefore$ In right angled triangle ABC, $\sin \angle BCA = \frac{AB}{AC} = \frac{5}{10} = \frac{1}{2} = \sin 30^\circ$

$$\therefore \angle BCA = 30^\circ$$

$$\therefore \angle CAB = 90^\circ - 30^\circ = \boxed{}$$



Application : 12. In a right angled triangle ABC, $\angle B$ is right angle. If $AB = 7$ cm. and $AC = 7\sqrt{2}$ cm., then let us write by calculating, the values of $\angle BCA$ and $\angle CAB$. [Let me do it myself]

Application : 13. Let us show that, $\sin^2 30^\circ + \cos^2 30^\circ = 1$

$$\sin^2 30^\circ + \cos^2 30^\circ = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1 \quad \text{[proved]}$$



Application : 14. Let us show that, $\tan^2 60^\circ + 1 = \sec^2 60^\circ$ [Let me do it myself]

Application : 15. If in an equilateral triangle ABC, AD is a median, then let us prove that, $\sin \angle BAD = \cos \angle DBA$.

In an equilateral triangle ABC, AD is a median.

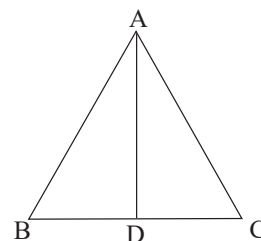
$\therefore AD \perp BC$ and $\angle BAD = \angle DAC$

ABD is a right angled triangle and $\angle DBA = 60^\circ$

[$\because$ ABC is an equilateral triangle] $\therefore \angle BAD = 30^\circ$

$$\therefore \sin \angle BAD = \sin 30^\circ = \frac{1}{2} \quad \text{and} \quad \cos \angle DBA = \cos 60^\circ = \frac{1}{2}$$

$$\therefore \sin \angle BAD = \cos \angle DBA \quad \text{[proved]}$$



Application : 16. Let us prove that, $\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ} = \cos 60^\circ$

$$\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ} = \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2} = \cos 60^\circ$$

Application : 17. Let us prove that, $\tan^2 60^\circ - 2\sin 60^\circ = 3 - \cot 30^\circ$ [Let me do it myself]

Application : 18. Let us determine the value of, $\frac{5\cos^2 \frac{\pi}{3} + 4\sec^2 \frac{\pi}{6} - \tan^2 \frac{\pi}{4}}{\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{6}}$



$$\begin{aligned} \frac{5\cos^2 \frac{\pi}{3} + 4\sec^2 \frac{\pi}{6} - \tan^2 \frac{\pi}{4}}{\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{6}} &= \frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ} \\ &= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{\frac{5}{4} + \frac{16}{3} - 1}{\frac{1}{4} + \frac{3}{4}} = \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{67}{12} = 5\frac{7}{12} \end{aligned}$$

Application : 19. $\sin(A+B)=1$ and $\cos(A-B)=1$, where $0^\circ \leq (A+B) \leq 90^\circ$ and $A > B$; let us determine the values of the angles A and B.

$$\sin(A+B)=1 = \sin 90^\circ$$

$$\therefore A+B = 90^\circ$$

$$\text{Again, } \cos(A-B)=1 = \cos 0^\circ$$

$$\therefore A-B = 0^\circ$$

$$A+B = 90^\circ$$

$$A-B = 0^\circ$$

$$\hline 2A = 90^\circ$$

$$\therefore A = 45^\circ \text{ and } B = 90^\circ - 45^\circ = 45^\circ$$



Application : 20. Let us determine, for what value of θ ($0^\circ \leq \theta \leq 90^\circ$), $\sin^2 \theta - 3\sin \theta + 2 = 0$ will be true.

$$\sin^2 \theta - 3\sin \theta + 2 = 0$$

$$\text{or, } \sin^2 \theta - 2\sin \theta - \sin \theta + 2 = 0$$

$$\text{or, } \sin \theta (\sin \theta - 2) - 1(\sin \theta - 2) = 0$$

$$\text{or, } (\sin \theta - 1)(\sin \theta - 2) = 0$$

$$\text{Either, } \sin \theta - 1 = 0 \quad \therefore \sin \theta = 1;$$

$$\text{or, } \sin \theta - 2 = 0 \quad \therefore \sin \theta = 2$$

Since, the value of $\sin \theta$ can not be greater than 1, hence $\sin \theta = 1 = \sin 90^\circ \quad \therefore \theta = 90^\circ$



Application : 21. Let us determine, for what value/values of θ ($0^\circ \leq \theta \leq 90^\circ$), $2\sin\theta\cos\theta = \cos\theta$ will be true.

$$\begin{aligned} 2\sin\theta\cos\theta &= \cos\theta \\ \text{or, } 2\sin\theta\cos\theta - \cos\theta &= 0 \\ \text{or, } \cos\theta(2\sin\theta - 1) &= 0 \\ \text{Either, } \cos\theta &= 0 \quad \text{or, } 2\sin\theta - 1 = 0 \\ \text{If } \cos\theta &= 0, \text{ then, } \cos\theta = 0 = \cos 90^\circ \therefore \theta = 90^\circ \\ \text{Again, if } 2\sin\theta - 1 &= 0 \text{ then, } \sin\theta = \frac{1}{2} = \sin 30^\circ \therefore \theta = 30^\circ \\ \therefore \theta &= 90^\circ \quad \text{or, } \theta = 30^\circ \text{ for these values } 2\sin\theta\cos\theta = \cos\theta. \end{aligned}$$



But if both sides of $2\sin\theta\cos\theta = \cos\theta$ are divided by $\cos\theta$, we get only $\theta = 30^\circ$, let us see, why other value could not be found.

$$\begin{aligned} \text{If both sides of } 2\sin\theta\cos\theta &= \cos\theta \text{ are divided by } \cos\theta, \text{ we get, } 2\sin\theta = 1 \\ \therefore \sin\theta &= \frac{1}{2} = \sin 30^\circ \therefore \theta = 30^\circ \end{aligned}$$

But here it is not given $\cos\theta \neq 0$, so, both sides can not be divided by $\cos\theta$. For this, one value of θ instead of two has been found.

Let us work out 23.2

- In the window of our house, there is a ladder at an angle of 60° with the ground. If the ladder is $2\sqrt{3}$ m. long, then let us calculate with figure, the height of our window from the ground.
- ABC is a right angled triangle with its $\angle B$ is 1 right angle. If $AB = 8\sqrt{3}$ cm. and $BC = 8$ cm., then let us write by calculating, the values of $\angle ACB$ and $\angle BAC$.
- In a right angled triangle ABC, $\angle B = 90^\circ$, $\angle A = 30^\circ$ and $AC = 20$ cm.. Let us determine the lengths of two sides BC and AB.
- In a right angled triangle PQR, $\angle Q = 90^\circ$, $\angle R = 45^\circ$; if $PR = 3\sqrt{2}$ m., then let us find out the lengths of two sides PQ and QR.
- Let us determine the values of :**
 - $\sin^2 45^\circ - \operatorname{cosec}^2 60^\circ + \sec^2 30^\circ$
 - $\sec^2 45^\circ - \cot^2 45^\circ - \sin^2 30^\circ - \sin^2 60^\circ$
 - $3\tan^2 45^\circ - \sin^2 60^\circ - \frac{1}{3}\cot^2 30^\circ - \frac{1}{8}\sec^2 45^\circ$
 - $\frac{4}{3}\cot^2 30^\circ + 3\sin^2 60^\circ - 2\operatorname{cosec}^2 60^\circ - \frac{3}{4}\tan^2 30^\circ$
 - $\frac{\frac{1}{3}\cos 30^\circ}{\frac{1}{2}\sin 45^\circ} + \frac{\tan 60^\circ}{\cos 30^\circ}$
 - $\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sin^2 30^\circ$
 - $\sec^2 60^\circ - \cot^2 30^\circ - \frac{2\tan 30^\circ \operatorname{cosec} 60^\circ}{1 + \tan^2 30^\circ}$
 - $\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} + \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$
 - $\frac{1 - \sin^2 30^\circ}{1 + \sin^2 30^\circ} \times \frac{\cos^2 60^\circ + \cos^2 30^\circ}{\operatorname{cosec}^2 90^\circ - \cot^2 90^\circ} \div (\sin 60^\circ \tan 30^\circ)$

6. Let us show that,

$$(i) \sin^2 45^\circ + \cos^2 45^\circ = 1 \quad (ii) \cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ \quad (iii) \frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \sqrt{3}$$

$$(iv) \sqrt{\frac{1+\cos 30^\circ}{1-\cos 30^\circ}} = \sec 60^\circ + \tan 60^\circ \quad (v) \frac{2\tan^2 30^\circ}{1-\tan^2 30^\circ} + \sec^2 45^\circ - \cot^2 45^\circ = \sec 60^\circ$$

$$(vi) \tan^2 \frac{\pi}{4} \sin \frac{\pi}{3} \tan \frac{\pi}{6} \tan^2 \frac{\pi}{3} = 1 \frac{1}{2} \quad (vii) \sin \frac{\pi}{3} \tan \frac{\pi}{6} + \sin \frac{\pi}{2} \cos \frac{\pi}{3} = 2 \sin^2 \frac{\pi}{4}$$

7. (i) If $x \sin 45^\circ \cos 45^\circ \tan 60^\circ = \tan^2 45^\circ - \cos 60^\circ$, then let us determine the value of x .

(ii) If $x \sin 60^\circ \cos^2 30^\circ = \frac{\tan^2 45^\circ \sec 60^\circ}{\operatorname{cosec} 60^\circ}$, then let us determine the value of x .

(iii) If $x^2 = \sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ$, then let us determine the value of x .

8. If $x \tan 30^\circ + y \cot 60^\circ = 0$ and $2x - y \tan 45^\circ = 1$, then let us write, by calculating the values of x and y .

9. If $A = B = 45^\circ$, then let us justify

$$(i) \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$(ii) \cos(A+B) = \cos A \cos B - \sin A \sin B$$

10. (i) In an equilateral triangle ABC , BD is a median. Let us prove that, $\tan \angle ABD = \cot \angle BAD$

(ii) In an isosceles triangle ABC , $AB = AC$ and $\angle BAC = 90^\circ$; the bisector of $\angle BAC$ intersects the side BC at the point D .

$$\text{Let us prove that, } \frac{\sec \angle ACD}{\sin \angle CAD} = \operatorname{cosec}^2 \angle CAD$$

11. Let us determine the value/values of θ ($0^\circ \leq \theta \leq 90^\circ$), for which $2\cos^2 \theta - 3\cos \theta + 1 = 0$ will be true.

Our friend Biskakh drew a right-angled triangle ABC on the board whose $\angle B$ is right angle.

11 Today, we shall try to find out the relations of trigonometric ratios of any acute angle of the right angled triangle drawn by Bishakh.

ABC is a right angled triangle whose $\angle B$ is right angle.

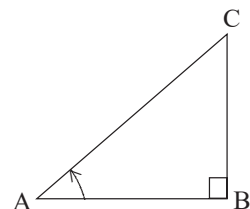
$\therefore$ By Pythagoras theorem, we get, $AB^2 + BC^2 = AC^2$ _____ (i)

Dividing both sides of the equation (i) with AC^2 , we get,

$$\frac{AB^2}{AC^2} + \frac{BC^2}{AC^2} = \frac{AC^2}{AC^2}$$

$$\text{or, } \left(\frac{AB}{AC}\right)^2 + \left(\frac{BC}{AC}\right)^2 = 1$$

$$\therefore \sin^2 A + \cos^2 A = 1 \text{ _____ (I)}$$



We are observing that, the relation (I) is applicable for all values of angle A , when $0^\circ \leq A \leq 90^\circ$

But what the relation (I) is called for?

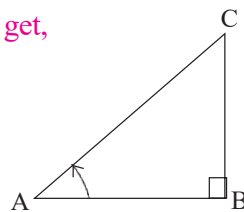
The relation (I) is an **Identify**.

Dividing both sides of the relation (i) by AB^2 and let us see what we get,

$$AB^2 + BC^2 = AC^2$$

$$\text{or, } \frac{AB^2}{AB^2} + \frac{BC^2}{AB^2} = \frac{AC^2}{AB^2}$$

$$\therefore 1 + \tan^2 A = \sec^2 A \text{ _____ (II)}$$



In the Identify (II) putting $A=0^\circ$ and $A=90^\circ$, let us see what we get

If $A=0^\circ$, the identify no. (II) is true, but if $A=90^\circ$, $\tan A$ is undefined.

$\therefore$ We got, For all values of A , where $0^\circ \leq A < 90^\circ$

$$1 + \tan^2 A = \sec^2 A \text{ _____ (II)}$$



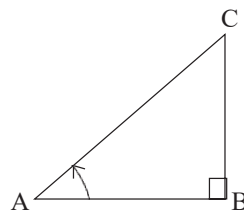
But dividing both sides of the relation (i) by BC^2 and let us see, what we get,

$$AB^2 + BC^2 = AC^2$$

$$\text{or, } \frac{AB^2}{BC^2} + \frac{BC^2}{BC^2} = \frac{AC^2}{BC^2}$$

$$\text{or, } \left(\frac{AB}{BC}\right)^2 + 1 = \left(\frac{AC}{BC}\right)^2$$

$$\therefore \cot^2 A + 1 = \operatorname{cosec}^2 A \text{ _____ (III)}$$



Putting $A=90^\circ$ and $A=0^\circ$ in identify no. (III), let us see, what we get,

If $A=90^\circ$, then identify no. (III) is true, but if $A=0^\circ$ $\cot A$ is undefined.

$\therefore$ We get, for all values of A , where $0^\circ < A \leq 90^\circ$

$$1 + \cot^2 A = \operatorname{cosec}^2 A \text{ _____ (III)}$$

12 Let us try to express each of the trigonometric ratios in terms of other trigonometric ratios with the help of identities (I), (II) and (III).

From (I) we get, $\sin^2 A + \cos^2 A = 1$

$$\therefore \sin A = \sqrt{1 - \cos^2 A} \text{ and } \cos A = \sqrt{1 - \sin^2 A}$$

Only positive value is taken, since $0^\circ \leq A \leq 90^\circ$

From (II) we get, $1 + \tan^2 A = \sec^2 A$

$$\therefore \sec A = \sqrt{1 + \tan^2 A} \text{ and } \tan A = \sqrt{\sec^2 A - 1}$$

Only positive value is taken, since $0^\circ \leq A < 90^\circ$

From (III), we get, $1 + \cot^2 A = \operatorname{cosec}^2 A$

$$\therefore \operatorname{cosec} A = \sqrt{1 + \cot^2 A} \text{ and } \cot A = \sqrt{\operatorname{cosec}^2 A - 1}$$

Only positive value is taken, since $0^\circ < A \leq 90^\circ$



Application : 22. If $0^\circ \leq \theta \leq 90^\circ$, let us write with reason whether $\sin \theta = 0.5$ and $\cos \theta = 0.6$ are possible or not.

$$\begin{aligned}\sin \theta &= 0.5 & \therefore \sin^2 \theta &= 0.25 \\ \cos \theta &= 0.6 & \therefore \cos^2 \theta &= 0.36 \\ \sin^2 \theta + \cos^2 \theta &= 0.25 + 0.36 = 0.61\end{aligned}$$

But for $0^\circ \leq \theta \leq 90^\circ$, $\sin^2 \theta + \cos^2 \theta = 1$

$\therefore \sin \theta = 0.5$ and $\cos \theta = 0.6$ are not possible.



Application : 23. For $0^\circ \leq \theta \leq 90^\circ$, let us write with reason whether $\sin \theta = \frac{\sqrt{3}}{2}$ and $\cos \theta = \frac{1}{3}$ are possible or not. [Let me do it myself]

Application : 24. If $0^\circ < \theta < 90^\circ$, then let us show that, $\sin \theta + \cos \theta > 1$

Let in right angled triangle ABC, $\angle B$ be right angle.

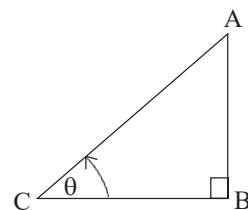
Let, $\angle BCA = \theta$ [where $0^\circ < \theta < 90^\circ$]

$$\therefore \sin \theta = \frac{AB}{AC} \text{ and } \cos \theta = \frac{BC}{AC}$$

$$\therefore \sin \theta + \cos \theta = \frac{AB}{AC} + \frac{BC}{AC} = \frac{AB+BC}{AC}$$

But in ΔABC , $AB+BC > AC$, so, $\frac{AB+BC}{AC} > 1$

$\therefore \sin \theta + \cos \theta > 1$



(i) $\tan \theta = \frac{\text{perpendicular}}{\text{base}}$ (a) If perpendicular $>$ base, then $\frac{\text{perpendicular}}{\text{base}} > 1$

(b) Again, if perpendicular $<$ base, then $\frac{\text{perpendicular}}{\text{base}} < 1$

(c) When perpendicular = base, $\frac{\text{perpendicular}}{\text{base}} = 1$

Therefore, the value of $\tan \theta$ can be greater than 1, less than 1 and equal to 1.

Is $\tan \theta = \sqrt{2}$ possible?

Since $\sqrt{2} > 1$, hence it may be possible that $\tan \theta = \sqrt{2}$.

(ii) $\operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{perpendicular}}$, since hypotenuse $>$ perpendicular, hence $\frac{\text{hypotenuse}}{\text{perpendicular}} > 1$

May $\operatorname{cosec} \theta = \sqrt{3}$ be possible?

Since, $\sqrt{3} > 1$, hence the value of $\operatorname{cosec} \theta$ may be $\sqrt{3}$.

Application : 25. I express $\cot \theta$ and $\sec \theta$ in term of $\sin \theta$,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\text{or, } \cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\text{Hence, } \cot \theta = \frac{\cos \theta}{\sin \theta} = \frac{\sqrt{1 - \sin^2 \theta}}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\sqrt{1 - \sin^2 \theta}}$$

Application : 26. I express $\cot \theta$ and $\operatorname{cosec} \theta$ in term of $\cos \theta$. [Let me do it myself]

Application : 27. If $\tan\theta = \frac{8}{15}$, then let us write by calculating, the value of $\sin\theta$.

$$\tan\theta = \frac{8}{15}$$

$$\text{We know, } \sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{8}{15}\right)^2 = 1 + \frac{64}{225} = \frac{289}{225}$$

$$\therefore \sec\theta = \frac{17}{15}$$

$$\therefore \cos\theta = \frac{15}{17}$$

$$\therefore \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{15}{17}\right)^2} = \sqrt{\frac{289 - 225}{289}} = \sqrt{\frac{64}{289}} = \frac{8}{17}$$

In another way, we get $\sin\theta = \frac{\tan\theta}{\sec\theta} = \frac{\frac{8}{15}}{\frac{17}{15}} = \frac{8}{17}$

Alternative proof : ABC is a right-angled triangle whose $\angle B$ is right angle. Let, $\angle ACB = \theta$

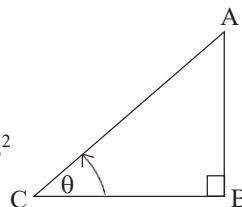
$$\therefore \tan\theta = \frac{\text{perpendicular}}{\text{base}} = \frac{8}{15}$$

Let, $AB = 8k$ units and $BC = 15k$ units (where $k > 0$)

$$\begin{aligned} \text{In } \triangle ABC, AC^2 &= AB^2 + BC^2 = (8k \text{ units})^2 + (15k \text{ units})^2 \\ &= 64k^2 \text{ units}^2 + 225k^2 \text{ units}^2 = 289k^2 \text{ units}^2 \end{aligned}$$

$$\therefore AC = 17k \text{ units}$$

$$\sin\theta = \frac{AB}{AC} = \frac{8k}{17k} \quad \therefore \sin\theta = \frac{8}{17}$$



Application : 28. If $\tan\theta = \frac{4}{3}$, then let us write by calculating, the value of $(\sin\theta + \cos\theta)$
[Let me do it myself]

Application : 29. If $\sin A = \frac{p}{q}$, then let us determine the value of each of $\tan A$, $\cot A$ and $\sec A$.

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}} = \frac{\frac{p}{q}}{\sqrt{1 - \left(\frac{p}{q}\right)^2}} = \frac{\frac{p}{q}}{\sqrt{\frac{q^2 - p^2}{q^2}}} = \frac{p}{q} \times \frac{q}{\sqrt{q^2 - p^2}} = \frac{p}{\sqrt{q^2 - p^2}}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\sqrt{1 - \sin^2 A}}{\sin A} = \frac{\sqrt{1 - \left(\frac{p}{q}\right)^2}}{\frac{p}{q}} = \frac{\sqrt{\frac{q^2 - p^2}{q^2}} \times \frac{q}{p}}{\frac{p}{q}} = \frac{\sqrt{q^2 - p^2}}{p}$$

$$\sec A = \frac{1}{\cos A} = \frac{1}{\sqrt{1 - \sin^2 A}} = \frac{1}{\sqrt{1 - \left(\frac{p}{q}\right)^2}} = \frac{1}{\sqrt{\frac{q^2 - p^2}{q^2}}} = \frac{q}{\sqrt{q^2 - p^2}}$$



Alternative method : PQR is a right-angled triangle, whose $\angle Q$ is right angle

Let, $\angle PRQ = A$

$$\sin A = \frac{\text{Perpendicular}}{\text{hypotenuse}} = \frac{p}{q}$$

Let, PQ = pk units, and PR = qk units, where $k > 0$,

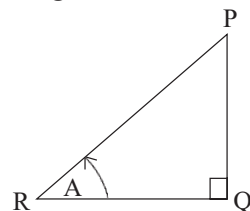
$$\text{In } \triangle PQR, QR^2 = PR^2 - PQ^2 = q^2k^2 - p^2k^2 = k^2(q^2 - p^2)$$

$$\therefore QR = k\sqrt{q^2 - p^2}$$

$$\tan A = \frac{PQ}{QR} = \frac{pk}{k\sqrt{q^2 - p^2}} = \frac{p}{\sqrt{q^2 - p^2}}$$

$$\sec A = \frac{PR}{QR} = \frac{qk}{k\sqrt{q^2 - p^2}} = \frac{q}{\sqrt{q^2 - p^2}}$$

$$\cot A = \frac{1}{\tan A} = \frac{1}{\frac{p}{\sqrt{q^2 - p^2}}} = \frac{\sqrt{q^2 - p^2}}{p}$$



Application : 30. If $\cot \theta = \frac{x}{y}$, then let us prove that $\frac{x \cos \theta - y \sin \theta}{x \cos \theta + y \sin \theta} = \frac{x^2 - y^2}{x^2 + y^2}$

$$\begin{aligned} \text{LHS} &= \frac{x \cos \theta - y \sin \theta}{x \cos \theta + y \sin \theta} \\ &= \frac{\frac{x \cos \theta}{\sin \theta} - \frac{y \sin \theta}{\sin \theta}}{\frac{x \cos \theta}{\sin \theta} + \frac{y \sin \theta}{\sin \theta}} \quad [\text{dividing numerator and denominator by } \sin \theta] \\ &= \frac{x \cot \theta - y}{x \cot \theta + y} = \frac{x \times \frac{x}{y} - y}{x \times \frac{x}{y} + y} = \frac{\frac{x^2 - y^2}{y}}{\frac{x^2 + y^2}{y}} = \frac{x^2 - y^2}{x^2 + y^2} = \text{RHS} \end{aligned}$$



Alternative method : (I) $\cot \theta = \frac{x}{y}$ or, $\frac{\cos \theta}{\sin \theta} = \frac{x}{y}$

$$\therefore \frac{\cos \theta}{x} = \frac{\sin \theta}{y} = k \quad (\text{Supposing}) \text{ where } k > 0$$

$$\therefore \cos \theta = xk \text{ and } \sin \theta = yk$$

$$\therefore \frac{x \cos \theta - y \sin \theta}{x \cos \theta + y \sin \theta} = \frac{x \times xk - y \times yk}{x \times xk + y \times yk} = \frac{k(x^2 - y^2)}{k(x^2 + y^2)} = \frac{x^2 - y^2}{x^2 + y^2}$$

This is to remember : from $\cot \theta = \frac{x}{y}$, we get $\frac{\cos \theta}{\sin \theta} = \frac{x}{y}$

But $\cos \theta = x$ and $\sin \theta = y$ are wrongly written. we can take $\cos \theta = xk$ and $\sin \theta = yk$, where $k > 0$.

Alternative method : (II) $\cot \theta = \frac{x}{y}$ or, $\frac{\cos \theta}{\sin \theta} = \frac{x}{y}$

$$\text{or, } \frac{x \cos \theta}{y \sin \theta} = \frac{x^2}{y^2} \quad (\text{multiplying both sides by } \frac{x}{y})$$

$$\text{or, } \frac{x \cos \theta + y \sin \theta}{x \cos \theta - y \sin \theta} = \frac{x^2 + y^2}{x^2 - y^2} \quad (\text{by componendo and dividendo})$$

$$\therefore \frac{x \cos \theta - y \sin \theta}{x \cos \theta + y \sin \theta} = \frac{x^2 - y^2}{x^2 + y^2}$$



Application : 31. If $\sin\theta + \cos\theta = \frac{7}{5}$ and $\sin\theta \cos\theta = \frac{12}{25}$, then let us determine the values of $\sin\theta$ and $\cos\theta$.

$$\begin{aligned}\sin\theta + \cos\theta &= \frac{7}{5} \\ \therefore \cos\theta &= \frac{7}{5} - \sin\theta \quad \text{--- (i)}\end{aligned}$$

Putting $\cos\theta = \frac{7}{5} - \sin\theta$ in the equation $\sin\theta \cos\theta = \frac{12}{25}$, we get

$$\begin{aligned}\sin\theta \times \left(\frac{7}{5} - \sin\theta\right) &= \frac{12}{25} \\ \text{or, } \frac{7\sin\theta}{5} - \sin^2\theta &= \frac{12}{25} \\ \text{or, } 7\sin\theta - 5\sin^2\theta &= \frac{12}{5} \\ \text{or, } 35\sin\theta - 25\sin^2\theta &= 12 \\ \text{or, } 25\sin^2\theta - 35\sin\theta + 12 &= 0 \\ \text{or, } 25\sin^2\theta - 20\sin\theta - 15\sin\theta + 12 &= 0 \\ \text{or, } 5\sin\theta(5\sin\theta - 4) - 3(5\sin\theta - 4) &= 0 \\ \text{or, } (5\sin\theta - 4)(5\sin\theta - 3) &= 0 \quad [\text{factorising}]\end{aligned}$$

$$\text{Either, } 5\sin\theta - 4 = 0 \quad \therefore \sin\theta = \frac{4}{5}$$

$$\text{or, } 5\sin\theta - 3 = 0 \quad \therefore \sin\theta = \frac{3}{5}$$

$$\therefore \text{ If } \sin\theta = \frac{4}{5}, \cos\theta = \frac{7}{5} - \frac{4}{5} = \frac{3}{5}$$

$$\text{Again, if } \sin\theta = \frac{3}{5}, \cos\theta = \frac{7}{5} - \frac{3}{5} = \frac{4}{5}$$

$$\therefore \left. \begin{array}{l} \sin\theta = \frac{4}{5} \\ \cos\theta = \frac{3}{5} \end{array} \right\} \text{ or, } \left. \begin{array}{l} \sin\theta = \frac{3}{5} \\ \cos\theta = \frac{4}{5} \end{array} \right\}$$

Alternative proof :

$$(\sin\theta - \cos\theta)^2 = (\sin\theta + \cos\theta)^2 - 4\sin\theta \cos\theta$$

$$= \left(\frac{7}{5}\right)^2 - 4 \times \frac{12}{25} = \frac{49}{25} - \frac{48}{25} = \frac{1}{25}$$

$$\therefore \sin\theta - \cos\theta = \pm \frac{1}{5}$$

$$\sin\theta + \cos\theta = \frac{7}{5}$$

$$\sin\theta - \cos\theta = \frac{1}{5}$$

$$\text{Adding we get, } 2\sin\theta = \frac{8}{5}$$

$$\therefore \sin\theta = \frac{4}{5} \quad \text{and} \quad \cos\theta = \frac{7}{5} - \frac{4}{5} = \frac{3}{5}$$

$$\therefore \sin\theta = \frac{4}{5}, \cos\theta = \frac{3}{5} \quad \text{or,} \quad \sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}$$

Again,

$$\sin\theta + \cos\theta = \frac{7}{5}$$

$$\sin\theta - \cos\theta = -\frac{1}{5}$$

$$\text{Adding we get, } 2\sin\theta = \frac{6}{5}$$

$$\therefore \sin\theta = \frac{3}{5} \quad \text{and} \quad \cos\theta = \frac{7}{5} - \frac{3}{5} = \frac{4}{5}$$



Application : 32. Let us express $1+2\sin\theta\cos\theta$ into a perfect square.

$$1+2\sin\theta\cos\theta = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= (\sin\theta + \cos\theta)^2$$



Application : 33. If $\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = 2\frac{51}{79}$, then let us determine the value of $\sin\theta$.

$$\frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta} = 2\frac{51}{79} = \frac{209}{79}$$

$$\text{or, } \frac{\sec\theta + \tan\theta + \sec\theta - \tan\theta}{\sec\theta + \tan\theta - \sec\theta + \tan\theta} = \frac{209 + 79}{209 - 79} \quad (\text{By the process of componendo and dividendo})$$

$$\text{or, } \frac{2\sec\theta}{2\tan\theta} = \frac{288}{130}$$

$$\text{or, } \frac{\tan\theta}{\sec\theta} = \frac{130}{288} = \frac{65}{144}$$

$$\text{or, } \frac{\sin\theta}{\cos\theta} \times \cos\theta = \frac{65}{144}$$

$$\therefore \sin\theta = \frac{65}{144}$$

Application : 34. If $\frac{5\cot\theta + \operatorname{cosec}\theta}{5\cot\theta - \operatorname{cosec}\theta} = \frac{7}{3}$, then let us determine the value of $\cos\theta$. [Let me do it myself]

Application : 35. If $x = a\cos\theta$ and $y = b\sin\theta$, then let us eliminate θ from these two relations and see what we shall get.

$$x = a\cos\theta \quad \therefore \cos\theta = \frac{x}{a}$$

$$\text{Again, } y = b\sin\theta \quad \therefore \sin\theta = \frac{y}{b}$$

$$\text{Since, } \sin^2\theta + \cos^2\theta = 1$$

$$\therefore \left(\frac{y}{b}\right)^2 + \left(\frac{x}{a}\right)^2 = 1$$

$$\text{Hence, } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



Application : 36. From the two relations $2x = 3\sin\theta$ and $5y = 3\cos\theta$, by eliminating θ , let us write the relation between x and y . [Let me do it myself]

Application : 37. Let us eliminate θ from the two relations $x\cos\theta = 3$ and $4\tan\theta = y$ and write the relation between x and y .

$$x\cos\theta = 3 \quad \text{or, } \cos\theta = \frac{3}{x} \quad \therefore \sec\theta = \frac{x}{3}$$

$$4\tan\theta = y \quad \text{or, } \tan\theta = \frac{y}{4} \quad \therefore \tan\theta = \frac{y}{4}$$

$$\text{Since, } \sec^2\theta - \tan^2\theta = 1$$

$$\text{Hence, } \left(\frac{x}{3}\right)^2 - \left(\frac{y}{4}\right)^2 = 1$$

$$\therefore \frac{x^2}{9} - \frac{y^2}{16} = 1$$



Application : 38. Let us eliminate θ from the two relations $x = a\sec\theta$, $y = b\tan\theta$. [Let me do it myself]

Application : 39. If $x = a\cos\theta + b\sin\theta$, $y = b\cos\theta - a\sin\theta$, then let us eliminate θ from these two relations.

$$\begin{aligned}x^2 + y^2 &= (a\cos\theta + b\sin\theta)^2 + (b\cos\theta - a\sin\theta)^2 \\&= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + b^2\cos^2\theta + a^2\sin^2\theta - 2ab\sin\theta\cos\theta \\&= a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) \\&= a^2 + b^2 \quad [\because \sin^2\theta + \cos^2\theta = 1] \\ \therefore x^2 + y^2 &= a^2 + b^2\end{aligned}$$

Application : 40. If $\sin\theta + \operatorname{cosec}\theta = 2$, then let us determine the value of $(\sin^{10}\theta + \operatorname{cosec}^{10}\theta)$.

$$\begin{aligned}\sin\theta + \operatorname{cosec}\theta &= 2 \\ \text{or, } \sin\theta + \frac{1}{\sin\theta} &= 2 \\ \text{or, } \frac{\sin^2\theta + 1}{\sin\theta} &= 2 \\ \text{or, } \sin^2\theta + 1 &= 2\sin\theta \\ \text{or, } \sin^2\theta - 2\sin\theta + 1 &= 0 \\ \text{or, } (\sin\theta - 1)^2 &= 0 \\ \text{or, } \sin\theta - 1 &= 0 \\ \therefore \sin\theta &= 1\end{aligned}$$

Hence, $\operatorname{cosec}\theta = 1$

$$\begin{aligned}\sin^{10}\theta + \operatorname{cosec}^{10}\theta &= (1)^{10} + (1)^{10} \\&= 1 + 1 \\&= 2\end{aligned}$$



Application : 41. If $\cos\theta + \sec\theta = 2$, then let us determine the value of $(\cos^{11}\theta + \sec^{11}\theta)$
[Let me do it myself]

Application : 42. If $0^\circ < \alpha \leq 90^\circ$, then let us determine the least value of $(4\operatorname{cosec}^2\alpha + 9\sin^2\alpha)$.

$$\begin{aligned}4\operatorname{cosec}^2\alpha + 9\sin^2\alpha &= (2\operatorname{cosec}\alpha)^2 + (3\sin\alpha)^2 \\&= (2\operatorname{cosec}\alpha - 3\sin\alpha)^2 + 2 \times 2\operatorname{cosec}\alpha \times 3\sin\alpha \\&= (2\operatorname{cosec}\alpha - 3\sin\alpha)^2 + 12 \times \frac{1}{\sin\alpha} \times \sin\alpha \\&= (2\operatorname{cosec}\alpha - 3\sin\alpha)^2 + 12\end{aligned}$$



The least value of any perfect square of the expression is 0 (Zero). So, the least value of $(2\operatorname{cosec}\alpha - 3\sin\alpha)$ is zero.

$\therefore$ The least value of $(4\operatorname{cosec}^2\alpha + 9\sin^2\alpha)$ is 12.

$$\begin{aligned}4\operatorname{cosec}^2\alpha + 9\sin^2\alpha &= (2\operatorname{cosec}\alpha + 3\sin\alpha)^2 - 2 \times 2\operatorname{cosec}\alpha \times 3\sin\alpha \\&= (2\operatorname{cosec}\alpha + 3\sin\alpha)^2 - 12\end{aligned}$$

The least value of $(2\operatorname{cosec}\alpha - 3\sin\alpha)^2$ is 0 (Zero). Hence the least value of $(4\operatorname{cosec}^2\alpha + 9\sin^2\alpha)$ is -12.

Then what will be the least value?

The sum of two squares of the expression can not be negative. Hence the least value of $(4\operatorname{cosec}^2\alpha + 9\sin^2\alpha)$ is 12.

Let us work out

23.3

1. (i) If $\sin\theta = \frac{4}{5}$, then let us write the value of $\frac{\operatorname{cosec}\theta}{1+\cot\theta}$ by determining it.
 (ii) If $\tan\theta = \frac{3}{4}$, then let us show that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = \frac{1}{2}$
 (iii) If $\tan\theta = 1$, then let us determine the value of $\frac{8\sin\theta+5\cos\theta}{\sin^3\theta-2\cos^3\theta+7\cos\theta}$.
2. (i) Let us express $\operatorname{cosec}\theta$ and $\tan\theta$ in term of $\sin\theta$.
 (ii) Let us write $\operatorname{cosec}\theta$ and $\tan\theta$ in term of $\cos\theta$.
3. (i) If $\sec\theta + \tan\theta = 2$, then let us determine the value of $(\sec\theta - \tan\theta)$.
 (ii) If $\operatorname{cosec}\theta - \cot\theta = \sqrt{2} - 1$, then let us write by calculating, the value of $(\operatorname{cosec}\theta + \cot\theta)$.
 (iii) If $\sin\theta + \cos\theta = 1$, then let us determine the value of $\sin\theta \times \cos\theta$.
 (iv) If $\tan\theta + \cot\theta = 2$, then let us determine the value of $(\tan\theta - \cot\theta)$.
 (v) If $\sin\theta - \cos\theta = \frac{7}{13}$, then let us determine the value of $\sin\theta + \cos\theta$.
 (vi) If $\sin\theta \cos\theta = \frac{1}{2}$, then let us write by calculating, the value of $(\sin\theta + \cos\theta)$.
 (vii) If $\sec\theta - \tan\theta = \frac{1}{\sqrt{3}}$, then let us determine the values of both $\sec\theta$ and $\tan\theta$.
 (viii) If $\operatorname{cosec}\theta + \cot\theta = \sqrt{3}$, then let us determine the values of both $\operatorname{cosec}\theta$ and $\cot\theta$.
 (ix) If $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = 7$, then let us write by calculating, the value of $\tan\theta$.
 (x) If $\frac{\operatorname{cosec}\theta + \sin\theta}{\operatorname{cosec}\theta - \sin\theta} = \frac{5}{3}$, then let us write by calculating, the value of $\sin\theta$.
 (xi) If $\sec\theta + \cos\theta = \frac{5}{3}$, then let us write by calculating, the value of $(\sec\theta - \cos\theta)$.
 (xii) Let us determine the value of $\tan\theta$ from the relation $5\sin^2\theta + 4\cos^2\theta = \frac{9}{2}$.
 (xiii) If $\tan^2\theta + \cot^2\theta = \frac{10}{3}$, then let us determine the values of $\tan\theta + \cot\theta$ and $\tan\theta - \cot\theta$ and from these let us write the value of $\tan\theta$.
 (xiv) If $\sec^2\theta + \tan^2\theta = \frac{13}{12}$, then let us write by calculating, the value of $(\sec^4\theta - \tan^4\theta)$.
4. (i) In ΔPQR , $\angle Q$ is right angle. If $PR = \sqrt{5}$ units and $PQ - RQ = 1$ unit, then let us determine the value of $\cos P - \cos R$.
 (ii) In ΔXYZ , $\angle Y$ is right angle. If $XY = 2\sqrt{3}$ units and $XZ - YZ = 2$ units then let us determine the value of $(\sec X - \tan X)$.
5. Let us eliminate ' θ ' from the relations :
 (i) $x = 2\sin\theta$, $y = 3\cos\theta$ (ii) $5x = 3\sec\theta$, $y = 3\tan\theta$
6. (i) If $\sin\alpha = \frac{5}{13}$, then let us show that, $\tan\alpha + \sec\alpha = 1.5$.
 (ii) If $\tan A = \frac{n}{m}$, then let us determine the values of both $\sin A$ and $\sec A$.

- (iii) If $\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$, then let us show that, $x\sin\theta = y\cos\theta$.
- (iv) If $\sin\alpha = \frac{a^2-b^2}{a^2+b^2}$, then let us show that, $\cot\alpha = \frac{2ab}{a^2-b^2}$.
- (v) If $\frac{\sin\theta}{x} = \frac{\cos\theta}{y}$, then let us show that, $\sin\theta - \cos\theta = \frac{x-y}{\sqrt{x^2+y^2}}$.
- (vi) If $(1+4x^2)\cos A = 4x$, then let us show that, $\operatorname{cosec} A + \cot A = \frac{1+2x}{1-2x}$.
7. If $x = a\sin\theta$ and $y = b\tan\theta$, then let us prove that, $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$.
8. If $\sin\theta + \sin^2\theta = 1$, then let us prove that, $\cos^2\theta + \cos^4\theta = 1$.
9. **V.S.A.**
- (A) **M.C.Q. :**
- (i) If $3x = \operatorname{cosec}\alpha$ and $\frac{3}{x} = \cot\alpha$, then the value of $3(x^2 - \frac{1}{x^2})$ is (a) $\frac{1}{27}$ (b) $\frac{1}{81}$ (c) $\frac{1}{3}$ (d) $\frac{1}{9}$
- (ii) If $2x = \sec A$ and $\frac{2}{x} = \tan A$, then the value of $2(x^2 - \frac{1}{x^2})$ is (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{8}$ (d) $\frac{1}{16}$
- (iii) If $\tan\alpha + \cot\alpha = 2$, then the value of $(\tan^{13}\alpha + \cot^{13}\alpha)$ is (a) 1 (b) 0 (c) 2 (d) none of these
- (iv) If $\sin\theta - \cos\theta = 0$ ($0^\circ \leq \theta \leq 90^\circ$) and $\sec\theta + \operatorname{cosec}\theta = x$, then the value of x is (a) 1 (b) 2 (c) $\sqrt{2}$ (d) $2\sqrt{2}$
- (v) If $2\cos 3\theta = 1$, then the value of θ is (a) 10° (b) 15° (c) 20° (d) 30°
- (B) **Let us write whether the following statements are true or false :**
- (i) If $0^\circ \leq \alpha < 90^\circ$, then the least value of $(\sec^2\alpha + \cos^2\alpha)$ is 2.
- (ii) The value of $(\cos 0^\circ \times \cos 1^\circ \times \cos 2^\circ \times \cos 3^\circ \times \dots \times \cos 90^\circ)$ is 1.
- (C) **Let us fill in the blanks :**
- (i) The value of $\left(\frac{4}{\sec^2\theta} + \frac{1}{1+\cot^2\theta} + 3\sin^2\theta\right)$ is _____
- (ii) If $\sin(\theta - 30^\circ) = \frac{1}{2}$, then the value of $\cos\theta$ is _____
- (iii) If $\cos^2\theta - \sin^2\theta = \frac{1}{2}$, then the value of $\cos^4\theta - \sin^4\theta$ is _____
10. **S.A.**
- (i) If $r\cos\theta = 2\sqrt{3}$, $r\sin\theta = 2$ and $0^\circ < \theta < 90^\circ$, then let us determine the values of both r and θ .
- (ii) If $\sin A + \sin B = 2$ where $0^\circ \leq A \leq 90^\circ$ and $0^\circ \leq B \leq 90^\circ$, then let us find out the value of $(\cos A + \cos B)$.
- (iii) If $0^\circ < \theta < 90^\circ$, then let us calculate the least value of $(9\tan^2\theta + 4\cot^2\theta)$.
- (iv) Let us calculate the value of $(\sin^6\alpha + \cos^6\alpha + 3\sin^2\alpha \cos^2\alpha)$.
- (v) If $\operatorname{cosec}^2\theta = 2\cot\theta$ and $0^\circ < \theta < 90^\circ$, then let us determine the value of θ .

24

TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLE

Every day we play in a big field beside which is Ramus' garden house. Often our ball falls into their garden. Today we have put a ladder leaning against their wall. We have decided that if the ball falls into their garden we can collect the ball from their garden with the help of ladder.



1 But putting the ladder in that way we see that the ladder, base and wall of the garden are in the shape of a right-angled triangle.

In the adjoining figure, let AB is the wall, which is perpendicular to the base BC and length of the ladder is AC. AC makes an angle θ with BC

$\therefore$ The right-angled triangle ABC is formed a right-angled triangle of which $\angle ABC = 90^\circ$



2 But what value of $\angle CAB$ will be?

If $\angle BCA = \theta$, then $\angle CAB = 90^\circ - \theta$

We understand that, $\angle BCA$ and $\angle CAB$ are [complementary / supplementary] to each other.

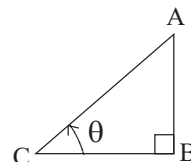
By drawing the figure, let us calculate, the trigonometric ratios of two complementary angles $\angle BCA$ and $\angle CAB$.

ABC is right-angled triangle of which $\angle B = 90^\circ$

Let, $\angle BCA = \theta \therefore \angle CAB = 90^\circ - \theta$

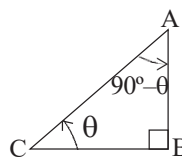
$$\therefore \sin \theta = \frac{AB}{AC}, \cos \theta = \frac{BC}{AC}, \tan \theta = \frac{\text{opposite}}{\text{adjacent}}, \csc \theta = \frac{AC}{AB}, \sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

and $\cot \theta = \frac{BC}{AB}$



3 Let us write the trigonometric ratios of an acute angle $(90^\circ - \theta)$ of right-angled triangle ABC.

$$\begin{aligned} \sin(90^\circ - \theta) &= \sin \angle CAB = \frac{\text{Perpendicular}}{\text{hypotenuse}} = \frac{BC}{AC} \\ \cos(90^\circ - \theta) &= \cos \angle CAB = \frac{\text{base}}{\text{hypotenuse}} = \frac{AB}{AC} \\ \tan(90^\circ - \theta) &= \tan \angle CAB = \frac{\text{Perpendicular}}{\text{base}} = \frac{BC}{AB} \\ \csc(90^\circ - \theta) &= \csc \angle CAB = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{AC}{BC} \\ \sec(90^\circ - \theta) &= \sec \angle CAB = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{AC}{AB} \\ \cot(90^\circ - \theta) &= \cot \angle CAB = \frac{\text{adjacent}}{\text{opposite}} = \frac{AB}{BC} \end{aligned}$$



4 Let us write what we get by comparing with trigonometric ratios of $\angle BCA$ and $\angle CAB$ of a right-angled triangle ABC.

We see that $\sin(90^\circ - \theta) = \cos \theta$

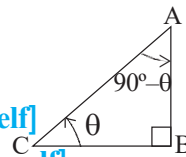
$$\tan(90^\circ - \theta) = \cot \theta$$

$$\sec(90^\circ - \theta) = \operatorname{cosec} \theta$$

$$\cos(90^\circ - \theta) = \sin \theta$$

$$\cot(90^\circ - \theta) = \boxed{} \text{ [Let me do it myself]}$$

$$\operatorname{cosec}(90^\circ - \theta) = \boxed{} \text{ [Let me do it myself]}$$



We understand that of the ladder makes an angle 60° with the base, i.e. if $\theta = 60^\circ$

$$\sin \theta = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\begin{aligned} \text{Again, } \cos(90^\circ - \theta) &= \cos(90^\circ - 60^\circ) \\ &= \cos 30^\circ = \frac{\sqrt{3}}{2} \end{aligned}$$

$$\therefore \text{ We get, } \sin \theta = \cos(90^\circ - \theta)$$

If $\theta = 45^\circ$, $\tan \theta = \cot(90^\circ - \theta)$ [Let me do it myself]

5 But if $\theta = 0^\circ$ and $\theta = 90^\circ$ let us see what we shall get.

$$\theta = 0^\circ \quad \tan 0^\circ = 0 = \cot(90^\circ - 0^\circ)$$

$$\text{and } \sec 0^\circ = 1 = \operatorname{cosec}(90^\circ - 0^\circ)$$

if $\theta = 90^\circ$, $\tan 90^\circ$, $\sec 90^\circ$ are undefined.

and if $\theta = 0^\circ$, $\cot 0^\circ$, $\operatorname{cosec} 0^\circ$ are undefined.



Application : 1. Let us find the value of $\frac{\cos 53^\circ}{\sin 37^\circ}$

We know that $\cos(90^\circ - \theta) = \sin \theta$

$$\cos 53^\circ = \cos(90^\circ - 37^\circ) = \sin 37^\circ$$

$$\therefore \frac{\cos 53^\circ}{\sin 37^\circ} = \frac{\sin 37^\circ}{\sin 37^\circ} = 1$$

Application : 2. Let us find the value of $\frac{\sec 49^\circ}{\operatorname{cosec} 41^\circ}$ [Let me do it myself]

Application : 3. Let us show that $\cos 55^\circ \cos 35^\circ - \sin 55^\circ \sin 35^\circ = 0$

$$\cos 55^\circ \cos 35^\circ - \sin 55^\circ \sin 35^\circ$$

$$= \cos(90^\circ - 35^\circ) \cos 35^\circ - \sin(90^\circ - 35^\circ) \sin 35^\circ$$

$$= \sin 35^\circ \cos 35^\circ - \cos 35^\circ \sin 35^\circ = 0 \text{ (proved)}$$

Application : 4. Let us show that $\sin 43^\circ \cos 47^\circ + \cos 43^\circ \sin 47^\circ = 1$ [Let me do it myself]

Application : 5. Let us prove that $\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ = \sqrt{3}$

$$\tan 7^\circ \tan 23^\circ \tan 60^\circ \tan 67^\circ \tan 83^\circ$$

$$= (\tan 7^\circ \tan 83^\circ) \tan 60^\circ (\tan 23^\circ \tan 67^\circ)$$

$$= \{\tan 7^\circ \times \tan(90^\circ - 7^\circ)\} \tan 60^\circ \{\tan 23^\circ \times \tan(90^\circ - 23^\circ)\}$$

$$= (\tan 7^\circ \cot 7^\circ) \tan 60^\circ (\tan 23^\circ \cot 23^\circ) = 1 \times \sqrt{3} \times 1 \quad (\because \tan \theta \times \cot \theta = 1)$$

$$= \sqrt{3} \text{ (proved)}$$

Application : 6. Let us show that $\sin^2 21^\circ + \sin^2 69^\circ = 1$

$$\sin 69^\circ = \sin(90^\circ - 21^\circ) = \cos 21^\circ$$

$$\therefore \sin^2 21^\circ + \sin^2 69^\circ = \sin^2 21^\circ + \cos^2 21^\circ = 1 \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$



Application : 7. Let us show that $\tan 15^\circ + \tan 75^\circ = \frac{\sec^2 15^\circ}{\sqrt{\sec^2 15^\circ - 1}}$



$$\begin{aligned} \tan 15^\circ + \tan 75^\circ &= \tan 15^\circ + \tan (90^\circ - 15^\circ) \\ &= \tan 15^\circ + \cot 15^\circ = \tan 15^\circ + \frac{1}{\tan 15^\circ} \quad [\because \sec^2 \theta = 1 + \tan^2 \theta] \\ &= \frac{\tan^2 15^\circ + 1}{\tan 15^\circ} = \frac{\sec^2 15^\circ}{\sqrt{\sec^2 15^\circ - 1}} \quad \text{and } \tan \theta = \sqrt{\sec^2 \theta - 1} \quad \text{(proved)} \end{aligned}$$

Application : 8. If two angles A and B are complementary to each other, let us show that, $(\sin A + \sin B)^2 = 1 + 2\sin A \sin B$

Angles A and B are complementary angles to each other.

$$\text{so, } A + B = 90^\circ$$

$$\therefore B = 90^\circ - A$$

$$\sin B = \sin(90^\circ - A) = \cos A$$

$$\begin{aligned} (\sin A + \sin B)^2 &= (\sin A + \cos A)^2 \\ &= \sin^2 A + \cos^2 A + 2\sin A \cos A \\ &= 1 + 2\sin A \cos A \\ &= 1 + 2\sin A \cos(90^\circ - B) \\ &= 1 + 2\sin A \sin B \quad \text{(proved)} \end{aligned}$$

Application : 9. If $\sec \theta = \operatorname{cosec} \phi$ and $0^\circ < \theta < 90^\circ$, $0^\circ < \phi < 90^\circ$ let us find the value of $\sin(\theta + \phi)$

$$\sec \theta = \operatorname{cosec} \phi \quad \text{or, } \sec \theta = \sec(90^\circ - \phi)$$

$$\text{or, } \theta = 90^\circ - \phi \quad \therefore \theta + \phi = 90^\circ \quad \therefore \sin(\theta + \phi) = \sin 90^\circ = 1$$

Application : 10. If $\tan 2A = \cot(A - 18^\circ)$ where $2A$ is a positive acute angle, let us find the value of A .

$$\tan 2A = \cot(A - 18^\circ)$$

$$\text{or, } \cot(90^\circ - 2A) = \cot(A - 18^\circ) \quad [\because \cot(90^\circ - \theta) = \tan \theta]$$

$$\text{or, } 90^\circ - 2A = A - 18^\circ$$

$$\text{or, } -2A - A = -90^\circ - 18^\circ \text{ or, } -3A = -108^\circ \quad \therefore A = 36^\circ$$



Application : 11. If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$ where $4A$ is a positive acute angle, let us find the value of A .

$$\sec 4A = \operatorname{cosec}(A - 20^\circ)$$

$$\text{or, } \operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ) \quad [\because \operatorname{cosec}(90^\circ - \theta) = \sec \theta]$$

$$\text{or, } 90^\circ - 4A = A - 20^\circ$$

$$\text{or, } -4A - A = -20^\circ - 90^\circ$$

$$\text{or, } -5A = -110^\circ \quad \therefore A = 22^\circ$$

Application : 12. If $\cos 43^\circ = \frac{x}{\sqrt{x^2 + y^2}}$ let us write by calculating the value of $\tan 47^\circ$

$$\cos 43^\circ = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\sin^2 43^\circ = 1 - \cos^2 43^\circ = 1 - \frac{x^2}{x^2 + y^2} = \frac{x^2 + y^2 - x^2}{x^2 + y^2} = \frac{y^2}{x^2 + y^2}$$

$\therefore \sin 43^\circ = \frac{y}{\sqrt{x^2 + y^2}}$ [As 43° is a positive acute angle, $\therefore$ The value of $\sin 43^\circ$ can not be negative]



$$\therefore \tan 47^\circ = \tan(90^\circ - 43^\circ) = \cot 43^\circ = \frac{\cos 43^\circ}{\sin 43^\circ} = \frac{\frac{x}{\sqrt{x^2 + y^2}}}{\frac{y}{\sqrt{x^2 + y^2}}} = \frac{x}{\sqrt{x^2 + y^2}} \times \frac{\sqrt{x^2 + y^2}}{y} = \frac{x}{y}$$

Alternative method :

ABC is a right-angled triangle of which $\angle B = 90^\circ$

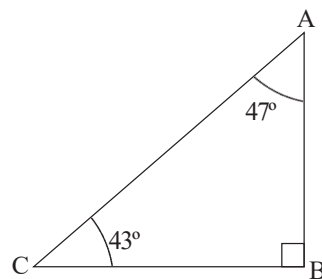
Let $\angle BCA = 43^\circ$; so, $\angle CAB = 90^\circ - 43^\circ = 47^\circ$

$$\cos 43^\circ = \frac{x}{\sqrt{x^2+y^2}} = \frac{BC}{AC}$$

let $BC = kx$ and $AC = k\sqrt{x^2+y^2}$ unit where $k > 0$.

$$\begin{aligned} \text{In right-angled } \triangle ABC, AB^2 &= AC^2 - BC^2 = (k\sqrt{x^2+y^2} \text{ unit})^2 - (kx \text{ unit})^2 \\ &= (k^2x^2 + k^2y^2 - k^2x^2) \text{ sq unit} = k^2y^2 \text{ sq unit} \\ \therefore AB &= ky \text{ unit} \end{aligned}$$

$$\therefore \tan 47^\circ = \frac{BC}{AB} = \frac{kx}{ky} = \frac{x}{y}$$



Application : 13. If $\tan 50^\circ = \frac{p}{q}$, let us find the value of $\cos 40^\circ$ [Let me do it myself]



Let us work out

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- Find the value : (i) $\frac{\sin 38^\circ}{\cos 52^\circ}$ (ii) $\frac{\operatorname{cosec} 79^\circ}{\sec 11^\circ}$ (iii) $\frac{\tan 27^\circ}{\cot 63^\circ}$
- Let us show that : (i) $\sin 66^\circ - \cos 24^\circ = 0$ (ii) $\cos^2 57^\circ + \cos^2 33^\circ = 1$
(iii) $\cos^2 75^\circ - \sin^2 15^\circ = 0$ (iv) $\operatorname{cosec}^2 48^\circ - \tan^2 42^\circ = 1$
(v) $\sec 70^\circ \sin 20^\circ + \cos 20^\circ \operatorname{cosec} 70^\circ = 2$
- If two angles α and β are complementary angles, let us show that
(i) $\sin^2 \alpha + \sin^2 \beta = 1$ (ii) $\cot \beta + \cos \beta = \frac{\cos \beta}{\cos \alpha} (1 + \sin \beta)$ (iii) $\frac{\sec \alpha}{\cos \alpha} - \cot^2 \beta = 1$
- If $\sin 17^\circ = \frac{x}{y}$, let us show that $\sec 17^\circ - \sin 73^\circ = \frac{x^2}{y\sqrt{y^2-x^2}}$
- Let us show that $\sec^2 12^\circ - \frac{1}{\tan^2 78^\circ} = 1$
- $\angle A + \angle B = 90^\circ$, let us show that $1 + \frac{\tan A}{\tan B} = \sec^2 A$
- Let us show that $\operatorname{cosec}^2 22^\circ \cot^2 68^\circ = \sin^2 22^\circ + \sin^2 68^\circ + \cot^2 68^\circ$
- If $\angle P + \angle Q = 90^\circ$, let us show that, $\sqrt{\frac{\sin P}{\cos Q}} - \sin P \cos Q = \cos P$
- let us prove that $\cot 12^\circ \cot 38^\circ \cot 52^\circ \cot 78^\circ \cot 60^\circ = \frac{1}{\sqrt{3}}$
- AOB is a diameter of a circle with centre O and C is any point on the circle, joining A.C; B,C; and O,C let us show that
(i) $\tan \angle ABC = \cot \angle ACO$ (ii) $\sin^2 \angle BCO + \sin^2 \angle ACO = 1$
(iii) $\operatorname{cosec}^2 \angle CAB - 1 = \tan^2 \angle ABC$
- ABCD is a rectangular figure, joining A, C let us prove that.
(i) $\tan \angle ACD = \cot \angle ACB$ (ii) $\tan^2 \angle CAD + 1 = \frac{1}{\sin^2 \angle BAC}$

12. **Very short answer type questions (V.S.A)**

(A) (M.C.Q.) :

- (i) The value of $(\sin 43^\circ \cos 47^\circ + \cos 43^\circ \sin 47^\circ)$ is
 (a) 0 (b) 1 (c) $\sin 4^\circ$ (d) $\cos 4^\circ$
- (ii) The value of $\left(\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ}\right)$ is
 (a) 0 (b) 1 (c) 2 (d) none of this
- (iii) The value of $\{\cos(40^\circ + \theta) - \sin(50^\circ - \theta)\}$ is
 (a) $2\cos \theta$ (b) $7\sin \theta$ (c) 0 (d) 1
- (iv) ABC is a triangle. $\sin\left(\frac{B+C}{2}\right) =$
 (a) $\sin \frac{A}{2}$ (b) $\cos \frac{A}{2}$ (c) $\sin A$ (d) $\cos A$
- (v) If $A+B=90^\circ$ and $\tan A = \frac{3}{4}$, value of $\cot B$ is
 (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

(B) Let us write whether the following statements are true or false :

- (i) The value of $\cos 54^\circ$ and $\sin 36^\circ$ are equal.
 (ii) The simplified value of $(\sin 12^\circ - \cos 78^\circ)$ is 1.

(C) Let us fill up the blanks :

- (i) The value of $(\tan 15^\circ \times \tan 45^\circ \times \tan 60^\circ \times \tan 75^\circ)$ is _____.
 (ii) The value of $(\sin 12^\circ \times \cos 18^\circ \times \sec 78^\circ \times \operatorname{cosec} 72^\circ)$ is _____.
 (iii) If A and B are complementary to each other, $\sin A =$ _____.

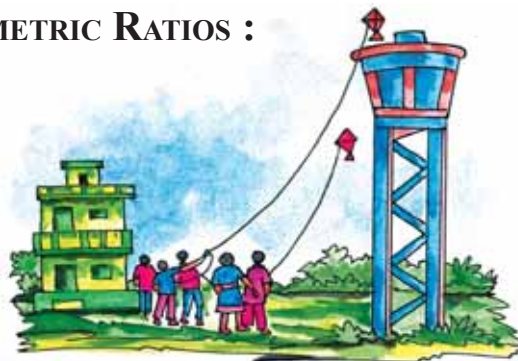
 13. **Short answer type questions :**

- (i) If $\sin 10\theta = \cos 8\theta$ and 10θ is a positive acute angle, let us find the value of $\tan 9\theta$.
 (ii) If $\tan 4\theta \times \tan 6\theta = 1$ and 6θ is a positive acute angle, let us find the value of θ .
 (iii) Let us find the value of $\frac{2\sin^2 63^\circ + 1 + 2\sin^2 27^\circ}{3\cos^2 17^\circ - 2 + 3\cos^2 73^\circ}$
 (iv) Let us find the value of $(\tan 1^\circ \times \tan 2^\circ \times \tan 3^\circ \dots \dots \dots \tan 89^\circ)$
 (v) If $\sec 5A = \operatorname{cosec} (A + 36^\circ)$ and $5A$ is a positive acute angle, let us find the value of A.

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APPLICATION OF TRIGONOMETRIC RATIOS : HEIGHTS & DISTANCES

Every afternoon, we play in a big field, which is situated Ramus' garden house. Some distance away a tank for that area is placed on a high pillar. Some times we fly kite in this field. Often the kite of Satish rises up at the highest-level from base and it rises above the water tank.



1 But let us see, by drawing, how shall we get the height of the water tank from the base.

We suppose that CE is the height of water tank and the water tank stands perpendicularly on the base line.

AB is my height and I stand perpendicularly on the base. I stand at a distance of BE from the water tank, the line segments AD and BE are parallel to each other. I see the vertex of water tank along the line AC.

2 What is this line AC called ?

This line AC is called **Line of Sight**.

We understand that the line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.

We see that my line of sight AC makes an angle θ with the horizontal line AD.

3 What is this angle θ called?

This angle θ is called **Angle of Elevation**.

We understand that when an observer standing on base, the point being viewed is above the horizontal level, the angle formed by the line of sight of an observer with horizontal line, is called the angle of elevation. In the angle of elevation head of an observer rises upwards.

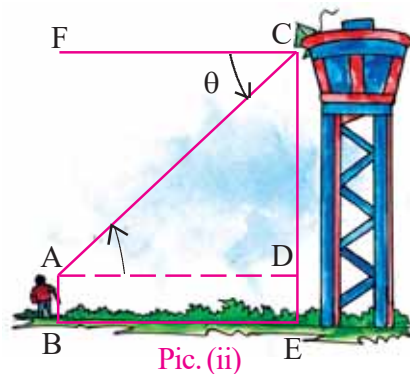
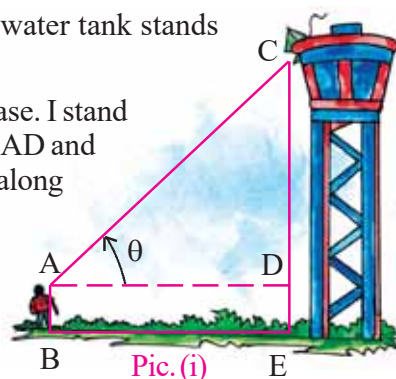
4 But if I would be on the tank at the tank at the point C and if I look down from this point at head of my friend standing on base AB, let us see by drawing a picture, what type of angle we shall get.

Let my friend stands on base AB I am looking the point A from the point C at the top of tower.

In this case my line of sight CA makes an angle θ with the line CF which is parallel to horizontal line AD.
[$\because \angle DAC = \theta$, $CF \parallel AD$, So, $\angle DAC = \angle FCA$]

5 What is this type of angle θ called ?

This angle is called **Angle of Depression**.



We understand that when the observer is looking down to an object, then the angle so formed by his line of sight with the horizontal line is called the angle of depression. In the angle of depression the head of an observer is lowered to look at the point being viewed.

6 Let us see how shall we get height of tank from picture no (i)

Height of water tank is $CE = CD + DE$

$$= CD + AB$$

$\angle D = 90^\circ$ of a right-angled triangle ADC

Angle of elevation $\angle DAC = \theta$

$\therefore$ For finding, with the help of trigonometric ratios it is needed to know the value of AC or AD .

7 If $\theta = 30^\circ$, i.e. the angle of elevation of top of tank from the point A is 30° and $AD = 120$ meter. i.e. if my distance from the water tank is 120 meter. Let us write by calculating what will be the height of water tank.

$\angle D = 90^\circ$ of right-angled triangle ACD ; The value of CD is to be determined. Hence we take such a trigonometric ratio, where there are CD and AD .

In right-angled triangle ACD , $\tan \theta = \frac{CD}{AD}$ or, $\tan 30^\circ = \frac{CD}{120}$ metre

$$\text{or, } \frac{CD}{120} \text{ metre} = \frac{1}{\sqrt{3}}$$

$$\therefore CD = \frac{120 \text{ m.}}{\sqrt{3}} = \frac{40 \times 3 \text{ m.}}{\sqrt{3}} = 40\sqrt{3} \text{ metre}$$

$\therefore$ Height of tank is $= [40\sqrt{3} \text{ metre} + \text{my height (AB)}]$

Let us write what we have done for determining the value of any height.

- (i) We have found the distance of which we shall find height from my position.
- (ii) We find the angle of elevation of vertex of which we shall find height.
- (iii) Now we shall find the height with the help of trigonometric ratios.



8 But if $\theta = 30^\circ$, and $AC = 150$ metre, let us find the height.

$\angle D = 90^\circ$ of a right angled triangle ACD . The Value of CD is to be determined.

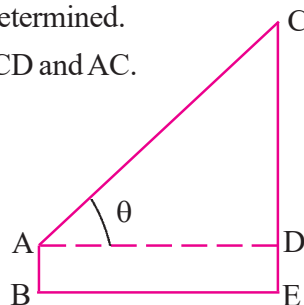
To find CD . We take such a trigonometric ratio where there are CD and AC .

In right angled $\triangle ACD$, $\sin 30^\circ = \frac{CD}{AC}$

$$\text{or, } \frac{1}{2} = \frac{CD}{150 \text{ m.}}$$

$$\text{or, } CD = \frac{150 \text{ m.}}{2} = 75 \text{ metre}$$

$\therefore$ Height of water tank is $= [75 \text{ metre} + AB]$



It should be remembered : Unless otherwise stated, in case of such problems the information regarding the angle of elevation and depression with respect to a person is given, the height of the person should be neglected while drawing the necessary diagram for solving the problem. In these problems, the observer is to be treated as a point. Let us consider that the tree, pillar, Light post etc stand perpendicularly on the base.

Application 1. A coconut tree stands on the bank of Rita's pond. Proceeding along the bank of the pond to a point which is at a distance of 12 metre with respect to this point, the angle of elevation of vertex of tree is 60° , Let us write by calculating the height of coconut tree on the bank of Rita's pond [$\sqrt{3} = 1.732$ approx]

Let AB represents the height of coconut tree which is perpendicular to the base. Moving along the bank of the pond to a point C from the point B, the angle of elevation of the point A with respect to C is 60° .

Joining A and C the right-angled triangle ABC is obtained where $\angle B = 90^\circ$ and $\angle ACB = 60^\circ$ with respect to the angle $\angle ACB$, the base $BC = 12$ metres. In right-angled triangle ABC, we get with the help of trigonometric ratios,

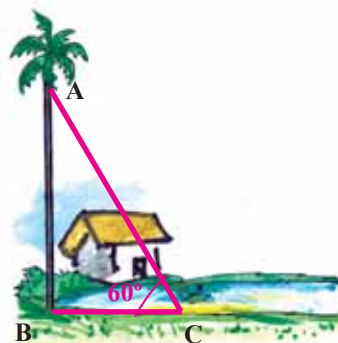
$$\tan \angle ACB = \tan 60^\circ = \frac{\text{Perpendicular}}{\text{base}} = \frac{AB}{BC} = \frac{AB}{12 \text{ m.}}$$

$$\text{or, } \tan 60^\circ = \frac{AB}{12 \text{ m.}}$$

$$\text{or, } \sqrt{3} = \frac{AB}{12 \text{ m.}}$$

$$\begin{aligned} \text{or, } AB &= 12\sqrt{3} \text{ metre} = 12 \times 1.732 \text{ metre (approx)} \\ &= 20.784 \text{ metre (approx)} \end{aligned}$$

$\therefore$ Height of coconut tree is = 20.784 metre (approx).



Application : 2. If the angle of elevation at the top of the coconut tree is 60° with respect to a point which is 20 metre away in the horizontal plane from the foot of the coconut tree, let us write by calculating height of coconut tree. [Let me do it myself]



Application : 3. Yesterday a light-post is bent at a point due to storm and the top of the light post meets the ground at a point 4 metre from the foot of the light post making an angle of 45° with the horizontal line. Let us write by calculating the length of the light-post. [$\sqrt{2} = 1.414$ approx]

Let the length of light post be AB, it is bent at the point O and top of the light post meets the ground at the point C.

$$\therefore AO = OC = x \text{ metre (let)}$$

$$\therefore AB = AO + OB = CO + OB$$

$\therefore$ A right-angled triangle OBC is formed of which $\angle B = 90^\circ$

$BC = 4$ metre, $OC = x$ metre.

$$\text{In right-angled triangle } \triangle OBC, \cos 45^\circ = \frac{BC}{OC} = \frac{4}{x}$$

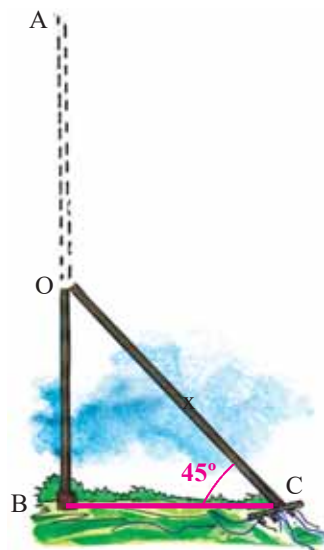
$$\text{or, } \frac{1}{\sqrt{2}} = \frac{4}{x} \therefore x = 4\sqrt{2}$$

$$\therefore OC = 4\sqrt{2} \text{ metre} = 4 \times 1.414 \text{ metre} = 5.656 \text{ metre (approx)}$$

$$\triangle OBC, \text{ in } \angle OCB = 45^\circ \therefore \angle BOC = 90^\circ - 45^\circ = 45^\circ$$

$$\therefore BO = CO = 4 \text{ metre}$$

$$\begin{aligned} \therefore \text{The length of light post} &= (4 + 5.656) \text{ metre (approx)} \\ &= 9.656 \text{ metre (approx)} \end{aligned}$$



Application : 4. If the angle of elevation of the sun is 60° the shadow of a palm tree is 12 metre in length. Let us find the height of the palm tree.

In the adjoining picture, AB is the length of height of palm tree and BC denotes the length of shadow when $\angle ACB = 60^\circ$

In right-angled triangle $\triangle ABC$, $\tan 60^\circ = \frac{AB}{BC}$

$$\text{or, } \sqrt{3} = \frac{AB}{12 \text{ m.}}$$

$$\text{or, } AB = 12\sqrt{3} \text{ metre}$$

$\therefore$ Height of palm tree is $12\sqrt{3}$ metre.

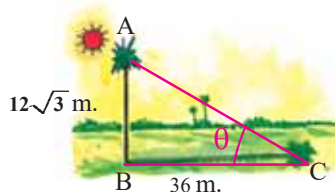
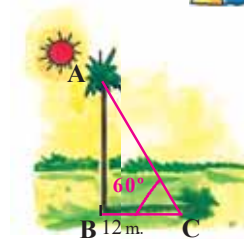
But if length of shadow of palm tree (of which length is $12\sqrt{3}$ metre) is 36 metre, let us write by calculating what will be the angle of elevation of the sun.

Let the length of shadow of palm tree $BC = 36$ and length of palm tree $AB = 12\sqrt{3}$ metre and angle of elevation of sun be θ . In right-angled triangle ABC.

$$\tan \theta = \frac{AB}{BC} = \frac{12\sqrt{3}}{36} = \frac{12\sqrt{3}}{12 \times 3} = \frac{1}{\sqrt{3}}$$

$$\therefore \tan \theta = \frac{1}{\sqrt{3}} = \tan 30^\circ \quad \therefore \theta = 30^\circ$$

$\therefore$ Then the angle of elevation of sun is 30° .



Application : 5. Let us find what will be the angle of elevation of the sun when length of shadow of a stick of 20 metre will be $12\sqrt{3}$ metre. [Let me do it myself]

Application : 6. Samiran observes a light post just on the opposite bank of the canal of Haskhali pole from the roof of three storied building of his house. If the angle of depression of the foot of the post at the eye of Samiran is 30° and height of building is 10 metre, Let us calculate the width of the canal. [$\sqrt{3} = 1.732$ (approx)]

Let three storied building be AB and post be CD stands on opposite side of canal of width BC. Samiran observed foot of the post CD at the point C from the point A at an angle of depression 30° .

$\therefore \angle EAC = 30^\circ$ [Let $AE \parallel BC$]

$\therefore$ We get a right-angled triangle ABC of which $\angle B = 90^\circ$, $AB = 10$ metre.

and $\angle ACB = \angle EAC$ [$\because AE \parallel BC$]

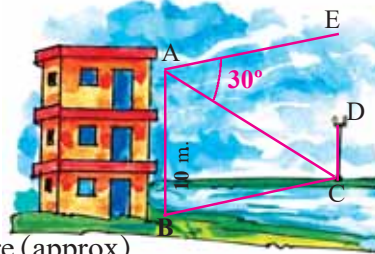
$\therefore \angle ACB = 30^\circ$

In right-angled triangle ABC $\tan 30^\circ = \frac{AB}{BC} = \frac{10 \text{ metre}}{BC}$

$$\text{or, } \frac{1}{\sqrt{3}} = \frac{10 \text{ metre}}{BC}$$

$$\therefore BC = 10\sqrt{3} \text{ metre} = 10 \times 1.732 \text{ metre (approx)} \\ = 17.32 \text{ metre (approx)}$$

$\therefore$ The width of the canal is 17.32 metre (approx).



Application : 7. But if a high multi-storied building stands on the bank of any river, let us see how shall we measure the height of multi-storied building standing on opposite side of the bank of river.



Let the height of multi-storied building = x metre.

If the top of the building from a point C along bank of river just opposite to the point B , the angle of elevation of the top of the building is 45° and D is a point on BC produced at a distance of 14 metres from this bank and the angle of elevation of the top of the building at the point D is found to be 30° , let us find the height of the building.

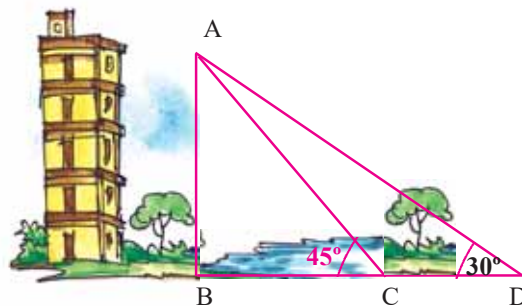
Let width of the river (BC) = y metre

$\therefore$ From right-angled triangle ABC we get

$$\tan 45^\circ = \frac{AB}{BC}$$

$$\text{or, } 1 = \frac{x}{y}$$

$$\therefore x = y$$



Again, we get from right-angled triangle ABD , $\tan 30^\circ = \frac{AB}{BD} = \frac{AB}{BC+CD}$

$$\therefore \frac{1}{\sqrt{3}} = \frac{x}{y+14}$$

$$\text{or, } y+14 = x\sqrt{3}$$

$$\text{or, } x+14 = x\sqrt{3} \quad [\because x=y]$$

$$\text{or, } x(\sqrt{3}-1) = 14$$

$$\therefore x = \frac{14}{\sqrt{3}-1} = \frac{14(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$$

$$= \frac{14(1.732+1)}{3-1}$$

$$= 7 \times 2.732 \text{ (approx)} = 19.124 \text{ (approx)}$$



$\therefore$ Height of the building is 19.124 metre (approx).

Application : 8. If the angle of elevation of the top of monument when observed from a point on the roof of a five-storied building of 18 metre height is 45° and the angle of depression the to foot of the monument, when observed from the same point is 60° , let us write by calculating the height of the monument. [$\sqrt{3} = 1.732$ (approx)]

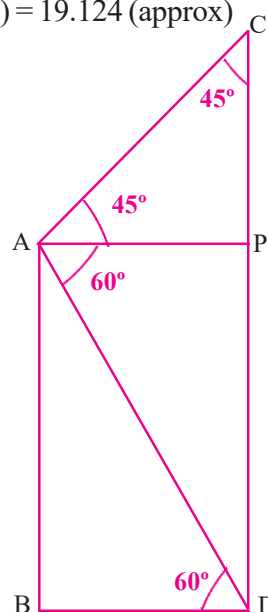
In the adjoining picture, let $AB=18\text{m}$ =height of the five-storied building and CD is the height of the Monument. The angle of elevation of the point C from the point A of AB is 45° and the angle of depression at the point D of the foot of the Monument is 60° .

$\therefore \angle PAC = 45^\circ$ and $\angle PAD = 60^\circ$ [let, $AP \parallel BD$]

$\angle PAD = \angle ADB$ [$\because AP \parallel BD$] $\therefore \angle ADB = 60^\circ$

Let the height of the Monument is $CD = X$ metre and $BD = Y$ metre $= AP$

$AB = 18$ metre. So, $CP = (x-18)$ metre.



From right-angled triangle $\triangle ABD$, we get, $\tan 60^\circ = \frac{AB}{BD}$

$$\text{or, } \sqrt{3} = \frac{18}{y} \quad \therefore y = \frac{18}{\sqrt{3}} = 6\sqrt{3}$$

We get from right-angled triangle $\triangle APC$ $\tan \angle PAC = \frac{CP}{AP}$



$$\text{or, } \tan 45^\circ = \frac{x-18}{y} = \frac{x-18}{6\sqrt{3}}$$

$$\text{or, } 1 = \frac{x-18}{6\sqrt{3}}$$

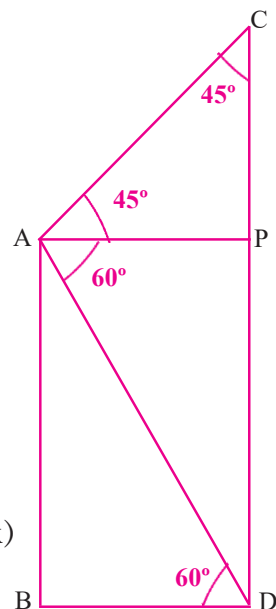
$$\text{or, } x-18 = 6\sqrt{3}$$

$$\text{or, } x = 18 + 6\sqrt{3} = 6(3 + \sqrt{3})$$

$$\text{or, } x = 6(3 + 1.732) \text{ (approx)}$$

$$\text{or, } x = 6 \times 4.732 \text{ (approx)} \quad \therefore x = 28.392 \text{ (approx)}$$

$\therefore$ Height of the Monument is 28.392 metre (approx).



Application : 9. From a point on the roof of a house of 11 metre height, it is observed that the angles of depression of the top and foot of lamp post are 30° and 60° ; respectively. Let us write by calculating height of lamp post.

In the adjoining figure, $AB = 11$ metre = height of the building

CD = height of lamp post = x metre (let)



The angle of depression from the point A of AB at the point C of the top of lamp post is 30° and angle of depression at the point D of foot of lamp post is 60° .

$\therefore \angle PAC = 30^\circ$ and $\angle PAD = 60^\circ$ [Let $AP \parallel BD$ and extended DC is CP]

$AB = PD = 11$ metre. $CD = x$ metre $\therefore PC = (11 - x)$ metre.

Let $BD = y$ metre = AP

From right-angled triangle $\triangle APD$ we get $\tan 60^\circ = \frac{PD}{AP} = \frac{11}{y}$

$$\text{or, } \sqrt{3} = \frac{11}{y} \quad \therefore y = \frac{11}{\sqrt{3}} \quad \text{————— (1)}$$

Again from right-angled triangle $\triangle APC$, we get, $\tan 30^\circ = \frac{PC}{AP}$

$$\text{or, } \frac{1}{\sqrt{3}} = \frac{11-x}{y}$$

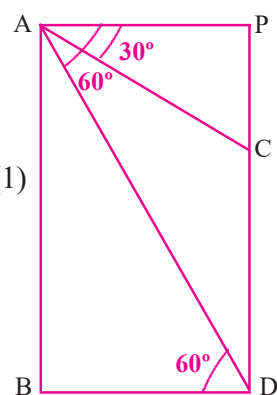
$$\text{or, } y = 11\sqrt{3} - x\sqrt{3}$$

$$\text{or, } \frac{11}{\sqrt{3}} = 11\sqrt{3} - x\sqrt{3} \quad [\text{from (1) we get}]$$

$$\text{or, } 11 = 33 - 3x$$

$$\text{or, } 3x = 22 \quad \therefore x = \frac{22}{3} = 7\frac{1}{3}$$

$\therefore$ The height of lamp post is $7\frac{1}{3}$ metre.



Application : 10. From a point at the top of a building of 60 metre height, it is observed that the angles of depression of the top and foot of tower are 30° and 60° respectively. Let us write by calculating the height of the tower. **[Let me do it myself]**

Application : 11. From a quay of a river of 600 metres wide, two boats start in two different directions to reach the opposite side of the river. The first boat moves making an angle of 30° with this bank and the second boat moves making an angle of 90° with direction of the first boat's movement and reaches the other bank of the river. let us calculate what will be the distance between the two boats when both of them reach the other side. [$\sqrt{3} = 1.732$ (approx)]

Let in the adjoining figure, XY be one bank of the river and the boats starts from a point O on this bank. Let OA be the direction of the first boat and OB be that of the second boat. So, A and B are the respective points where the first and second boats reach the opposite bank.

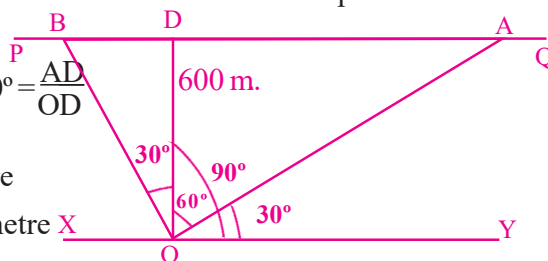
$\therefore \angle YOA = 30^\circ$, $\angle AOB = 90^\circ$; we draw perpendicular OD on AB from the point O.

$\therefore \angle AOD = 60^\circ$ and $\angle DOB = 30^\circ$

From right-angled triangle AOD, we get, $\tan 60^\circ = \frac{AD}{OD}$

$$\text{or, } \sqrt{3} = \frac{AD}{600 \text{ metre}}$$

$$\therefore AD = 600\sqrt{3} \text{ metre}$$



Again, we get from right-angled triangle BOD, we get, $\tan 30^\circ = \frac{BD}{OD}$

$$\text{or, } \frac{1}{\sqrt{3}} = \frac{BD}{600 \text{ metre}}$$

$$\text{or, } BD = \frac{600 \text{ m}}{\sqrt{3}} = \frac{600\sqrt{3} \text{ m}}{3}$$

$$= 200\sqrt{3} \text{ metre}$$

$$AD + BD = (600\sqrt{3} + 200\sqrt{3}) \text{ metre}$$

$$AB = 800\sqrt{3} \text{ metre} = 800 \times 1.732 \text{ metre (approx)} = 1385.6 \text{ metre (approx)}$$

$\therefore$ The distance between the two boats when they reach the other bank is 1385.6 metre (approx).

Application : 12. Two pillars of equal heights are on the either side of a road, which is 150 metre wide. The angles of elevation of the top of the pillars are 60° and 30° respectively at a point on the road between the pillars. Let us find the height of each pillar.

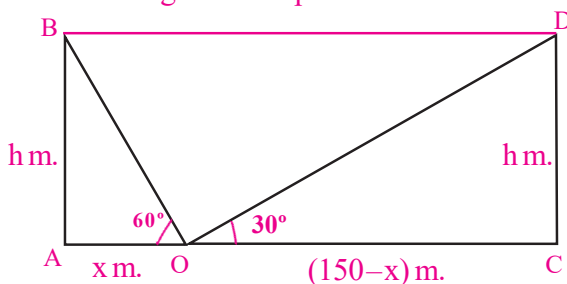
Let, AB and CD are two equal pillars.

Let, $AB = CD = h$ metre.

Let, O is a fixed point on the road AC.

Let, $OA = x$ m. $\therefore OC = (150 - x)$ m.

$\therefore \angle AOB = 60^\circ$ and $\angle COD = 30^\circ$



From right-angled triangle AOB, we get, $\tan 60^\circ = \frac{AB}{AO} = \frac{h}{x}$

$$\text{or, } \sqrt{3} = \frac{h}{x} \quad \therefore x = \frac{h}{\sqrt{3}} \quad \text{———— (i)}$$



Again we get from right-angled triangle COD

$$\tan 30^\circ = \frac{CD}{OC} = \frac{h}{150-x}$$

$$\text{or, } \frac{1}{\sqrt{3}} = \frac{h}{150-x}$$

$$\text{or, } 150-x = h\sqrt{3}$$

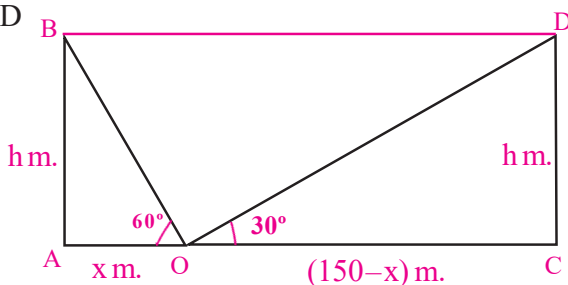
$$\therefore x = 150 - h\sqrt{3} \text{ ————— (ii)}$$

$$\therefore \text{Comparing equation (i) and (ii) we get, } \frac{h}{\sqrt{3}} = 150 - h\sqrt{3}$$

$$\text{or, } h = 150\sqrt{3} - 3h$$

$$\text{or, } 4h = 150\sqrt{3} \quad \therefore h = \frac{150\sqrt{3}}{4} = \frac{75\sqrt{3}}{2}$$

$$\therefore \text{The height of each pillar is } \frac{75\sqrt{3}}{2} \text{ metre.}$$



Application : 13. A bird was flying in a line parallel to the ground from north to south. It was at a height of 200 metre. Susovan, standing in the midst of a field, first observed the bird in the north at an angle of 30° from a point on the ground. After 3 minutes, he again observed it in the south at an angle of 45° . Let us write by calculating the speed of the bird in kilometers per hour to the nearest integer. ($\sqrt{3} = 1.732$ (approximately))

Let, from the point P, Susovan observed the bird at an angle of 30° at the point A and after 3 minutes he observed the bird at an angle of 45° at the point B.

Let, $AO = x$ m. and $BO = y$ m.

PO is perpendicular on AB from the point P. $\therefore PO = 200$ metre

$$\therefore \angle XPA = 30^\circ, \therefore \angle APO = 90^\circ - 30^\circ = 60^\circ;$$

$$\therefore \angle BPY = 45^\circ, \therefore \angle BPO = 90^\circ - 45^\circ = 45^\circ$$

In right-angled triangle APO.

$$\tan \angle APO = \tan 60^\circ = \frac{x}{200}$$

$$\text{or, } \sqrt{3} = \frac{x}{200} \quad \therefore x = 200\sqrt{3}$$

In right-angled triangle BPO.

$$\tan \angle BPO = \tan 45^\circ = \frac{y}{200}$$

$$\text{or, } 1 = \frac{y}{200} \quad \therefore y = 200$$

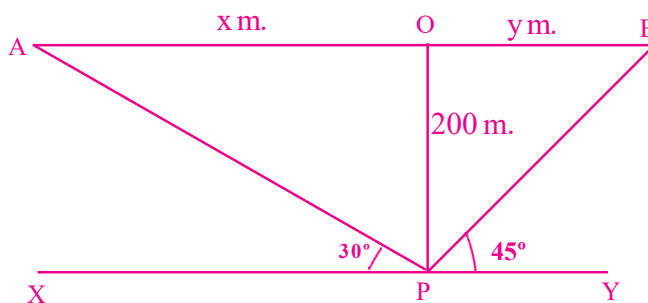
$$\text{So, } x+y = 200\sqrt{3} + 200 = 200(\sqrt{3} + 1) = 200 \times 2.732 = 546.4$$

In 3 minute the bird flies 546.4 metre

In 1 minute the bird flies $\frac{546.4 \text{ m.}}{3}$

In 60 minutes the bird flies $\frac{546.4}{3} \times 60 \text{ metre} = 10928 \text{ metre} = 10.928 \text{ Km.}$

$\therefore$ The speed of the bird is 11 km/hour. (in nearest integer)



1. If the angle of elevation of the top of a coconut tree from a point on the ground is 60° and the point is 20 metre away from the foot of the tree, let us find the height of the tree.
2. The length of the shadow of a tower is 9 metres when the angle of elevation of sun is 30° . Let us write by calculating the height of the tower.
3. A kite is flying with a thread of 150 metres length from a field. If the thread of kite makes an angle 60° with the horizontal line, let us write by calculating the height of the kite from the ground.
4. A palm tree stands on the bank of a river. A post is fixed in the land on the other bank just opposite to the palm tree. On moving $7\sqrt{3}$ metres from the post along the bank, it is found that the tree makes an angle of 60° at that point with respect to this bank. Let us find the width of the river.
5. A telegraph post is bent at a point above the ground due to storm. Its top just meets the ground at a distance of $8\sqrt{3}$ metres from its foot and makes an angle of 30° . Let us write by calculating at what height the post is bent and what is the height of the post.
6. Two houses stand just on the opposite sides of a road of our village. A ladder that stands against the wall of the first house is at a distance of 6 metres from the house and makes an angle of 30° with the horizontal line. But if the ladder stands against the wall of the second house keeping its foot at the same point, it makes an angle of 60° with the horizontal line.
 - (i) Let us find the length of the ladder.
 - (ii) Let us write by calculating, the distance of the foot of the ladder from the foot of the wall of the second house.
 - (iii) Let us find the width of the road.
 - (iv) Let us find the height where the top of the ladder is fixed against the wall of the second house.
7. If the angle of elevation of the top of a chimney from a point on the horizontal plane passing through the foot of the chimney is 60° and the angle of elevation from another point on the same plane at a distance of 24 metres from the first point is 30° , let us write by calculating the height of the chimney.

[Let us find the approximate value correct upto three decimal places. assuming $\sqrt{3} = 1.732$ (approx)]
8. The length of the shadow of a post becomes 3 metre smaller when the angle of elevation of the sun increases from 45° to 60° . Let us find the height of the post.

[Let us find the approximate value correct upto three decimal places, assuming $\sqrt{3} = 1.732$ (approx)]
9. When the top of a chimney of a factory is seen from a point on the roof of the three storied building of $9\sqrt{3}$ metre height, the angle of elevation is 30° . If the distance between the factory and chimney is 30 metres, let us write by calculating height of the chimney.
10. The bottom of a light house and bottom of the mast of two ships are on the same straight line and the angles of depression from the light house at the bottom of mast of two ships are 60° and 30° respectively. If the distance of the points at the bottom of the light house and the bottom of the mast of first ship is 150 metres, let us write by calculating, how far will the mast of other ship from the light house be and what will the height of light house be.
11. From a point on the roof of five-storied building the angle of elevation of the top of a monument and that of angle of depression of the foot of the monument are 60° and 30° respectively. If the height of the building is 16 metres, let us write by calculating the height of the monument and the distance of the building from the monument.

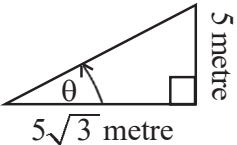
12. I am flying a kite having the length of thread 250 metres; when the thread makes an angle 60° with the horizontal line, and when the thread makes an angle of 45° with the horizontal line. let us write by calculating in each case what is the height of kite from me. Let us find in which of the two cases will the kite be at a greater height from the other.
13. A passenger of an aeroplane observes that Howrah station is at one side of the plane and Saheed minar is just on the opposite side. The angles of depression of Howrah station and Saheed minar from the passenger of aeroplane are 60° and 30° respectively. If the aeroplane is at a height of $545\sqrt{3}$ metres at that time, let us find the distance between Howrah station and Saheed minar.
14. The length of the flag at the roof of three-storied building is 3.3 metres. From any point of road, the angles of elevation of the top and foot of the flagpost are 50° and 45° . Let us write by calculating the height of three-storied building. [Let, $\tan 50^\circ = 1.192$]
15. The heights of two towers are 180 metres and 60 metres respectively. If the angle of elevation of the top of the first tower from the foot of the second tower is 60° , let us write by calculating what is the angle of elevation of the top of the second tower from the foot of the first.
16. The length of shadow of a tower standing on the ground is found to be 60 metres more when the angle of elevation of sun changes from 45° to 30° , let us find the height of the tower.
17. From a point on the same plane along the horizontal line passes through the foot of a chimney, the angle of elevation of the top of the chimney is 30° and the angle of elevation of the top of the chimney is 60° at a point on the same straight line proceeding 50 metres nearer to the chimney. Let us write by calculating the height of the chimney.
18. A vertical post of 126 dm height was bent at some point above the ground and it just touched the ground making an angle of 30° with the ground. Let us write by calculating at what height was the post bent and at what distance did it meet the ground from the foot of the post.
19. Mohit, standing in the midst of a field, observes a flying bird in his north at an angle of elevation of 30° and after 2 minutes he observes the bird in his south at an angle of elevation of 60° . If the bird flies in a straight line all along at a height of $50\sqrt{3}$ metres, let us find its speed in kilometre per hour.
20. Amitadidi standing on a railway overbridge of $5\sqrt{3}$ metres height observed the engine of the train from one side of the bridge at an angle of depression of 30° . But just after 2 seconds, she observed the engine at an angle of depression of 45° from the other side of the bridge. Let us find the speed of the train in metres per second.
21. A bridge is situated at right angle to the bank of the river. If one moves away a certain distance from the bridge along this side of the river, the other end of the bridge is seen at an angle of 45° and if someone moves a further distance of 400 metres in the same direction, the other end is seen at an angle of 30° . Let us find the length of the bridge.
22. A house of 15 metres height, stands on one side of a park and from a point on the roof of the house the angle of depression of the foot of the chimney of brick kiln of the other side is 30° and the angle of elevation of the top of the chimney of brick kiln is 60° . Let us write the height of the chimney and the distance between the brick kiln and the house.
23. If the angle of depression of two consecutive mile stones on a road from an aeroplane are 60° and 30° respectively, let us find the height of the aeroplane, (i) when the two mile stones stand on opposite side of the aeroplane, (ii) when the two mile stones stand on the same side of the aeroplane.

24. **Very short answer type question (V.S.A.)**

(A) M.C.Q. :

- (i) If the angle of elevation of the top of mobile tower from a distance of 10 metres from its foot is 60° , then the height of the tower is

(a) 10 metres (b) $10\sqrt{3}$ metres (c) $\frac{10}{\sqrt{3}}$ metres (d) 100 metres

- (ii)  Value of θ is (a) 30° (b) 45° (c) 60° (d) 75°

- (iii) At what angle an observer observes a box lying on ground from the roof of three-storied building, so that the height of building is equal to the distance of the box from the building, then the angle is

(a) 15° (b) 30° (c) 45° (d) 60°

- (iv) Height of tower is $100\sqrt{3}$ metre. The angle of elevation of the top of a tower from a point at a distance of 100 metre of foot of tower is

(a) 30° (b) 45° (c) 60° (d) none of this

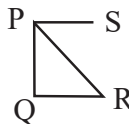
- (v) If the length of shadow on the ground of a post is $\sqrt{3}$ times of its height, the angle of elevation of the sun is

(a) 30° (b) 45° (c) 60° (d) none of these

(B) Let us write whether the following statements are true or false :-

- (i) In $\triangle ABC$, $\angle B = 90^\circ$, if $AB = BC$, then $\angle C = 60^\circ$.

- (ii) PQ is the height of a building, QR is the base, the angle of depression from a point P at the point R is $\angle SPR$;
So, $\angle SPR = \angle PRQ$.



(C) Let us fill in the blanks.

- (i) If the angle of elevation of sun increases from 30° to 60° , the length of the shadow of a post _____. (decreases/increases)

- (ii) If the angle of elevation of sun is 45° , then the length of shadow and length of post are _____.

- (iii) If the angle of elevation of sun is _____ 45° , the length of shadow of tower will be less than the height of tower.

25. **Short answer type questions (S.A.) :**

- (i) If the angle of elevation of a kite is 60° and the length of thread is $20\sqrt{3}$ metre, let us calculate the height of kite above the ground.
- (ii) AC is the hypotenuse with length of 100 metres of a right-angled triangle ABC and if $AB = 50\sqrt{3}$ metre, let us find the value of $\angle C$.
- (iii) A tree breaks due to storm and its top touches the ground in such a manner that the distance from the top of the tree to the base of the tree and present height are equal. Let us calculate how much angle is made by the top of the tree with the base.
- (iv) In right-angled triangle ABC, $\angle B = 90^\circ$, D is such a point on AB that $AB : BC : BD = \sqrt{3} : 1 : 1$, let us find the value of $\angle ACD$.
- (v) If the ratio between length of shadow of a tower and height of tower is $\sqrt{3} : 1$, let us find the angle of elevation of the sun.

At the crossing of chowmatha of village Berhampur there are two tea stalls having good quality of tea. One is Bimal uncle's tea stall and the other is Asha aunty's tea stall. There are gathering of customers nearly all time of the day.

Sutapa has made a table of accounts of per day profit of last month of each of that two tea stalls.



The two tables of accounts made by Sutapa are,

The table of everyday profit (in `) of last month in Bimal uncle's tea stall is

321, 352, 388, 410, 480, 400, 475, 415, 345, 360
445, 390, 552, 495, 570, 530, 436, 580, 510, 462
437, 491, 498, 460, 372, 463, 458, 515, 464, 428

The table of every day profit (in `) of last month in Asha aunty's tea stall is

315, 420, 480, 530, 580, 315, 360, 440, 480, 465
530, 465, 580, 360, 420, 360, 480, 530, 580, 360
440, 420, 360, 480, 530, 420, 580, 530, 465, 420



1 But how shall we understand that the tea stall which has more average profit at last month from the above data i.e. let us see how shall we compare data obtained from two tea stalls.

We shall determine such a special number for each data so that it will be a representative of the whole data.

2 What is called this special number for each data ?

A special number of any data which represents the whole data, usually occupies nearly a central position. So, that special number is called **Measure of Central tendency** of data.

3 What do we mean by central position ?

If the numbers of data are arranging in ascending order then the middle number / or the positions of nearby numbers is called the central position.

There are three measures of central tendency. (i) Mean (ii) Median (iii) Mode

4 Let us find the mean profit of data of Bimal uncle's tea stall obtained by Sutapa and let us see what we shall get.

Last month in 30 days profit of Bimal uncle = $x_1 + x_2 + x_3 + \dots + x_{30}$

Where, $x_1 = ` 321, x_2 = ` 352, x_3 = ` 388, \dots, x_{30} = ` 428$

$$\therefore \text{Total Profit} = (321 + 352 + 388 + \dots + 428) \\ = ` 13502$$

$$\therefore \text{Mean profit of Bimal uncle is} = \frac{13502}{30} = ` 450.06 = ` 450 \text{ (approx) } |$$



The mean obtained by this method is called Arithmetic mean. Usually we consider arithmetic mean as mean only

If $x_1, x_2, x_3, \dots, x_n$ be the n values of a variable x , and if their arithmetic mean be $\bar{x}$, then

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}$$

$\bar{x}$ is read as **x-bar**.

5 But how shall we write it briefly

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \frac{1}{n} \sum_{i=1}^n x_i$$

Here, $\sum$ sign is a Greek capital letter and is called 'Sigma' which means summation.



Then what will be $\sum_{i=1}^{10} x_i$?

$$\sum_{i=1}^{10} x_i = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

What will be $\sum_{i=1}^{10} (10 \times i)$?

$$\begin{aligned} \sum_{i=1}^{10} (10 \times i) &= (10 \times 1) + (10 \times 2) + \dots + (10 \times 10) \\ &= 10 \times (1 + 2 + 3 + \dots + 10) \\ &= 10 \times 55 = 550 \end{aligned}$$

I form a frequency table from data of profit obtained by Sutapa from Asha aunty's tea stall.

Table-1 (Table – 1)

Amount of profit (in `) (x_i)	315	360	420	440	465	480	530	580	Total
Number of days (f_i)	2	5	5	2	3	4	5	4	30

Here how shall we get the average profit of Asha aunty for the last month. i.e. Let us see, how shall we determine the value of mean from frequency distribution table. i.e. let us find the arithmetic mean of ungrouped data.

Amount of profit ` (x_i)	Number of days(f_i)	$x_i f_i$
315	2	630
360	5	1800
420	5	2100
440	2	880
465	3	1395
480	4	1920
530	5	2650
580	4	2320
Total	$\sum f_i = 30$	$\sum f_i x_i = 13695$



$\therefore$ In the last month the average profit of Asha aunty's tea stall = $\frac{\sum f_i x_i}{\sum f_i} = \frac{13695}{30} = 456.50$

$\therefore$ We are observing that, according to the data obtained by Sutapa, the mean profit of Asha aunty was more in the last month.

But what is called the arithmetic mean which I obtained from the frequency distribution table.

This is called weighted mean of the variable x .

We understand that if $f_1, f_2, f_3, \dots, f_n$ be respective frequencies of n values $x_1, x_2, x_3, \dots, x_n$ of a variable x , then the weighted mean will be $\bar{x}$,

$$\text{if } \bar{x} = \frac{x_1 f_1 + x_2 f_2 + x_3 f_3 + \dots + x_n f_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum_{i=1}^n f_i x_i}{\sum_{i=1}^n f_i}$$

Application 1. I have prepared a table of height of 40 students of Shakil's class in our higher secondary school of our village. Let us find the mean of their heights.

Height of students	2	6	8	12	7	3	2
Number of students	90	97	110	125	134	140	148

Height of students (cm.) (x_i)	Number of students (f_i)	$x_i f_i$
90	2	$90 \times 2 = 180$
97	6	$97 \times 6 = \boxed{}$
110	8	$110 \times 8 = \boxed{}$
125	12	$\boxed{}$
134	7	$\boxed{}$
140	3	$\boxed{}$
148	2	$\boxed{}$
Total	$\sum f_i = 40$	$\sum f_i x_i = 4796$



$$\therefore \text{Mean height of students} = \frac{\sum f_i x_i}{\sum f_i} = \frac{4796}{40} \text{ cm.} = 119.9 \text{ cm.}$$

Application : 2. The marks obtained by 30 students of class of Bishakha in Geography are

61, 78, 80, 77, 80, 69, 73, 61, 82, 78, 79, 72, 78, 62, 80
71, 82, 73, 66, 73, 62, 80, 74, 78, 62, 80, 66, 70, 79, 75

Let us find the arithmetic mean of marks obtained Geography [Let me do it myself]

Let us write grouped data in table-I of profit of ungrouped data made by Sutapa of Asha aunty's tea stall where the length of class interval is 50.

Let us find the arithmetic mean of profit from adjoining grouped data and let us see what I shall get.

In adjoining frequency distribution table how shall I get a number which will represent class interval of each class.

Table – 2

Profit amount (Rs.)	number of days (f_i)
300 - 350	2
350 - 400	5
400 - 450	7
450 - 500	7
500 - 550	5
550 - 600	4
Total	30

It is assumed that the most of the numbers of the frequency of each class interval in grouped data of frequency distribution table is centred around that mean of class size. So the class mark of each class is assumed as the representative of that class.

$$\text{class mark} = \frac{\text{Upper class limit} + \text{lower class limit}}{2}$$

Application : 3. Let us find the mean of profit of Asha aunty's tea stall in the last month from the table 2 and let us see what I shall get.

Table – 3

Profit amount (`)	Number of days (frequency f_i)	Class mark (x_i)	$x_i f_i$
300 - 350	2	325	650
350 - 400	5	375	1875
400 - 450	7	425	2975
450 - 500	7	475	3325
500 - 550	5	525	2625
550 - 600	4	575	2300
Total	$\sum f_i = 30$		$\sum f_i x_i = 13750$

$$\therefore \text{Arithmetic mean of profit} = \frac{\sum f_i x_i}{\sum f_i} = \frac{13750}{30} = \text{`} 458.33 \text{ (approx)}$$

6 What is called this new method of finding arithmetic mean.

The new method of finding the mean is known as the Direct Method.



We see that, the mean of profit of tea shop of Asha aunty we have got from ungrouped data ` 456.50 and from grouped data we have got the mean of profit ` 458.33 (approx)

7 But the results obtained are different. Why this is so and which is more accurate?

In grouped data of table-2, mid-point of class interval is assumed, so we have got mean in approximate value as ` 458.33 from grouped data.

But the value of mean ` 456.50 obtained from ungrouped data is exact mean.

8 But when the numerical values of x_i and f_i are large, then calculation of the product of x_i and f_i becomes tedious and time consuming and there will be a possibility of mistake.



So, let us see how we shall calculate it more easily and in less time.

So, at first let us choose a fixed x_i from x_i 's and subtract it from each x_i and find the arithmetic mean.

9 But what is called that fixed x_i ?

That fixed x_i subtracted from each x_i 's is called assumed mean and generally it is denoted by "a"

Again, to make the calculation of finding this arithmetic mean more easily, this assumed mean "a" is to be considered x_i

which lies generally in the centre of $x_1, x_2, \dots, x_n$.

$\therefore$ In Table-3, let $a=425$ or $a=475$

Application : 4. Let us find the arithmetic mean assuming $a=425$, $d_i = x_i - a = x_i - 425$ from Table – 1.

Table – 4

Amount of profit (`)	Number of days (frequency f_i)	Class mark (x_i)	($d_i = x_i - a$) $d_i = (x_i - 425)$	$d_i f_i$
300 - 350	2	325	-100	-200
350 - 400	5	375	-50	-250
400 - 450	7	425 = a	0	0
450 - 500	7	475	50	350
500 - 550	5	525	100	500
550 - 600	4	575	150	600
Total	$\sum f_i = 30$			$\sum f_i d_i = 1000$

$\therefore$ We get from above table, $\bar{d} = \frac{\sum f_i d_i}{\sum f_i} = \frac{1000}{30} = 33.33$ (approx)

10 What this $\bar{d}$ is called?

d_i is the deviation of 'a' from each x_i 's and $\bar{d}$ is the mean of deviation.

11 Now let us find the relation between $\bar{x}$ and $\bar{d}$

$$\begin{aligned}\bar{d} &= \frac{\sum f_i d_i}{\sum f_i} = \frac{\sum f_i (x_i - a)}{\sum f_i} \\ &= \frac{\sum f_i x_i - \sum f_i a}{\sum f_i} \\ &= \frac{\sum f_i x_i}{\sum f_i} - \frac{\sum f_i a}{\sum f_i} = \frac{\sum f_i x_i}{\sum f_i} - \frac{a \sum f_i}{\sum f_i} \quad [\because a \text{ is constant}]\end{aligned}$$

$$\bar{d} = \bar{x} - a$$

$$\text{or } \bar{x} = a + \bar{d}$$

$$\text{or } \bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$$

Arithmetic mean of profit = $a + \bar{d} = (425 + 33.33)$ (approx) = 458.33 (approx)

$\therefore$ Last month Asha aunty got average profit of ` 458.33 (approx) from her tea stall.

12 What this method of comparing casilly to determine the arithmetic mean is called?

The method discussed above is called the Assumed mean method or short method or deviation method.

In finding arithmetic mean we can use any one method of Direct method or Asumed mean method. When the numerical value of data is large, it is easy to calculate arithmetic mean by Assumed mean method.

Taking $a=325$; [or, $a=375$ or, $a=475$ or, $a=525$ or, $a=575$] of Table-3, I find the arithmetic mean using assumed mean method and verifying that the arithmetic mean will be same in each case by hands on trial. (Let me do it myself)

But at the time of finding arithmetic mean by assumed mean method, we observe that in Table-4, each number in column 4 is multiple of 50; So dividing the numbers in the column 4 by 50, it will easy to calculate with respect to previous calculation. i.e. we divide by the class size (h) to each d_i .

Application : 5. Let us find $\bar{u} = \frac{\sum f_i u_i}{\sum f_i}$ from Table-4, where $u_i = \frac{x_i - a}{h}$, here $a=425$ and $h=50$

Amount of profit (`)	Number of days (frequency f_i)	Class mark (x_i)	($d_i = x_i - a$) $d_i = x_i - 425$	$u_i = \frac{x_i - a}{50}$	$f_i u_i$
300 - 350	2	325	-100	-2	-4
350 - 400	5	375	-50	-1	-5
400 - 450	7	425	0	0	0
450 - 500	7	475	50	1	7
500 - 550	5	525	100	2	10
550 - 600	4	575	150	3	12
Total	$\sum f_i = 30$				$\sum f_i u_i = 20$

$$\therefore \text{We get } \bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{20}{30} = \frac{2}{3}$$

13 Let us find the relation between $\bar{u}$ and $\bar{x}$.

$$\bar{u} = \frac{\sum f_i u_i}{\sum f_i} = \frac{\sum f_i \left(\frac{x_i - a}{h} \right)}{\sum f_i} \quad \left[\because u_i = \frac{x_i - a}{h} \right]$$

$$= \frac{1}{h} \times \frac{\sum f_i (x_i - a)}{\sum f_i}$$

$$= \frac{1}{h} \times \left[\frac{\sum f_i x_i}{\sum f_i} - \frac{a \sum f_i}{\sum f_i} \right]$$

$$\bar{u} = \frac{1}{h} \times (\bar{x} - a)$$

$$\text{or } h \bar{u} = \bar{x} - a$$

$$\text{So, } \bar{x} = a + h \bar{u}$$

$$\therefore \bar{x} = a + h \frac{\sum f_i u_i}{\sum f_i}$$

We understand that the mean profit of tea shop of Asha aunty in last month by this method

$$= \text{` } (425 + 50 \times \frac{2}{3}) \\ = \text{` } 458.33 \text{ (approx)}$$

$\therefore$ We get same value of arithmetic mean.

14 What this method of finding arithmetic mean in comparitably easy method is called?

The method of determining the arithmetic mean as discussed above is called the **Step-deviation method**.

15 Is Step-deviation method is always convenient for finding arithmetic mean?

If the class sizes of each class interval are equal or, all d_i 's have a common factor, the step



deviation method will be convenient.

- ∴ We get, (i) The mean values obtained by all three methods– Direct Method, Assumed Mean Method, and Step Deviation Method– are equal.
- (ii) The assumed Mean Method and Step Deviation method are just simplified forms of the Direct Method.
- (iii) The step deviation method will be convenient to apply if all the class sizes are equal.
- (iv) Again, if the class sizes are unequal, then for determining arithmetic mean, assumed mean method can be used but step deviation method can not be used.

Application : 6. Satish and I have written the data of electric consumption of 50 families of our village in a table. This table is given below.

Electric consumption (unit)	85–105	105–125	125–145	145–165	165–185	185–205
Number of families	3	12	18	10	5	2

Let us calculate arithmetic mean of electric consumption of 50 families by using three methods of finding arithmetic mean. At first determining the class mark, let us write the given data.

Amount of electric consumption (unit)	Number of families (frequency f_i)	Class mark (x_i)
85 - 105	3	95
105 - 125	12	115
125 - 145	18	135
145 - 165	10	155
165 - 185	5	175
185 - 205	2	195
Total	$\sum f_i = 50$	



Let $a = 155$ and here length of class interval $h = 20$

∴ By putting $d_i = x_i - 155$ and $u_i = \frac{x_i - 155}{20}$, let us write in table below.

Amount of electric consumption (unit)	Number of family (f_i)	Class mark (x_i)	$d_i = x_i - 155$	$u_i = \frac{x_i - 155}{20}$	$f_i x_i$	$f_i d_i$	$f_i u_i$
85 - 105	3	95	-60	-3	285	-180	-9
105 - 125	12	115	-40	-2	1380	-480	-24
125 - 145	18	135	-20	-1	2430	-360	-18
145 - 165	10	155	0	0	1550	0	0
165 - 185	5	175	20	1	875	100	5
185 - 205	2	195	40	2	390	80	4
Total	50				6910	-840	-42

∴ From above table, we get $\sum f_i = 50$, $\sum f_i x_i = 6910$, $\sum f_i d_i = -840$ and $\sum f_i u_i = -42$

∴ We get from direct method, arithmetic mean = $\frac{\sum f_i x_i}{\sum f_i} = \frac{6910}{50} \text{ unit} = 138.2 \text{ unit}$

We get from assumed mean method, arithmetic mean = $a + \frac{\sum f_i d_i}{\sum f_i} = 155 + \frac{(-840)}{50} \text{ unit}$
 $= 155 - 16.8 = 138.2 \text{ unit}$

Again, we get from step deviation method, arithmetic mean = $a + \frac{\sum f_i u_i}{\sum f_i} \times h$
 $= 155 + \left(\frac{-42}{50}\right) \times 20 \text{ unit}$
 $= 155 - 16.8 = 138.2 \text{ unit}$

∴ We see the means of electric consumption of 50 families of our village obtained by all the three methods are the same i.e. 138.2 unit.

Application : 7. The weaver Ramesh has many looms. I have written the weekly income (in `) in case of 35 weavers in the table below



Income (in `)	2500 - 3000	3000 - 3500	3500 - 4000	4000 - 4500	4500 - 5000
frequency	3	6	9	12	5

let us calculate arithmetic mean of income.

Assuming, $a=3750$ and $h=500$, let us find arithmetic mean by step-deviation method.

Income (in `)	frequency (f_i)	Class mark (x_i)	$u_i = \frac{x_i - 3750}{500}$	$f_i u_i$
2500 - 3000	3	2750	-2	-6
3000 - 3500	6	3250	-1	-6
3500 - 4000	9	3750	0	0
4000 - 4500	12	4250	1	12
4500 - 5000	5	4750	2	10
Total	$\sum f_i = 35$			$\sum f_i u_i = 10$

∴ Arithmetic mean of income = $\text{` } 3750 + \text{` } 500 \times \frac{\sum f_i u_i}{\sum f_i}$
 $= \text{` } 3750 + \text{` } \left[500 \times \frac{10}{35}\right] = \text{` } \boxed{}$



Application : 8. Let us calculate arithmetic mean of data from the following distribution table by applying any method. [Let me do it myself]

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60
Frequency	7	5	6	12	8	2

Activity

I form a grouped distribution table of the weight of 40 friends of my class and let us find the arithmetic mean of weight of these 40 friends by using three methods. [Let me do it myself]



Application : 9. If the arithmetic mean of the following frequency distribution table is 54, let us find the value of k .

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
Frequency	7	11	k	9	13

Class Interval	Frequency (f_i)	Class mark (x_i)	$f_i x_i$	$u_i = \frac{x_i - 50}{20}$	$f_i u_i$
0 - 20	7	10	70	-2	-14
20 - 40	11	30	330	-1	-11
40 - 60	k	$50 = a$	$50k$	0	0
60 - 80	9	70	630	1	9
80 - 100	13	90	1170	2	26
Total	$\sum f_i = 40 + k$		$\sum f_i x_i = 2200 + 50k$		$\sum f_i u_i = 10$

$$\therefore \text{Required mean} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2200 + 50k}{40 + k}$$

$$\text{By the condition, } \frac{2200 + 50k}{40 + k} = 54$$

$$\text{or, } 2200 + 50k = 2160 + 54k$$

$$\text{or, } 50k - 54k = 2160 - 2200$$

$$\text{or, } -4k = -40$$

$$\therefore k = 10$$

$$\text{Alternatively } a + \frac{\sum f_i u_i}{\sum f_i} \times h$$

$$54 = 50 + \frac{10}{40 + k} \times 20$$

$$\text{or, } 4 = \frac{200}{40 + k}$$

$$\text{or, } 40 + k = 50$$

$$\therefore k = 10$$



Application : 10. If the arithmetic mean of the following frequency distribution table is 25; let us find the value of K . [Let me do it myself]

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	K	15	16	6

Application : 11. Mariya has made a cumulative frequency distribution table of the marks obtained by different competitors in drawing competition of their village. The table is

Marks	0 or greater than 0	10 or greater than 10	20 or greater than 20	30 or greater than 30	40 or greater than 40	50 or greater than 50
Number of competitors	40	36	22	11	2	0

Let us find the arithmetic mean of the obtained marks in drawing competition.

At first we express greater than cumulative frequency distribution table into simple frequency distribution table. We see that 40 students obtained 0 or more than 0.

and 36 students obtained 10 or more than 10.

∴ The number of students who have scored from 0 to 10 is $(40-36)=4$.

Similarly the number of students scored from 10 to 20 is $(36-22)=14$.

∴ The frequency distribution table is :

Marks	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Number of competitors	4	14	11	9	2

Let us find arithmetic mean considering assumed mean as 25 by the step deviation method

Class-limit(marks)	Number of competitors (f_i)	Class marks (x_i)	$u_i = \frac{x_i - 25}{10}$	$f_i u_i$
0 - 10	4	5	-2	-8
10 - 20	14	15	-1	-14
20 - 30	11	25	0	0
30 - 40	9	35	1	9
40 - 50	2	45	2	4
Total	$\sum f_i = 40$			$\sum f_i u_i = -9$

$$\text{Required arithmetic mean} = 25 + 10 \times \frac{\sum f_i u_i}{\sum f_i} = 25 + 10 \left(\frac{-9}{40} \right) = 25 - 2.25 = 22.75$$

∴ Mean scores of 40 students is 22.75



Application : 12. Let us find arithmetic mean of the following frequency distribution table.

Class limit	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79
Frequency	12	20	14	6	5	3

In given frequency distribution table the class intervals are formed in exclusive form.

At first we convert exclusive class interval into inclusive class form and then let us find the arithmetic mean. Let assumed mean be 44.5, here class length (h) = 10

Class interval	Class boundary	Frequency (f_i)	Class mark (x_i)	$[u_i = \frac{x_i - a}{h}]$ $u_i = \frac{x_i - 44.5}{10}$	$f_i u_i$
20 - 29	19.5 - 29.5	12	24.5	-2	-24
30 - 39	29.5 - 39.5	20	34.5	-1	-20
40 - 49	39.5 - 49.5	14	44.5	0	0
50 - 59	49.5 - 59.5	6	54.5	1	6
60 - 69	59.5 - 69.5	5	64.5	2	10
70 - 79	69.5 - 79.5	3	74.5	3	9
	Total	$\sum f_i = 60$			$\sum f_i u_i = -19$

$$\therefore \text{Required arithmetic mean} = 44.5 + h \times \frac{\sum f_i u_i}{\sum f_i} = 44.5 + \left(10 \times \frac{-19}{60} \right) = \boxed{}$$

[Let me calculate it myself]

Application :13. Let us find the arithmetic mean from the following distribution table [Let me do it myself]

Class interval	25 - 29	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54
Frequency	10	12	15	5	3	5

Application : 14. Let us find the average marks of 52 students of class X in a school by using direct method and assumed mean method from the table given below.



Number of students	4	7	10	15	8	5	3
Marks	30	33	35	40	43	45	48

Let assumed mean (a) = 40

Marks (x_i)	number of student (f_i)	$f_i x_i$	$(d_i = x_i - a)$ $d_i = (x_i - 40)$	$f_i d_i$
30	4	120	-10	-40
33	7	231	-7	-49
35	10	350	-5	-50
40=a	15	600	0	0
43	8	344	3	24
45	5	225	5	25
48	3	144	8	24
Total	$\sum f_i = 52$	$\sum f_i x_i = 2014$		$\sum f_i d_i = -66$

The mean by direct method = $\frac{2014}{52} = 38.73$ (approx)

The mean by assumed mean method = $a + \frac{\sum f_i d_i}{\sum f_i}$

$$= 40 + \frac{-66}{52}$$

$$= 40 - \frac{66}{52}$$

$$= (40 - 1.27) \text{ (approx)}$$

$$= 38.73 \text{ (approx)}$$

Let us work out 26.1

1. I have written ages of my 40 friends in the table given below.

Age (years)	15	16	17	18	19	20
Number of friends	4	7	10	10	5	4

Let us find average age of my friends by direct method.

2. I have written member of 50 families of our village in the table given below.

Number of members	2	3	4	5	6	7
Number of families	6	8	14	15	4	3

Let us write the average member of 50 families by the method of assumed mean.

3. If the arithmetic mean of the data given below is 20.6, let us find the value of 'a'

Variable (x_i)	10	15	a	25	35
Frequency (f_i)	3	10	25	7	5

4. If the arithmetic mean of the distribution given below is 15, let us find the value of p.

Variable	5	10	15	20	25
Frequency (f)	6	p	6	10	5

5. Rahamatchacha will go to retail market. for selling mangoes kept in 50 packing boxes. Let us write the number of boxes contained varing number of mangoes in the table given below.

Number of mangoes	50 - 52	52 - 54	54 - 56	56 - 58	58 - 60
Number of boxes	6	14	16	9	5

Let us write the mean number of mangoes kept in 50 packing boxes. [using any method]

6. Mohidul has written ages of 100 patients of village hospital. Let us write by calculating average age of 100 patients. [using any method]

Age (years)	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Number of patients	12	8	22	20	18	20

7. Let us find the mean of the following data by direct method.

(i)	Class interval	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
	Frequency	4	6	10	6	4

(ii)	Class interval	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
	Frequency	10	16	20	30	13	11

8. Let us find the mean of the following data by assumed mean method.

(i)	Class interval	0 - 40	40 - 80	80 - 120	120 - 160	160 - 200
	Frequency	12	20	25	20	13

(ii)	Class interval	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
	Frequency	4	10	8	12	6

9. Let us find the mean of the following data by step deviation method.

(i)	Class interval	0 - 30	30 - 60	60 - 90	90 - 120	120 - 150
	Frequency	12	15	20	25	8

(ii)	Class interval	0 - 14	14 - 28	28 - 42	42 - 56	56 - 70
	Frequency	7	21	35	11	16

10. If the mean of the following frequency distribution table is 24, let us find the value of p.

Class interval (number)	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Number of students	15	20	35	p	10

11. Let us see the ages of the persons present in a meeting and determine their average age from the following table :

Age (Years)	30 - 34	35 - 39	40 - 44	45 - 49	50 - 54	55 - 59
Number of persons	10	12	15	6	4	3

12. Let us find the mean of the following data.

Class interval	5 - 14	15 - 24	25 - 34	35 - 44	45 - 54	55 - 64
Frequency	3	6	18	20	10	3

13. Let us find the mean of obtaining marks of girl students if their cumulative frequencies are as follows.

Class interval (marks)	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50
Number of girl students	5	9	17	29	45

14. Let us find the mean of the obtaining marks of 64 students from the table given below.

Class interval (marks)	1 - 4	4 - 9	9 - 16	16 - 17
Students.	6	12	26	20

Yesterday we, the ten friends had spent a lot of time at the bank of the lake of Satragachi with fun and merriment. At the time of singing, reciting and gossiping, we observed the movement of migratory birds. My friends Mita and Sajal had arranged for some food also. We contributed money according to our capacity for the arrangement of food. Each of ten of us paid ` 10, ` 15, ` 14, ` 20, ` 12, ` 18, ` 12, ` 24, ` 100 and ` 200.



16 But let us see by calculating how much average money we paid.

$$\text{We paid average} = \frac{10+15+14+20+12+18+22+24+100+200}{10}$$

$$= \text{` } \boxed{}$$

Arithmetic mean is one measure of central tendency.



17 But we see that arithmetic mean is ` 435. But it is not near to central position. If the value of given data one or two are excessively large and small than the other values, then the arithmetic mean does not remain near to central position in many cases.

18 Which method shall we apply for this data?

In this cases we use **Median** as measure of central tendency.

19 But what do we mean by median?

Median is one of the measures of central tendency. The ungrouped data is arranged in ascending (or descending) order of magnitude and the arithmetic mean of the two middle values is the median of data.

We understood. arranging the given data in ascending order of magnitude, we have.

10, 12, 14, 15, 18, 20, 22, 24, 100, 200

20 But after arranging the given data in ascending order of magnitude, we get two middle terms 18 and 20.



How shall we get Median?

Let, $x_1, x_2, x_3, \dots, x_n$ be the n values of a variable x arranged in ascending order of magnitude we have

$x_1, x_2, x_3, \dots, x_n$ [Where, $x_1 < x_2 < x_3 < \dots < x_n$]

[Few may be equal among the values] |

(i) If n is odd, Median is the value of $\left(\frac{n+1}{2}\right)$ th observation.

$$\therefore \text{Median} = x_{\frac{n+1}{2}}, \text{ when } n \text{ is odd.}$$

(ii) If n is even the median is the mean of the $\left(\frac{n}{2}\right)$ th and the $\left(\frac{n}{2}+1\right)$ th observations.

$$\therefore \text{Median} = \frac{x_{\frac{n}{2}} + x_{\frac{n}{2}+1}}{2}, \text{ When } n \text{ is even.}$$

We understand that, 10, 12, 14, 15, 18, 20, 22, 24, 100, 200 in this case number of terms = 10

∴ In this case median is the mean of the values of $\left(\frac{10}{2}\right)$ th term and $\left(\frac{10}{2} + 1\right)$ th term.

$$\therefore \text{Required median} = \frac{18+20}{2} = 19$$

Application : 14. I have written weight of some of my friends in table below, let us find their median.

32kg, 30kg, 40kg, 65kg, 54kg, 38kg, 36kg, 45kg, 50kg, 52kg, 40kg

Arranging their weights in ascending order we get.

30 kg., 32 kg., 36 kg., 38 kg., 40 kg., 40 kg., 45 kg., 50 kg., 52 kg., 54 kg., 65 kg.

Here, $n = 11$ i.e. n is odd.

$$\text{Median of weight} = \left(\frac{n+1}{2}\right)\text{th value} = \left(\frac{11+1}{2}\right)\text{th value} = 6\text{th value} = 40 \text{ kg.}$$

Application : 15. We have recorded the number of days my friends were present in school in this month. These are 20 days, 25 days, 10 days, 18 days, 21 days, 18 days, 16 days, 22 days. Let us find the median of the number of days my friends present in school in this month.

We arrange the present days of my friends in this month in the school in ascending order, we get 10 days, 16 days, 18 days, 18 days, 20 days, 21 days, 22 days, 25 days

$$\begin{aligned} \therefore \text{Required median} &= \frac{1}{2} \left\{ \left(\frac{8}{2}\right)\text{th value} + \left(\frac{8}{2} + 1\right)\text{th value} \right\} \\ &= \frac{1}{2} \{ 4\text{th value} + 5\text{th value} \} \\ &= \frac{1}{2} [18 \text{ days} + 20 \text{ days}] = 19 \text{ days} \end{aligned}$$



Application : 16. The points obtained by two kabadi teams in different matches are given below. Let us determine their median

(i) 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

(ii) 6, 7, 8, 8, 9, 10, 15, 15, 16, 17, 19, 25 [Let me do it myself]

Application : 17. There are 100 balls in Niyamat chacha's shop having the diameters of 6 different types of lengths. The frequency distribution of length of diameters of those balls are as follows.

Diameter (mm.)	44	45	46	47	48	49
Frequency (number of balls)	12	15	23	20	15	15

Let us find the median of diameter of 100 balls.

Here, $n = 100$ i.e. n is even.

$$\begin{aligned} \therefore \text{Median} &= \text{The mean of } \left(\frac{n}{2}\right)\text{th and } \left(\frac{n}{2} + 1\right)\text{th observation.} \\ &= \text{The mean of 50th and 51th observation} \end{aligned}$$



At first we make the observation, less than type cumulative frequency table of given frequency distribution table for finding the median of length of diameter of the ball.

Length of diameter (mm)	Number of balls
up to 44	12
up to 45	$12+15=27$
up to 46	$27+23=50$
up to 47	$50+20=70$
up to 48	$70+15=85$
up to 49	$85+15=100$



∴ In the given frequency distribution table we get cumulative frequency column by adding.

Length of diameter (mm.)	Frequency	Cumulative frequency (less than type)
44	12	12
45	15	27
46	23	50
47	20	70
48	15	85
49	16	$100=n$

We see from the above table, 50 th observation is 46

and 51th observation is 47

$$\therefore \text{Median} = \frac{46+47}{2} = 46.5$$

∴ We understand that the length of diameter about 50% balls less than 46.5mm. and the length of diameter about 50% balls is greater than 46.5 mm. of Niyamat Chacha's shoap.



Application : 18. Let us find the median from the following frequency distribution table.

Variable (x_i)	25	26	27	28	29	30	31	32	33
Frequency (f_i)	4	2	4	7	6	5	5	4	2

At first we make the less than type cumulative frequency distribution table of given frequency distribution table.

Variable(x_i)	Frequency (f_i)	Cumulative frequency (less than type)
25	4	4
26	2	6
27	4	10
28	7	17
29	6	23
30	5	28
31	5	33
32	4	37
33	2	$39=n$

Here, $n = 39$ i.e. n is odd.

$$\begin{aligned}\therefore \text{Median} &= \left(\frac{n+1}{2}\right) \text{ the observation.} \\ &= \frac{39+1}{2} \text{ the observation.} = 20 \text{ the observation.}\end{aligned}$$



We see from the above table, the value of observation from 18th to 23th are same i.e. 29.

$\therefore$ Required Median = 20 th term = 29

Application: 19. Let us find the median from the following distribution table. [Let me do it myself]

Variable (x_i)	1	2	3	4	5	6
Frequency (f_i)	8	12	16	19	21	24

An I.Q. test has been done of 100 students of our school. The following is the distribution of I.Q.

I.Q. (x_i)	75 - 85	85 - 95	95 - 105	105 - 115	115 - 125	125 - 135
Number of students (f_i)	9	12	27	30	17	5

Let us find the median of I.Q. of the above distribution of 100 students.

We see that the given frequency distribution table is a grouped frequency distribution table.

We make a less than type cumulative frequency distribution table of this grouped frequency distribution a table.

I.Q. (x_i)	cumulative frequency (less than type)
75 or greater than 75 but less than 85	9
less than 95	$9 + 12 = 21$
less than 105	$21 + 27 = 48$
less than 115	$48 + 30 = 78$
less than 125	$78 + 17 = 95$
less than 135	$95 + 5 = 100$



$\therefore$ We get less than type cumulative frequency distribution table of given frequency distribution table.

I.Q. (x_i)	Frequency (f_i)	Cumulative frequency (less than type)
75 - 85	9	9
85 - 95	12	21
95 - 105	27	48
105 - 115	30	78
115 - 125	17	95
125 - 135	5	$100 = n$

Here, total frequency = $n = 100$

At first we locate the class whose cumulative frequency is equal to $\frac{n}{2}$ or greater than $\frac{n}{2}$ in the above frequency distribution table.

$$\frac{n}{2} = \frac{100}{2} = 50$$

∴ The cumulative frequency of class 105-115 is 78 which is just greater than 50.



21 But what this class of 105-115 of the above frequency distribution table is called?

Class 105-115 is the **Median Class**.

But how shall we find the median class of data from given frequency distribution table?

After finding the median class, we use the following formula for calculating the Median.

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h$$

Where, l = lower limit of median class. n = number of observation, f = frequency of median class. cf = cumulative frequency of class preceding the median class. h = class size of the median class.

We have understood, the required Median = $l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h$ [Here, $l = 105$, $n = 100$,

$$cf = 48, f = 30, h = 10]$$

$$\begin{aligned} &= 105 + \left[\frac{\frac{100}{2} - 48}{30} \right] \times 10 \\ &= 105 + \frac{50 - 48}{30} \times 10 \\ &= 105 + 0.66 = 105.66 \end{aligned}$$

We have understood, one half of the I.Q of the students is less than 105.66 and the other half of the I.Q of the students is more than 105.66.

Application : 20. Some marks awarded on handicrafts work of 50 girl students of our village.

I have written awarded marks in the table below.

Marks (x_i)	0-4	5-9	10-14	15-19	20-24	25-29	30-34	35-39
Number of girl students (f_i)	4	5	7	8	7	5	6	3

Let us find the median of the above frequency distribution table.

Here the frequency table of class interval are in exclusive form.

∴ At first we transform it into inclusive form and let us write cumulative frequency less than type.

Marks (x_i)	-0.5-4.5	4.5-9.5	9.5-14.5	14.5-19.5	19.5-24.5	24.5-29.5	29.5-34.5	34.5-39.5
Number of students (f_i)	4	5	7	8	7	5	6	3
Cumulative frequency (less than type)	4	9	16	24	31	36	42	45 = n

Here, $n=45$, $\therefore \frac{n}{2} = 22.5$

The cumulative frequency just greater than 22.5 is 24 and the corresponding class (14.5-19.5)

$\therefore$ Median class = (14.5 - 19.5)

$\therefore$ Required Median = $l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h$ [Here, $l = 14.5$, $n=45$, $f=8$, $cf=16$, $h=5$]

$$= 14.5 + \left[\frac{\frac{45}{2} - 16}{8} \right] \times 5$$

$$= 14.5 + 4.06$$

$$= 18.56$$



$\therefore$ So, one half of the students obtained marks less than 18.56 and the other half of the students obtained marks more than 18.56.

Application : 21. Let us see the following frequency distribution table and let us find the median.:

[Let me do it myself]

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	8	10	24	16	15	7

Application : 22. Let us find the median from frequency distribution table given below.

Marks obtained	less than 10	less than 20	less than 30	less than 40	less than 50	less than 60
Number of students.	8	15	29	42	60	70

We get frequency distribution table from given cumulative frequency distribution table.

Marks obtained (x_i)	Frequency (f_i) [number of students]	Cumulative frequency (less than type)
less than 10	8	8
10 - 20	7	15
20 - 30	14	29
30 - 40	13	42
40 - 50	18	60
50 - 60	10	70 = n

$n = 70$, $\therefore \frac{n}{2} = 35$

The corresponding class of the cumulative frequency just greater than 35 is (30-40).

$\therefore$ Required median class is (30-40)



$\therefore$ Required median = $l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h$ [Here, $l = 30$, $n = 70$, $cf = \square$, $f = 13$, $h = 10$]

$$= \square \text{ [Let me do it myself]}$$

$\therefore$ Required median is 34.6

Application: 23. If the median of the following data is 525, let us find the value of x and y, when the sum of frequencies is 100

Class interval	Frequency
0 - 100	2
100 - 200	5
200 - 300	x
300 - 400	12
400 - 500	17
500 - 600	20
600 - 700	y
700 - 800	9
800 - 900	7
900 - 1000	4



We make a cumulative frequency (less than type) distribution table of given data.

Class interval	Frequency (f_i)	Cumulative frequency (less than type)
0 - 100	2	2
100 - 200	5	7
200 - 300	x	7 + x
300 - 400	12	19 + x
400 - 500	17	36 + x
500 - 600	20	56 + x
600 - 700	y	56 + x + y
700 - 800	9	65 + x + y
800 - 900	7	72 + x + y
900 - 1000	4	76 + x + y = n

Given $n = 100$, So, $76 + x + y = 100 \therefore x + y = 24$ _____ (i)

Again, Median = 525

$\therefore$ Median class is (500 - 600)

$$\therefore 525 = l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h \quad [l = 500, n = 100, cf = 36 + x, f = 20, h = 100]$$

$$\text{or, } 525 = 500 + \left[\frac{50 - (36 + x)}{20} \right] \times 100$$

$$\text{or, } 525 - 500 = (14 - x)5$$

$$\text{or, } 5(14 - x) = 25$$

$$\text{or, } 14 - x = 5 \therefore x = 9$$

We get from (i), $x + y = 24$

$$\text{or, } y = 24 - x = 24 - 9 = 15 \therefore x = 9 \text{ and } y = 15$$



Application : 24. If the median of the following data is 28.5, and total frequency is 100, let us find the value of x and y. [Let me do it myself]

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	5	x	20	15	y	5

Application : 25. Let us find the median of the following data.

Marks obtained	0-10	10-30	30-60	60-70	70-90
Number of students	15	25	30	4	10



Marks obtained (x_i)	Number of students	Cumulative frequency (less than type)
0 - 10	15	15
10 - 30	25	$15 + 25 = 40$
30 - 60	30	$40 + 30 = 70$
60 - 70	4	$70 + 4 = 74$
70 - 90	10	$74 + 10 = 84 = n$

$$n = 84, \therefore \frac{n}{2} = 42$$

(30-60) is the class whose cumulative frequency 70 is just greater than 42.



But here we see that, all the class lengths are not equal. How much shall we take the length of class?

$\therefore$ h = length of median class, if the lengths of all class interval are not equal, we shall take the length of median class.

$$\therefore \text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h \quad [\because l = 30, \frac{n}{2} = 42, cf = 40, f = 30, h = 30]$$

$$= 30 + \left[\frac{42 - 40}{30} \right] \times 30 = 30 + 2 = 32 \text{ marks.}$$

So, half of the students scored less than 32 marks, and the other half of the students scored more than 32 marks.

Let us workout 26.2

- The selling prices (in `) of Madhu uncle's shop per day in the last week were 107, 201, 92, 52, 113, 75, 195; let us find the median of the selling prices.
- If the age (in year) of some animals are 6, 10, 5, 4, 9, 11, 20, 18; let us find the median of ages.
- The marks obtained by 14 students are 42, 51, 56, 45, 62, 59, 50, 52, 55, 64, 45, 54, 58, 60; let us find the median of the obtaining marks.
- Today the scores of our cricket match of our locality are

7	9	10	11	11	8	7	7	10	6	9
7	9	9	6	6	8	8	9	8	7	8

Let us find the Median of scores in our cricket match.

- Let us find the median of the weights from the following frequency distribution table of 70 students.

Weight (kg)	43	44	45	46	47	48	49	50
Number of students	4	6	8	14	12	10	11	5

- Let us find the median of length of diameter from the following frequency distribution table of length of diameter of pipe.

Length of diameter (mm)	18	19	20	21	22	23	24	25
Frequency	3	4	10	15	25	13	6	4

7. Let us find the median.

x	0	1	2	3	4	5	6
f	7	44	35	16	9	4	1

8. The frequency distribution table of expenditures of tiffin allowances of 40 students is given below.

Expenditure for tiffin (in `)	35-40	40-45	45-50	50-55	55-60	60-65	65-70
Students	3	5	6	9	7	8	2

Let us find the median of tiffin allowance.

9. Let us find the median of heights of students from the table below.

Height(cm)	135-140	140-145	145-150	150-155	155-160	160-165	165-170
No. of students	6	10	19	22	20	16	7

10. Let us find the median of data from the following frequency distribution table.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	7	10	15	10	8	5

11. Let us find the median of given data.

Class interval	5-10	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	5	6	15	10	5	4	3	2

12. Let us find the median of given data.

Class interval	1-5	6-10	11-15	16-20	21-25	26-30	31-35
Frequency	2	3	6	7	5	4	3

13. Let us find the median of given data.

Class interval	51-60	61-70	71-80	81-90	91-100	101-110
Frequency	4	10	15	20	15	4

14. Let us find the median of given data.

Marks	Number of Students
less than 10	12
less than 20	22
less than 30	40
less than 40	60
less than 50	72
less than 60	87
less than 70	102
less than 80	111
less than 90	120

15. If the median of the following data is 32, let us determine the values of x and y when the sum of the frequencies is 100

Class interval	Frequency
0-10	10
10-20	x
20-30	25
30-40	30
40-50	y
50-60	10

To day many leaves of tree of different shapes have been collected and brought in the science practical room of our school. Our elder sisters and brothers will observe those leaves.



We, some of our friends, have decided, that by measuring the lengths of leaves of tree of the practical room, we shall form a frequency distribution table. The frequency distribution table of lengths of leaves of tree is :

Length of leaves of tree	120-130	130-140	140-150	150-160	160-170	170-180	180-190
Number of leaves	4	6	10	14	6	6	4

We form a less than type cumulative frequency distribution table of lengths of leaves of tree

Length of leaves of tree mm. (aprox) (x)	Cumulative frequency (less than type)
less than 120	0
less than 130	4
less than 140	10
less than 150	20
less than 160	34
less than 170	40
less than 180	46
less than 190	50

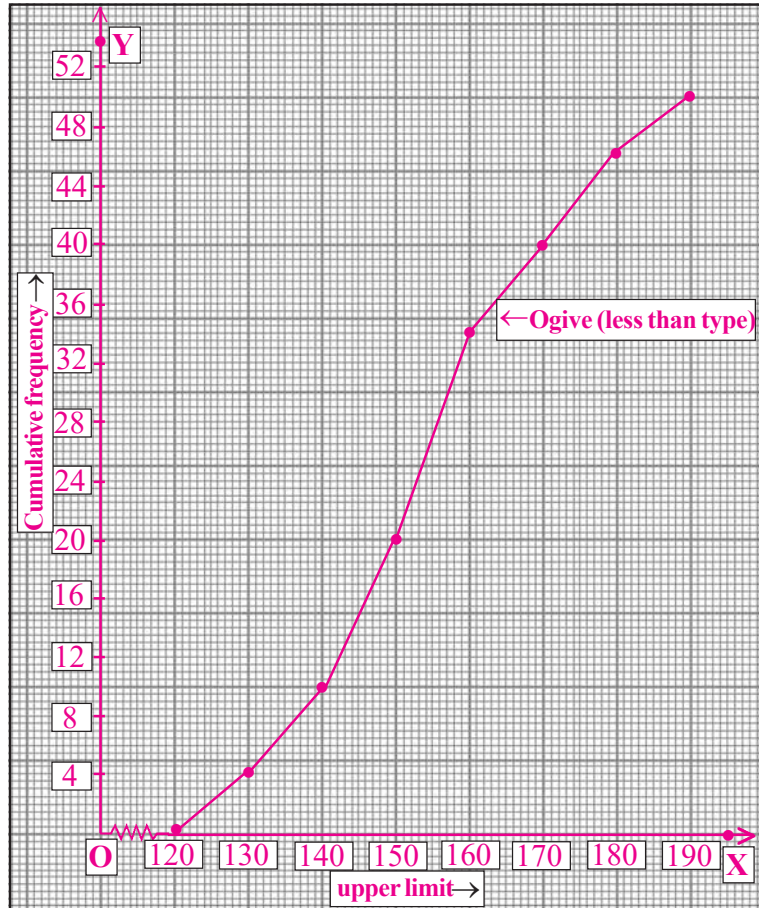
We see from above cumulative frequency table 130, 140, 150, 190 are the upper class limit of classes respectively.

But is it possible to draw the graph of above less then type cumulative frequency distribution table?

We mark the upper limits of the class intervals on the horizontal axis (x – axis) and their corresponding

cumulative frequencies on the vertical axis (y-axis) choosing a convenient scale.

The scale of measures of both the axes can be taken separately if it is required. We understand, if we plot the points (130, 4), (140, 10), (150, 20), (160, 34), (170, 40), (180, 46), (190, 50) of the above cumulative frequency table on a graph paper. Now we observe that the frequency less than 120 is 0. So as a special point we plot (120, 0) on the graph paper and join the points by free hand without scale, let us see what we shall get on a graph paper. We take the scale, along x-axis 1 small div. = 1 mm., along y-axis, 2 small div = 1 leaf of tree then by plotting the points and joining the points we shall get a curve line.



22 What the graph thus we get of cumulative frequency is called?

The graph we get is called cumulative frequency curve or an Ogive of less than type.



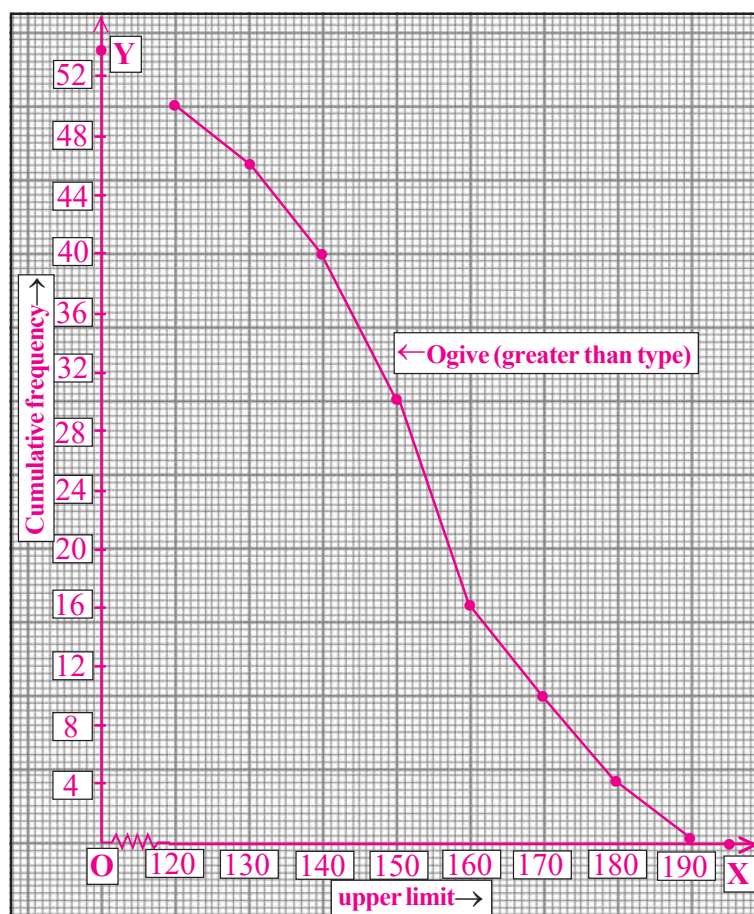
Let us find the cumulative frequency of greater than type from the frequency distribution of lengths of leaves of tree.

Length of leaves of tree (mm) (approx) (x)	Cumulative frequency of (greter than type)
more than 120	50
more than 130	46
more than 140	40
more than 150	30
more than 160	16
more than 170	10
more than 180	4
more than 190	0

We see from cumulative frequency distribution table of greater than type that 120, 130, 140, 150, 160, 170 and 180 are respectively the lower limits of class intervals.

But how shall we draw the graph of cumulative frequency of greater than type?

We mark the lower limit of each class interval on the horizontal axis i.e. along x-axis and their corresponding cumulative frequencies on the vertical axis i.e. along y-axis by choosing a convenient scale. [Here also the measures of scale of both axes may be taken separately]



Let us try to draw the graph of the above cumulative frequency of greater than type and let us see what I shall get.

On a graph paper taking the scale along x-axis, 1 small div = 1 mm. and along y-axis, 2 small div = 1 leaf of tree, by plotting the points (120, 50), (130, 46), (140, 40), (150, 30), (160, 16), (170, 10) and (180, 4). Here we observe that the frequency more than 190 is 0. So as a special point we plot (190, 0) on the graph paper and joining we get a curved line.

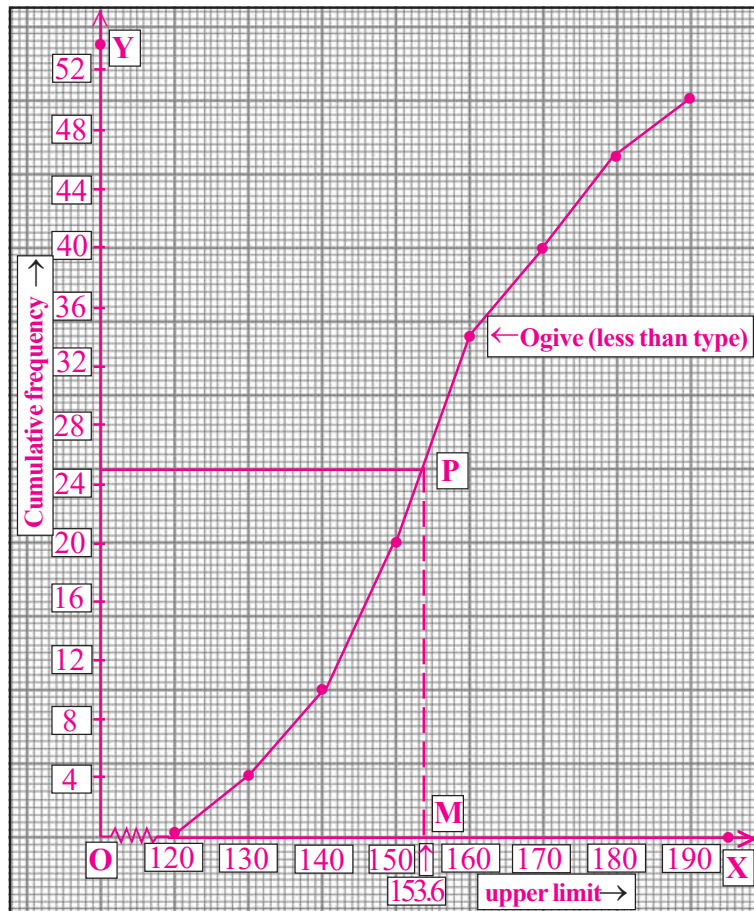
The curve which we get is the **cumulative frequency curve** (of greater than type) or Ogive (of greater than type)

We see that, both the Ogives denote the frequency distribution of same data.

But, let us see, is it possible to obtain the median from any one Ogive .



Length of leaf of tree (mm.) (approx) (x)	Cumulative frequency (less than type)
less than 120	0
less than 130	4
less than 140	10
less than 150	20
less than 160	34
less than 170	40
less than 180	46
less than 190	50



Here, number of leaves $n = 50$

$$\therefore \frac{n}{2} = 25$$

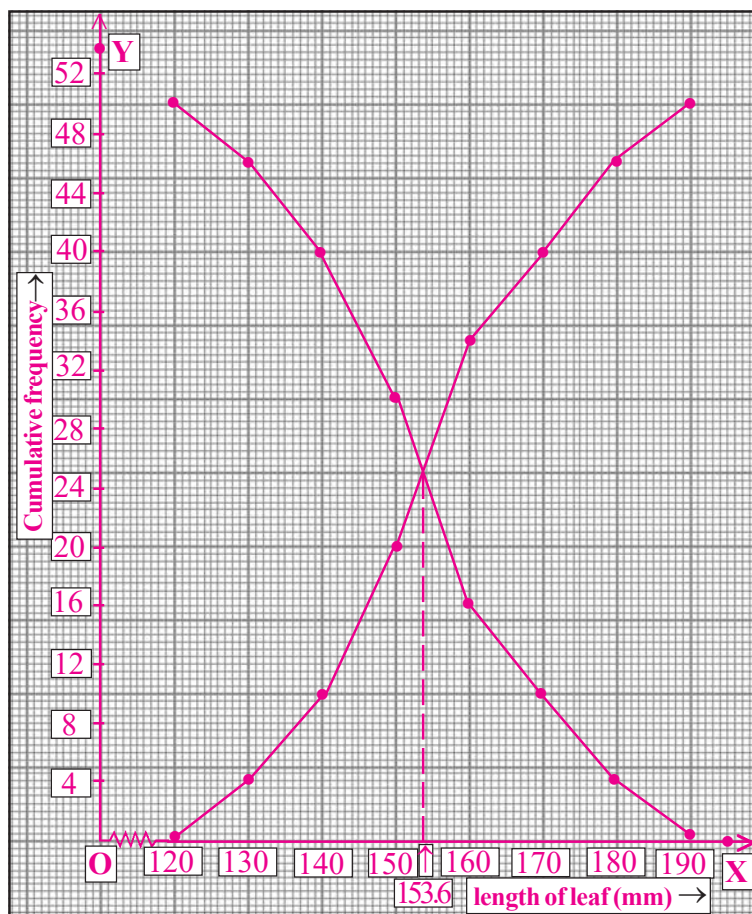
$\therefore$ After finding the point (0,25) on the y-axis of the Ogive, we draw a line from this point parallel to the x-axis which intersects the Ogive at the point P. From the point P we draw a perpendicular PM on the x-axis which intersects x-axis at the point M.

The value of abscissa of this point M is the required **median**.

$$\therefore \text{Median} = 153.6 \text{ mm.}$$

My friend Anowara has drawn two types of Ogives of lengths of leaves of tree on her copy in same graph paper.

But how shall we get the value of the median of two types of Ogives drawn by Anowara.



If the perpendiculars drawn on x-axis from the intersecting point of both Ogives, the abscissa of the point of intersection of this perpendicular with the x-axis will be the median.

We see that the two types of Ogives intersect each other at the point P.

We draw perpendicular PM from the point P on the x-axis which intersects x-axis at the point M.

The abscissa of the point M is 153.6 mm (aprox) is the median.



Application 26 : Let us draw Ogive of the following frequency distribution and let us find median from this Ogive.

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	7	10	23	50	6	4

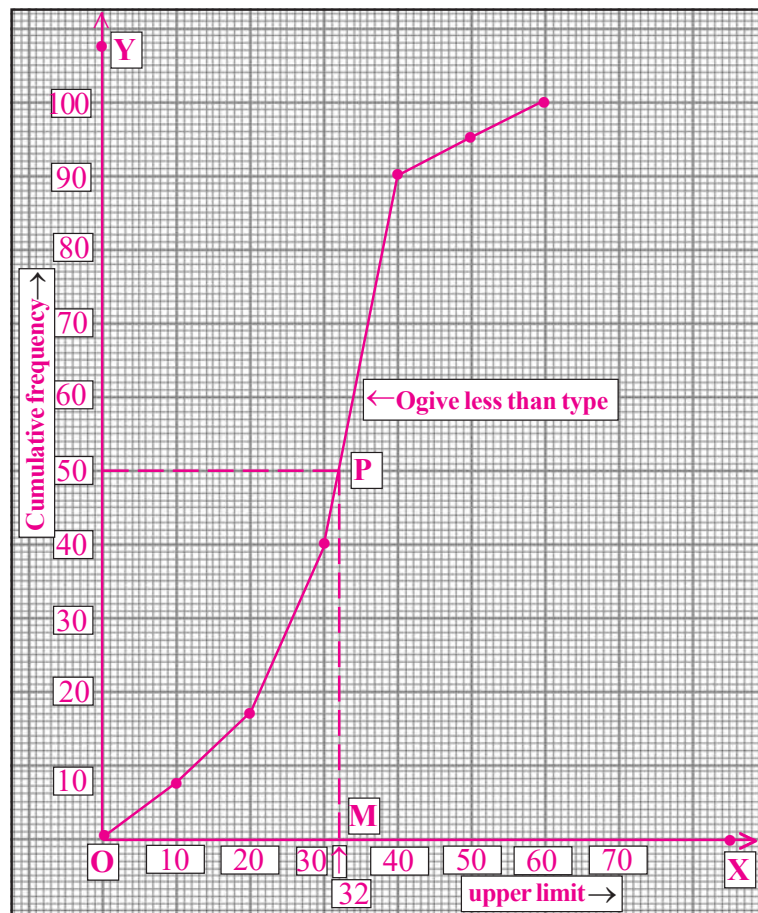
At first we form a less than type cumulative frequency distribution table.

Class interval	less than 10	less than 20	less than 30	less than 40	less than 50	less than 60
Cumulative frequency (less than type)	7	17	40	90	96	100

On graph paper we take along x-axis, 1 small div. = 1 unit, along y-axis 1 small div. = 1 unit, we plot the points (10, 7), (20, 17), (30, 40), (40, 90) (50, 96) and (60, 100) and as a special point we plot (0, 0) and join and we get Ogive (less than type). Here total frequency

$$(n) = 100$$

$$\therefore \frac{n}{2} = 50$$



$\therefore$ We take a point (0,50) on y-axis and draw a line parallel to x-axis through this point meeting the Ogive at the point P.

We draw perpendicular PM on x-axis from the point P, intersecting x-axis at the point M.

We see that the co-ordinates of the point P are (32,0)

$\therefore$ From Ogive, we get the median = 32

Alternatively : We make a table of less than type cumulative frequency and greater than type cumulative frequency of the given frequency distribution table.

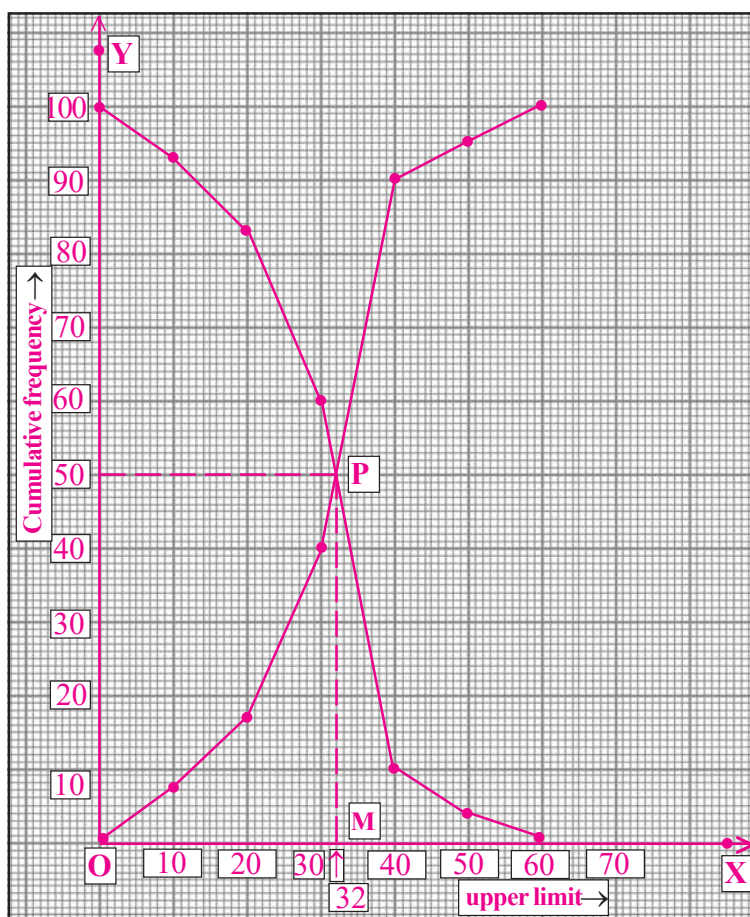
Class	Frequency
0-10	7
10-20	10
20-30	23
30-40	50
40-50	6
50-60	4

Class	Greater than type cumulative frequency
more than 0	100
more than 10	93
more than 20	83
more than 30	60
more than 40	10
more than 50	4
more than 60	0

Class	Cumulative frequency (less than type)
less than 0	0
less than 10	7
less than 20	17
less than 30	40
less than 40	90
less than 50	96
less than 60	100



Assuming on graph paper along x-axis 1 small div = 1 unit, along y axis 1 small div = 1 unit we draw greater than type Ogive and less than type Ogive. For drawing greater than type Ogive plotting the points (0, 100), (10, 93), (20, 83), (30, 60), (40, 10), (50, 4) and special point (60, 0) on graph paper we join them.



The greater than type Ogive and less than type Ogive intersects each other at the point P. We draw perpendicular PM on x-axis from the point P, which intersects the x-axis at the point M.

We see, that the co-ordinates of the point M are (32, 0)

∴ Required median = 32



Let us verify it by finding out median with the help of the formula. **[Let me do it myself]**

Application 27 : The following frequency distribution table shows the marks awarded to 200 examinees in the medical entrance examination.

Marks awarded	400-450	450-500	500-550	550-600	600-650	650-700	700-750	750-800
Number of examinees	20	30	28	26	24	22	18	32

Let us draw Ogive and let us find the median with the help of it. Let us verify, by finding out with the help of the formula.

At first we make a less than type cumulative frequency distribution table from the given data.

Marks obtained	less than 400	less than 450	less than 500	less than 550	less than 600	less than 650	less than 700	less than 750	less than 800
(Less than type) Cumulative frequency	0	20	50	78	104	128	150	168	200

On graph paper assuming along x-axis 1 small div = 5 marks and along y-axis 1 small div = 2 examinees, we plot the points (450, 20), (500, 50), (550, 78), (600, 104), (650, 128), (700, 150), (750, 168), (800, 200) and special point (400, 0) on graph paper and join them, we get less than type ogive.

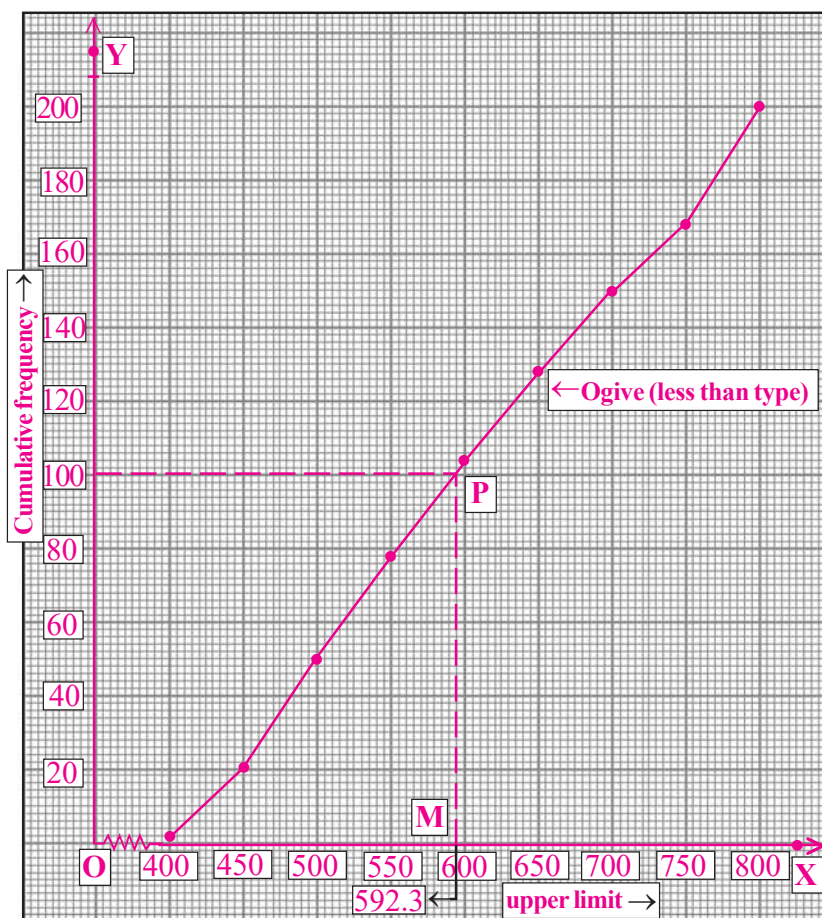
Here total examinees =

$$(n) = 200$$

$$\therefore \frac{n}{2} = 100$$

$\therefore$ We draw a line parallel to x-axis through the point (0, 100), intersected

the Ogive at the point P. We draw perpendicular PM on the line OX from the point P intersecting x-axis at the point M. The co-ordinates of the point M are (592.3, 0). $\therefore$ Median = 592.3



Alternatively I draw less than type Ogive and greater than type Ogive and let us find the median with the help of their intersecting point. [Let me do it myself]

With the help of the formula,



$$\begin{aligned}\text{Median} &= l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times h \\ &= 550 + \frac{100-78}{26} \times 50 \\ &= 550 + \frac{22 \times 50}{26} \\ &= 550 + \frac{550}{13} = 550 + 42.3 = 592.3\end{aligned}$$

Application 28 : Let us draw less than type Ogive and greater than type Ogive and let us find the median of the following frequency distribution table. [Let me do it myself]

Class	50-55	55-60	60-65	65-70	70-75	75-80	80-85
Frequency	2	8	12	24	34	16	4

Let us work out

26.3

1. The following distribution table shows the daily profit (in `) of 100 shops of our village.

Profit of each shop (in `)	0-50	50-100	100-150	150-200	200-250	250-300
Number of Shops	10	16	28	22	18	6

Making cumulative frequency table (less than type) of given frequency table, let us draw Ogive on graph paper.

2. The following data shows the weight of 35 students of the class of Nivedita.

Weight (kg.)	less than 38	less than 40	less than 42	less than 44	less than 46	less than 48	less than 50	less than 52
Number of students	0	4	6	9	12	28	32	35

Making frequency (less than type) distribution table, let us draw Ogive on graph paper and hence let us find the median from the graph. Let us find the median by using formula and verify it.

3.

Class	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	4	10	15	8	3	5

Making cumulative frequency (greater than type) distribution table of given data, let us draw Ogive on graph paper.

4.

Class	100-120	120-140	140-160	160-180	180-200
Frequency	12	14	8	6	10

Drawing less than type Ogive and greater than type Ogive of given data along same axes, on the graph paper, let us find the median from the graph.

Today is 5th September i.e. Teachers' day. Like every year, this year also we will celebrate the day in a special manner. This year we, the students of class X have decided to go to the students of lower classes and take class in presence of the teachers. We will spend the day joyfully through different games, song, dance, recitation, drawing, acting and quiz etc.



Dividing 36 students of class VI into three equal groups we told each student of the first group to write the answers of ten funny riddles.

23 Let us see the number of correct answers written by each of the 12 students of the first group.

The number of correct answers given by each of the 12 students of the first group is

4, 6, 5, 4, 7, 2, 3, 4, 2, 4, 5, 4

We form a frequency distribution table of data of above correct answers

Number of correct answers (x_i)	2	3	4	5	6	7
No. of students (f_i)	2	1	5	2	1	1



We see from the above table, that number of students given '4 correct answers' is maximum i.e. frequency of 4 is maximum.

24 What, in above frequency distribution table, the observations which occurred maximum number of times, i.e. the observation which is of maximum frequency is called?

In a data the observation having the maximum frequency is called the mode of this data.

Mode is another measure of central tendency. It is easy to calculate. Mode is generally used in the shop of garments, shoes etc. because garments and shoes are made according to the demand of majority of people. e.g. in a case of special company the demand of size 5 of girls' shoe is more i.e. we understand from the above table that 4 is the mode of the given data.

We supplied another 10 funny riddle questions to the second group of 12 students of class VI. The number of correct answers are

2, 4, 3, 5, 2, 5, 8, 2, 5, 9, 5, 2

Making frequency distribution table of above data, let us find the mode.

No. of correct answers (x_i)	2	3	4	5	8	9
No. of students (f_i)	4	1	1	4	1	1

We see from the above table, 4 students have given 2 correct answers and 4 students have given 5 correct answers.

25 But in this case what will be the value of the mode of the given data.

In this case, the number of modes is 2.

∴ The values of mode are 2 and 5.



26 But what the mode of data having 2 values is called?

When a distribution has one mode, it is called unimodal. It is bimodal when it has two modes. The number of correct answers of riddles given by the 3rd group of 12 students of class VI are
4, 5, 6, 7, 6, 7, 5, 4, 5, 4, 6, 7

I make a frequency distribution table of the above data and let us find the mode

No. of correct answers (x_i)	4	5	6	7
No. of students (f_i)	3	3	3	3



In above table 4, 5, 6 and 7 each occurs 3 times.

$\therefore$ The values of mode of given data are 4, 5, 6 and 7. Since all numbers are mode therefore there is no mode of the given data. i.e the data is tetramodal. So a distribution is called multimodal when the number of maximum observations of a given data is equal.

Application : 29. At the end of the day we, the ten friends, have collected subscriptions among ourselves for having some food. We gave

` 16, ` 15, ` 11, ` 12, ` 15, ` 10, ` 15, ` 10, ` 15, ` 10

Arranging our subscriptions (in `) in ascending order, we get,

10, 10, 10, 11, 12, 15, 15, 15, 15, 16

$\therefore$ In given data the maximum occurrence (i.e maximum frequency) is **[Let me do it myself]**

$\therefore$ The mode of given data is 15.

Application : 30. Let us find the mode of the data given below.

(i) 2, 3, 5, 6, 2, 4, 2, 8, 9, 4, 5, 4, 7, 4, 4

(ii) 11, 27, 18, 26, 13, 12, 9, 15, 4, 9

(iii) 102, 104, 117, 102, 118, 104, 120, 104, 122, 102

(iv) 5, 9, 18, 27, 15, 5, 8, 10, 16, 5, 7, 5

(i) Let us write the numbers of given data in ascending order in magnitude.

2, 2, 2, 3, 4, 4, 4, 4, 4, 5, 5, 6, 7, 8, 9

We see, 4 occurs maximum number of times.

$\therefore$ The mode of data = 4

Let us find the mode of data (ii), (iii) and (iv) **[Let me do it myself]**



Application : 31. The following frequency distribution table shows the number of present days of 100 students of our class in the last month.

No. of present days	6-10	10-14	14-18	18-22	22-26
No. of students	8	28	34	18	12



Let us find the mode of present days of frequency distribution table.

27 But in the case of a grouped frequency distribution it is not possible to determine the mode by looking at the frequencies. Here how shall we determine the mode ?

In the case of grouped frequency distribution table at first we shall locate a class, frequency of which is maximum.

28 What the class having maximum frequency is called?

A class with the maximum frequency is called the Modal class.

We understand that, the modal class of above frequency distribution table is (14-18)

29 But how shall we get mode of data with the help of determining modal class ?

The mode is a value inside the modal class, and we shall get required mode with the help of formula given below.

$$\text{Required mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

Here, l = lower limit of the modal class.

h = length of the modal class.

f_1 = Frequency of the modal class.

f_0 = Frequency of the class preceding the modal class .

f_2 = Frequency of the class succeeding the modal class.

We understand that, the mode of present days of 100 students in the last month

$$= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h \quad [\text{Here, } l = 14, f_1 = 34, f_0 = 28, f_2 = 18, h = 4]$$

$$= 14 + \left(\frac{34 - 28}{2 \times 34 - 28 - 18} \right) \times 4$$

$$= 14 + \frac{6 \times 4}{22} = 14 + \frac{12}{11} = 15.09 \text{ days [approx]}$$

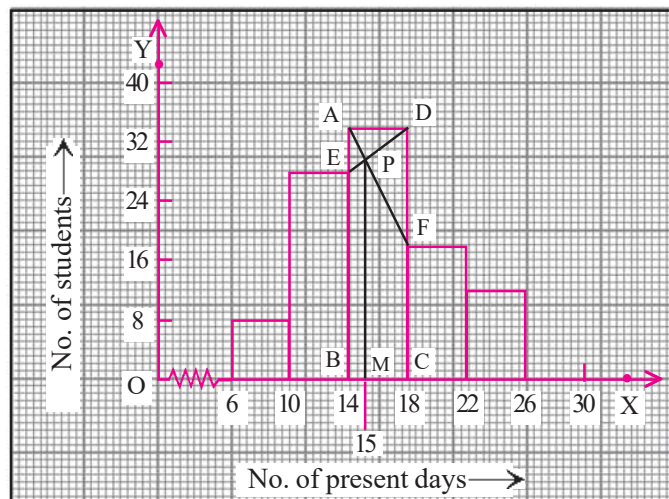
But let us see, how shall we get mode of above frequency distribution table with the help of histogram.

[This is not included in evaluation]

At first we express the above frequency distribution through the histogram. The modal class is a rectangle ABCD. (in picture beside). We join AF and ED which intersect each other at the point P. A perpendicular PM drawn on x-axis from the point P, intersects the x-axis at the point M. The co-ordinates of M are (15,0)

∴ Required mode = 15 (approx)

∴ We get approximately same mode from frequency distribution of given data with the help of histogram.



Application : 32. Let us find the mode from the following frequency distribution table.

Class interval	4-8	8-12	12-16	16-20	20-24	24-28	28-32
Frequency	9	10	18	14	10	6	3

The maximum frequency is 18 in the given frequency distribution table.

∴ Modal class is (12-16)

∴ Mode of the data = $l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$ [Here, $l = 12$, $h = 4$, $f_1 = 18$, $f_0 = 10$, $f_2 = 14$]

$$= 12 + \left(\frac{18 - 10}{2 \times 18 - 10 - 14} \right) \times 4$$

$$= 12 + \frac{8}{36 - 24} \times 4$$

$$= 12 + \frac{8 \times 4}{12} = 12 + \frac{8}{3} = 14.66 \text{ [Approx]}$$



∴ Required mode = 14.66 [Approx]

Application : 33. Let us find the mode of frequency distribution table given below.

[Let me do it myself]

Class interval	3-6	6-9	9-12	12-15	15-18	18-21	21-24
Frequency	2	6	12	24	21	12	3

Application : 34. The given frequency distribution table shows the age of 200 members of development committee of our locality.

Age (Years)	20-30	30-40	40-50	50-60	60-70
No. of members	30	38	70	42	20

Let us find the arithmetic mean, median and mode of the above frequency distribution table

Age (years)	Class mark (x_i)	Frequency (No. of members f_i)	Cumulative frequency (less than type)	$u_i = \frac{x_i - 45}{10}$	$f_i u_i$
20 - 30	25	30	30	-2	-60
30 - 40	35	38	68	-1	-38
40 - 50	45	70	138	0	0
50 - 60	55	42	180	1	42
60 - 70	65	20	200	2	40
Total		$\sum f_i = 200$			$\sum f_i u_i = -16$

Let assumed mean = 45

Required Arithmetic Mean = $A + \frac{\sum f_i u_i}{\sum f_i} \times h$

$$= \left\{ 45 + \left(\frac{-16}{200} \right) \times 10 \right\} \text{ years}$$

$$= \left(45 - \frac{4}{5} \right) \text{ years}$$

$$= 44.2 \text{ years}$$



$$n = 200 \quad \therefore \frac{n}{2} = 100$$

$\therefore$ The median lies in the class interval (40-50)

$$\therefore \text{Required median} = l + \left(\frac{\frac{n}{2} - cf}{f} \right) \times h$$

$$= 40 + \frac{\frac{200}{2} - 68}{70} \times 10 = 40 + \frac{32}{7} = 40 + 4.57 = 44.57 \text{ years [Approx]}$$

We see from above frequency distribution table

Modal class is (40-50)

$$\therefore \text{Required mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 40 + \frac{70 - 38}{2 \times 70 - 38 - 42} \times 10 = 40 + \frac{32}{60} \times 10 = 45.33 \text{ years [Approx]}$$

Application : 35. Let us find the mode from the following frequency distribution table .

Value	less than 10	less than 20	less than 30	less than 40	less than 50	less than 60	less than 70	less than 80
Frequency	4	16	40	76	96	112	120	125

At first we make a frequency distribution table from the given cumulative frequency distribution table.

Class limit (value)	Frequency
less than 10	4
10 - 20	$16 - 4 = 12$
20 - 30	$40 - 16 = 24$
30 - 40	$76 - 40 = 36$
40 - 50	$96 - 76 = 20$
50 - 60	$112 - 96 = 16$
60 - 70	$120 - 112 = 8$
70 - 80	$125 - 120 = 5$

The modal class is (30-40)

$$\therefore \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 30 + \frac{36 - 24}{2 \times 36 - 24 - 20} \times 10$$

$$= 30 + \frac{12}{72 - 44} \times 10$$

$$= 30 + \frac{12}{28} \times 10 = 30 + \frac{30}{7}$$

$$= 30 + 4.29$$

$$= 34.29 \text{ [Approx]}$$

[$\therefore$ h is the length of modal class, i.e at the time of finding mode the class length of all classes may not be equal, so in this case of less than 10, the class (0-10) is not taken, it is written as less than 10]

It is not included for evaluation.

There is an empirical relationship among three measures of central tendency,

$$\text{Arithmetic mean} - \text{Mode} = 3 (\text{Arithmetic mean} - \text{Median})$$

$$\therefore \text{Mode} = 3 \times \text{median} - 2 \times \text{arithmetic mean}$$



Let us work out 26.4

- Paid amounts of a day of our 16 friends for going to school and other expenditures are 15, 16, 17, 18, 17, 19, 17, 15, 15, 10, 17, 16, 15, 16, 18, 11
Let us find the mode of paid amounts of a day of our friends.
- Heights (cm) of some students of our class are given below.
131, 130, 130, 132, 131, 133, 131, 134, 131, 132, 132, 131, 133,
130, 132, 130, 133, 135, 131, 135, 131, 135, 130, 132, 135, 134, 133
Let us find the mode of heights of students.
- Let us find the mode of data given below:
 - 8, 5, 4, 6, 7, 4, 4, 3, 5, 4, 5, 4, 4, 5, 5, 4, 3,
3, 5, 4, 6, 5, 4, 5, 4, 5, 4, 2, 3, 4
 - 15, 11, 10, 8, 15, 18, 17, 15, 10, 19, 10, 11,
10, 8, 19, 15, 10, 18, 15, 3, 16, 14, 17, 2
- The frequency distribution table shows the selling prices of shoes of a special company of shoe shop of our village.

Size (x_i)	2	3	4	5	6	7	8	9
Frequency (f_i)	3	4	5	3	5	4	3	2

Let us find the mode of the above frequency distribution table.

- Let us find the mode from the following frequency distribution table of ages of examinees of an entrance examination.

Age (year)	16-18	18-20	20-22	22-24	24-26
No. of examinees	45	75	38	22	20

- Let us see the frequency distribution table of obtaining marks in a periodical examination of 80 students in a class and let us find the mode

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of students	2	6	10	16	22	11	8	5

- Let us find the mode of frequency distribution table given below.

Class interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35
Frequency	5	12	18	28	17	12	8

- Let us find the mode of frequency distribution table given below.

Class interval	45-54	55-64	65-74	75-84	85-94	95-104
Frequency	8	13	19	32	12	6

[Hints : In the case of finding mode, we take lower limit of modal class, so we first correct the class limit in inclusive form of class limit.]

9. **Very short answer type question :**(A) **M.C.Q :**

- (i) The median of a given frequency distribution is found graphically with the help of (a) Frequency curve (b) Frequency polygon (c) Histogram (d) Ogive
- (ii) If the mean of numbers 6, 7, x, 8, y, 14 is 9, then (a) $x+y=21$ (b) $x+y=19$ (c) $x-y=21$ (d) $x-y=19$
- (iii) If 35 is removed from the data 30, 34, 35, 36, 37, 38, 39, 40 then the median increases by (a) 2 (b) 1.5 (c) 1 (d) 0.5
- (iv) If the mode of data 16, 15, 17, 16, 15, x, 19, 17, 14 is 15, then the value of x is (a) 15 (b) 16 (c) 17 (d) 19
- (v) If the median of arranging the ascending order of data 8, 9, 12, 17, $x+2$, $x+4$, 30, 31, 34, 39, is 15, then the value of x is (a) 22 (b) 12 (c) 20 (d) 24

(B) **Let us write true or false of the following statements :**

- (i) Value of mode of data 2, 3, 9, 10, 9, 3, 9 is 10
- (ii) Median of data 3, 14, 18, 20, 5 is 18

(C) **Let us fill in the blanks:**

- (i) Mean, median, mode are the measures of _____.
- (ii) If mean of $x_1, x_2, x_3, \dots, x_n$ is $\bar{x}$ mean of $ax_1, ax_2, ax_3, \dots, ax_n$ is _____.
- (iii) At the time of finding arithmetic mean, the lengths of all classes are _____.

10. **Short answer type question :**

(i)

Class	65-85	85-105	105-125	125-145	145-165	165-185	185-205
Frequency	4	15	3	20	14	7	14

Let us find the difference between upper class limit in median class and lower class limit of modal class of the above frequency distribution table.

- (ii) The following frequency distribution shows the time taken to complete 100 metre hurdle race of 150 athletics :

Time (seconds)	13.8-14	14-14.2	14.2-14.4	14.4-14.6	14.6-14.8	14.8-15
No. of athletics	2	4	5	71	48	20

Let us find the difference between the upper class limit of modal class and lower class limit of modal class.

- (iii) The mean of a frequency distribution is 8.1, if $\sum f_i x_i = 132 + 5k$ and $\sum f_i = 20$, let us find the value of k.
- (iv) If $u_i = \frac{x_i - 25}{10}$, $\sum f_i u_i = 20$ and $\sum f_i = 100$, let us find the value of $\bar{x}$.

(v)

Marks	less than 10	less than 20	less than 30	less than 40	less than 50	less than 60
No. of students	3	12	27	57	75	80

Let us write the modal class from the above frequency distribution table.

(LET'S MATCH)

Chapter - 1

Let me do it myself :

Application : 1. (iv) does not form quadratic equation (v) is not a quadratic equation

4. $x^2 + 2x - 24 = 0$, breadth = x metre.

Let us work out 1.1 [Page : 4]

1. (i), (iii) 2. (i) 3. x^3 4. (i) 2 (ii) 1 (iii) $2x^2 + 9 = 0$ (iv) $6x^2 + 13x + 8 = 0$; 6, 13, 8

5. (i) $x^2 + x - 42 = 0$; where x is a number.

(ii) $x^2 - 36 = 0$; where $2x - 1$ and $2x + 1$ are consecutive odd numbers.

(iii) $x^2 + x - 156 = 0$; where x and $x + 1$ are consecutive numbers.

6. (i) $x^2 + 3x - 108 = 0$; where, width = x metre.

(ii) $x^2 + 4x - 320 = 0$; where price of x kg sugar is ₹ 80.

(iii) $x^2 + 5x - 750 = 0$; where at first velocity of train x length.

(iv) $x^2 + 100x - 33600 = 0$; where cost price ₹ x.

(v) $5x^2 - 21x - 20 = 0$; where velocity of boat x length at steam water.

(vi) $x^2 - x - 6 = 0$; where Mohim can do work in x how.

(vii) $x^2 - 5x + 6 = 0$; where 10th place number is x

(viii) $2x^2 + 85x - 225 = 0$; where, the width of road x metre.

Let me do it myself :

Application : 8. -4 13. $\frac{a+b}{ab} \oplus \frac{2}{a+b}$ 15. $x = -9$ and $x = 9$

Let us work out -1.2 [Page : 10]

1. (i) no (ii) no (iii) no (iv) yes 2. (i) $-\frac{1}{6}$ (ii) $2a^2$ 3. $a = 3$, $b = -6$

4. (i) $y = -6$ and $y = 6$ (ii) $x = 3$ and $x = -3$ (iii) $x = -6$ and $x = 22$

(iv) $x = -3$ and $x = 3$ (v) $x = -6$ and $x = 6$ (vi) $x = -\frac{1}{5}$ and $x = \frac{1}{2}$

(vii) $x = \frac{1}{2}$ and $x = 2$ (viii) $x = 0$ and $x = \frac{2}{3}$ (ix) $x = -9$ and $x = 7$

(x) $x = -4$ and $x = 3$ (xi) $x = -1$ and $x = 1$ (xii) $x = 0$ and $x = 1$

(xiii) $x = -7$ and $x = 0$ (xiv) $x = -\frac{68}{9}$ and $x = 3$ (xv) $x = 6$ and $x = 9$

(xvi) $x = -a$ and $x = -b$ (xvii) $x = 2a$ and $x = 3a$ (xviii) $x = -(a+b)$ and $x = a$

(xix) $x = -2$ and $x = 7$ (xx) $x = 0$ and $x = \frac{2ab - bc - ca}{a + b - 2c}$

(xxi) $x = \sqrt{3}$ and $x = 2$

Let me do it myself :

Application : 18. 8 or 9

Let us work out -1.3 [Page : 13]

1. 6 and 9 2. 15 metre 3. 3 4. 20 km/h 5. length = 50 m and breadth = 40 metre
 6. 5 and 9 7. 20 minute. and 25 minute. 8. 6 days 9. ` 30
 10. (A)(i) b (ii) c (iii) b (iv) a (v) b (B)(i) false (ii) false
 (C)(i) linear (ii) $x^2 - 2x + 1 = 0$ (iii) 0 and 6
 11. (i) $a = -4$ (ii) 3 (iii) 1 (iv) $\frac{1}{x} - x = \frac{9}{20}$; where x is real fraction (v) $a=1, b=12$

Let me do it myself :

Application : 26. 11 and 13 **30.** (v) $-\frac{1}{2}$ (vi) 1 and $\frac{7}{2}$ (vii) 1 and $\sqrt{2}$

Let us work out -1.4 [Page : 22]

1. (i) No, so, it is not a quadratic equation with are variable (ii) quadratic equation with one variable, (iii) -2
 2. (i) -4 and $\frac{1}{3}$ (ii) -1 and -1 (iii) $\frac{1}{8}$ and $\frac{3}{2}$ (iv) -1 and $\frac{1}{3}$ (v) no real root.
 (vi) $-\frac{1}{2}$ and $\frac{3}{5}$ (vii) no real root (viii) $\frac{3-\sqrt{2}}{5}$ and $\frac{3+\sqrt{2}}{5}$ (ix) $-\frac{7}{8}$ and $\frac{3}{2}$
 3. (i) 10 cm., 24 cm. and 26 cm. (ii) 3 (iii) 9 metre/second (iv) 9 metre (v) 10
 (vi) 18 (vii) $1\frac{3}{5}$ km./h (viii) 60 length/h (ix) ` 80

Let me do it myself :

Application : 31. (iv) Real and unequal **33.** $\frac{25}{2}$ **36.** (ii) $\text{sum} = \frac{9}{4}$, $\text{product} = -25$ **38.** 3
41. $\frac{3abc-b^3}{c^3}$

Let us work out -1.5 [Page : 29]

1. (i) Real and unequal. (ii) real and equal (iii) There are no real root. (iv) There are no real root.
 2. (i) $-4 \notin 4$ (ii) $\frac{25}{24}$ (iii) 16 (iv) $\frac{9}{8}$ (v) 2 and $\frac{1}{2}$ (vi) $-\frac{1}{2}$ and 1
 3. (i) $x^2 - 6x + 8 = 0$ (ii) $x^2 + 7x + 12 = 0$ (iii) $x^2 + x - 12 = 0$ (iv) $x^2 - 2x - 15 = 0$
 4. -3 **8.** (i) $\frac{34}{25}$ (ii) $-\frac{98}{125}$ (iii) $\frac{2}{3}$ (iv) $\frac{98}{75}$ **10.** $x^2 + px + 1 = 0$ **11.** $x^2 + x + 1 = 0$
12. (A)(i) c (ii) c (iii) a (iv) d (v) c (B)(i) False (ii) True (C)(i) 2:3 (ii) a (iii) 0
13. (i) $x^2 - 14x + 24 = 0$ (ii) $-\frac{2}{3}$ (iii) ± 8 (iv) $-\frac{1}{2}$ (v) 12

Chapter - 2**Let me do it myself :**

Application : 2. ` 3081 **4.** ` 1200 **5.** ` 1500, ` 1700

10. ` 93.75 ` 593.75; ` 0.01 ` 146.01; ` 456.50, ` 5021.50

13. ` 400, ` 7300 **16.** ` 840, ` 10,000 **19.** $3\frac{1}{2}$ years, 2 years **23.** (i) 5 (ii) 2

26. ` 400, ` 8 **30.** ` 90,000, ` 97,500 **32.** ` 6087.50

34. ` 3,00,000, ` 2,00,000, ` 1,20,000

Let us work out -2 [Page : 46]

1. ` 7200 2. ` 48 3. ` 1060 4. ` 3584 5. ` 8000 6. ` 37800
 7. $16\frac{2}{3}$ Years 8. $6\frac{1}{4}$ 9. ` 170 10.  $6\frac{1}{4}$  11.  $3\frac{1}{2}$  Years 12. 5 Years 13. ` 5000, ` 6
 14. 3:4 15. $9\frac{1}{2}$ 16. In bank ` 60000 and in post office ` 40000
 17. ` 6000 and ` 4000 18. ` 20837.50 19. 9 years 20. ` 80,000 and ` 60,000
 21. (A) (i) c (ii) c (iii) b (iv) c (v) b (B) (i) True (ii) false (C) (i)
 (ii) $\frac{\text{prt}}{100}$ (iii) $12\frac{1}{2}\%$ 22. (i) 16 (ii) ` 24000 (iii) 8 (iv)  $6\frac{2}{3}$  (v) ` 240

Chapter - 3

Let us work out -3.1 [Page : 52]

1. AO, CO, PO, QO 2. (i) infinite (ii) Diameter (iii) sector (iv) centre (v) equal (vi) radius
 (vii) big 4. (i) True (ii) True (iii) True (iv) True (v) False (vi) False (vii) False (viii) True

Let me do it myself :

Application : 6. 30 cm. 8. 7 cm.

Let us work out -3.2 [Page : 65]

1. 3 cm. 2. 24 cm. 3. 5.8 cm. 4. 3 cm. 5. 30 cm. 6. 3.25 cm. 10. 13 cm.
 16. (A) (i) c (ii) b (iii) b (iv) a (v) b (B) (i) false (ii) true (iii) false
 (C) (i) 1:1 (ii) passes through centre 17. (i) 16 cm. (ii) 9.6 cm. (iii) 6 cm. (iv) 8 cm.
 (v) 10 cm.

Chapter - 4

Let me do it myself :

Application : 2. 1440 sq cm. 4. 864 sq cm. 7. 9 metre 10. 640 sq cm., length of diagonal
 $8\sqrt{6}$ cm. 18. 9 hours.

Let us work out -4 [Page : 74]

2. Planes are — ABCD, EFGH, ADFE, BCGH, ABHE, DCGF,
 Sides are — AB, BC, CD, DA, EF, FG, GH, HE, AE, DF, BH, CG.
 Vertices are — A, B, C, D, E, F, G, H.
 3. $5\sqrt{2}$ metre. 4. 512 cubic cm. 5. 150 metre. 6. 96 sq cm. 7. 125 cubic cm.
 8. 216 cubic cm. 9. 6 cm. 10. 8:1 11. 352 sq cm. 12. 200 gm 13. 75
 14. 75 cm. 15. 18 ltr 16. 26.25 cubic cm., 1.5 cm. 17. 2 dcm. 18. $1\frac{5}{9}$ metre.
 19. 2 metre 20. length 20 dcm. and breadth 15 dcm. 21. 1 metre, 8 dcm.
 22. 5 dcm. ` 775 50 paise.
 23. 7 hrs 24 minute, 5 dcm. 24. (A) (i) b (ii) b (iii) d (iv) c (v) d (B) (i) false
 (ii) true (C) (i) 4 (ii) $\sqrt{2}$ (iii) cube 25. (i) 6 (ii) 3.5 (iii) 125 (iv) 6 cm.
 (v) 96 sq m.

Chapter - 5

Let me do it myself :

Application : 4. $y:x$ 6. 14 and 21 9. $1:p^2q^2r^2$ 11. $24:35$ 13. $20:36:45$ 17. $50:19$
19. $247:778$ 21. 9

Let us work out -5.1 [Page : 82]

1. (i) $2:9$, Ratio of lesser inequalities (ii) $3:5$, Ratio of lesser inequalities (iii) $1:1$, Ratio of equality (iv) $20:1$, Ratio of greater inequalities
2. (i) $1000p:q$ (ii) convert in to same unit (iii) Ratio of lesser inequalities (iv) $1:abc$ (v) $y^2:xz$ (vi) $1:xyz$
3. (i) $36:77$ (ii) $1:1$ 4. (i) $48:49$ (ii) $16:35$ (iii) $3:4:6$ (iv) $8:12:21$
5. (i) $9:10$ (iii) 86 6. (i) $40:19$ (ii) $9:4$ 8. (i) $\frac{8}{5}$ (ii) $\frac{bm-an}{m-n}$ (iii) 1

Let me do it myself :

Application : 25. Proportional 26. Proportional 29. Proportional 31. 84 33. 20
34. $5:10::6:12$; $6:5::12:10$; $10:5::12:6$ 36. 3 40. Rs. 48 42. $16pq^3$ 44. 1.5
46. xyz

Let us work out -5.2 [Page : 87]

1. (i) 12 (ii) 75 2. (i) $\frac{3}{20}$ (ii) 22.8 kg. (iii) $\frac{yz^3}{x}$ (iv) p^3+q^3 3. (i) 20 (ii) 1.5
(iii) $\frac{q^2r^2}{p^3}$ (iv) $(x+y)^4(x-y)^2$ 4. (i) 20 (ii) 4.5 (iii) x^2y^2 (iv) x^2-y^2
5. Inverse proportion of each other 6. 2 7. 162 8. 3 9. 6 10. $\frac{qr-ps}{q+r-p-s}$

Let us work out -5.3 [Page : 97]

6. (vi) 2 12. (A) (i) a (ii) c (iii) c (iv) c (v) d (B) (i) True (ii) True
(C) (i) 4 (ii) $12\frac{1}{2}$ (iii) $\frac{6}{5}$ 13. (i) 11 (ii) $\frac{7}{8}$ (iii) $17:7$ (iv) $x=18, y=8$ (v) $3:2$

Chapter - 6

Let me do it myself :

Application : 3. ` 1102.50 5. ` 1576.25 7. ` 102.50 10. ` 2300
12. ` 34719.24
14. ` 3200 19. ` 6000, 7% 21. 8% 23. 2 years

Let us work out -6.1 [Page : 112]

1. ` 5886.13(approx.) 2. ` 6298.56 3. ` 247.20 4. ` 8850.87 5. ` 90405
6. ` 15000 7. ` 8,000 8. ` 25,000 9. ` 25,000 10. ` 67.50 11. ` 76.25

12. ` 16,000 13. ` 30,000 14. ` 933.60 15. ` 565 16. ` 1250, 4%
 17. ` 70,000, 6% 18. ` 489.60 19. ` 480.57 20. 8% 21. 2 Years
 22. 10% 23. 2 Years 24. 3 Years 25. ` 252.20, ` 1852.20

Let me do it myself :

Application : 27. ` 102900 29. 506250

Let us work out - 6.2 [Page : 117]

1. 10609 2. 84896640 3. ` 72900 4. 3200 5. 12000 6. 532.4 qu. 7. 20 metre.
 8. ` 3429.50 9. 58.32 kg. 10. 3000 11. 1536 12. ` 131220 13. 75
 14. 33750 15. 40960 16. (A) (i) c (ii) b (iii) b (iv) d (v) a (B) (i) false (ii) true
 (C) (i) equal (ii) uniform rate (iii) decrease 17. (i) 5 (ii) $2n$ (iii) ` 6000 (iv) $v(1 - \frac{r}{100})^{-n}$
 (v) $p(1 + \frac{r}{100})^{-n}$

Chapter - 7

Let me do it myself : 7.1 1. cyclic, $\widehat{AQB}$ 2. ST, $\angle SLT$, $\widehat{SNT}$

Let me do it myself : 7.2 3. (a) (i) angle at the centre (ii) $\angle APB$ (iii) $\widehat{AQB}$, cyclic (iv) $\widehat{APB}$, not (v) $\angle ADB$, $\widehat{AQB}$

(b) (i) $\angle ACB$, $\angle ADB$ (ii) $\angle ACB$, $\angle ADB$, not

Let me do it myself :

Application : 5. (i) $x=60$ (ii) $y=120$

Let us work out -7.1 [Page : 126]

1. 65° , 25° 2. 125° 3. 144° 4. 55° , 110° 5. 160° 14. (A) (i) d (ii) a (iii) b (iv) c (v) c
 (B) (i) false (ii) true (C) (i) half (ii) equal (iii) 120° 15. (i) $x=120$, $y=80$ (ii) 40°
 (iii) 120° (iv) 5 cm. (v) 40°

Let me do it myself :

Application : 8. (ii) $x=60$

Let us work out -7.2 [Page : 132]

1. 40° , 60° , 180° 2. $\angle BAC = 45^\circ$, $\angle APC = 45^\circ$ 12. (A) (i) d (ii) d (iii) c
 (iv) b (v) d (B) (i) true (ii) false (C) (i) equal (ii) concyclic (iii) equal
 13. (i) $\angle CDE = 30^\circ$ (ii) 78° (iii) 80° (iv) 64° (v) 4 cm.

Let us work out -7.3 [Page : 138]

1. (ii) 10. (A) (i) d (ii) c (iii) d (iv) c (v) c (B) (i) false (ii) true (C) (i) Right angle
 (ii) obtuse angle (iii) angular angle 11. (i) 4 cm. (ii) 2.5 cm. (iii) $2r$ cm. (iv) 30° (v) 60°

Chapter - 8

Let me do it myself :

Application : 2. 616 sq m. 4. 187 square dkm 6. 6.4 dcm, ` 28.05

11. 2.541 cubic dcm, 12.705 kg. 13. 188.65 cubic dcm. 16. 360 cm.

Let us work out -8 [Page : 146]

1. (i) 3 (ii) 1;2 3. 18 cm. 4. 1232 cubic dcm.; ` 1100 5. 325 gm. 6. 4.05 dcm.

7. 6.468 ltr. 8. 2310 kl.ltr 9. 3.2 cm. 10. 14 m., 6 m. 11. 180 cm.

12. 5.6 dcm., 25 dcm. 13. ` 5829.12 14. 5 dcm. 15. 7 dcm.

16. 15840 ltr.; 21120 ltr. 17. (i) 277.2 cubic dcm. (ii) 55.44 cubic dcm. 18. 168

19. (A) (i) c (ii) b (iii) b (iv) c (v) a (B) (i) false (ii) true (C) (i) 1b (ii) 5 (iii) 4

20. (i) 7 m. (ii) 2 (iii) 396 cubic cm. (iv) 9:32 (v) $62\frac{1}{2}\%$ decreases.

Chapter - 9

Let me do it myself :

Application : 4. $\sqrt{48}, \sqrt{27}, \sqrt{75}$ 6. $3\sqrt{2}, -\sqrt{2}$, pure surd 9. $4\sqrt{3}$ 11. $21+5\sqrt{5}$

Let us work out - 9.1 [Page : 153]

1. (i) $5\sqrt{7}$ (ii) $8\sqrt{7}$ (iii) $6\sqrt{3}$ (iv) $5\sqrt{5}$ (v) $5\sqrt{119}$ 5. $5\sqrt{3}-\sqrt{2}$

6. (a) $\sqrt{5}-\sqrt{3}$ (b) $4-2\sqrt{3}$ (c) $4+2\sqrt{3}+\sqrt{5}+\sqrt{7}$ (d) $15-4\sqrt{11}$ (e) $\sqrt{7}-10$
(f) $5+\sqrt{3}$ and $5-\sqrt{3}$ [another answer accepted]

Let me do it myself :

Application : 16. $6\sqrt{2}+2\sqrt{14}-2\sqrt{10}-3-\sqrt{7}+\sqrt{5}$ 18. $\sqrt{7}, k\sqrt{7}$ (k is a nonzero rational number)

20. $7+\sqrt{3}, -7-\sqrt{3}$ 22. $\sqrt{15}-\sqrt{3}, \sqrt{3}-\sqrt{15}$ 25. (i) $2-\sqrt{3}$ (ii) $5+\sqrt{2}$

(iii) $-\sqrt{5}-7$ (iv) $6-\sqrt{11}$ (v) $-\sqrt{5}$ 28. $\frac{4\sqrt{15}}{15}, \frac{\sqrt{42}}{2}$ 30. (i) $14+8\sqrt{3}$
(ii) $4+\sqrt{15}$

Let us work out -9.2 [Page : 157]

1. (a) 3 (b) $\sqrt{2}$ (c) $15\sqrt{15}$ (d) 3 (e) $\pm\sqrt{23}$ 2. (a) $7\sqrt{2}$ (b) 12 (c) 15

(d) $3\sqrt{2}+\sqrt{10}$ (e) -1 (f) $24+8\sqrt{6}+2\sqrt{15}+3\sqrt{10}$ (g) 4 3. (a) 5 (b) $\sqrt{2}$ (c) $\sqrt{3}$

(d) $2-\sqrt{5}$ (e) $\sqrt{14}$ (f) $2+\sqrt{3}$ 4. $9+4\sqrt{5}; -2+\sqrt{7}$

5. (i) $-\sqrt{5}+\sqrt{2}; \sqrt{5}-\sqrt{2}$ (ii) $13-\sqrt{6}; -13+\sqrt{6}$ (iii) $-\sqrt{8}-3; \sqrt{8}+3$

(iv) $\sqrt{17}+\sqrt{15}; -\sqrt{17}-\sqrt{15}$ 6. (i) $\sqrt{2}+\sqrt{3}$ (ii) $\frac{1}{5}(\sqrt{10}-\sqrt{5}+\sqrt{30})$ (iii) $2+\sqrt{3}$

(iv) $\frac{1}{4}(3\sqrt{7}+\sqrt{35}+3\sqrt{3}+\sqrt{15})$ (v) $\frac{1}{19}(6\sqrt{10}+2\sqrt{5}+3\sqrt{2}+1)$ (vi) $5+2\sqrt{6}$

7. (i) $6+\sqrt{10}-3\sqrt{2}-\sqrt{5}$ (ii) $-(4+\sqrt{6})$ (iii) $\sqrt{3}$

8. (i) $-\frac{(5+2\sqrt{5})}{2}$ (ii) $\frac{1}{2}(9\sqrt{2}-8\sqrt{5})$

Let me do it myself :

Application : 32. $\frac{2}{5}$ 34. $6\sqrt{3}-4\sqrt{2}$ 36. $2\sqrt{2}, 18\sqrt{3}, 4\sqrt{6}$

Let us work out - 9.3 [Page : 161]

1. (a) (i) 1 (ii) 0 2. (a) $\frac{\sqrt{6}}{3}$ (b) 0 (c) $4\sqrt{6}$ (d) 0 3. 0

4. (i) $2\sqrt{6}$ (ii) $2\sqrt{7}$ (iii) 26 (iv) $50\sqrt{7}$ 5. $(4x^2-2), x=\pm 2$

6. (i) $1\frac{1}{3}$ (ii) $\frac{5\sqrt{5}}{27}$ (iii) $1\frac{5}{8}$ (iv) $\frac{9\sqrt{5}}{20}$

7. (a) (i) $2\sqrt{3}$ (ii) 14 (iii) $30\sqrt{3}$ (iv) 2 (b) 37 9. $\sqrt{5}+\sqrt{3}$

10. (A) (i) c (ii) a (iii) b (iv) a (v) b (B) (i) true (ii) false (C) (i) irrational (ii) $-\sqrt{3}-5$ (iii) irrational

11. (i) 6 (ii) $\sqrt{10}+\sqrt{8}$ (iii) $a+\sqrt{b} \ncong a-\sqrt{b}$ [where a is rational number & is purely quadratic $\sqrt{b}$] (iv) $2\sqrt{2}$ (v) 1

Chapter - 10

Let us work out -10 [Page : 169]

1. $\angle SQP = 65^\circ, \angle RSP = 70^\circ$ 2. 35° 3. $40^\circ, 25^\circ$ 17. (A) (i) c (ii) c (iii) d (iv) c (v) d (B) (i) false (ii) true (C) (i) cyclic (ii) rectangular (iii) cyclic

18. (i) $x = 60$ (ii) $\angle QBC = 100^\circ, \angle BCP = 96^\circ$ (iii) $\angle DPC = 40^\circ, \angle BQC = 20^\circ$ (iv) $\angle BED = 80^\circ$ (v) $\angle BED = 20^\circ$

Chapter - 11

Let me do it myself : 11 1. chord 2. centre 3. equal 4. radius

Chapter - 12

Let me do it myself :

Application : 2. ` 1540 4. $1437\frac{1}{3}$ cubic cm. 7. 10.5 cm. 10. 1:4

Let us work out -12 [Page : 183]

1. 1386 sq cm. 2. 2.8 cm. 3. $179\frac{2}{3}$ cubic cm. 4. $11498\frac{2}{3}$ cubic cm. 5. 1:9 6. 9 cm.

7. 38.808 cubic cm.; 55.44 sq cm. 8. 8 9. 6 cm. 10. ` 970 20 paise 11. 36:25

12. $(2\sqrt{2}-1):1$ 13. 2460.92 sq cm. 14. 512 15. (A) (i) a (ii) b (iii) c (iv) a (v) a

(B) (i) False (ii) True (C) (i) Sphere (ii) 1 (iii) 12

16. (i) 4.5 unit (ii) 6 cm. (iii) $2:\sqrt{3}$ (iv) 36π (v) 125

Chapter - 13

Let me do it myself :

Application : 4. $\frac{9}{2}$, $y = \frac{9}{2}\sqrt{x}$, $x = \frac{256}{81}$ 5. yes, $x \propto \frac{1}{y}$ 10. $1\frac{1}{2}$ hrs. 19. $5\frac{2}{5}$ day 27. 8

Let us work out -13 [Page : 195]

1. $A \propto B$, $\frac{5}{2}$ 2. $x \propto \frac{1}{y}$ 3. (i) 168 km. (ii) 6 (iii) 10 4. (i) $x=4$ (ii) 2
 (iii) $x = \frac{3y}{10z}$, 9 5. (iii) $a \propto \frac{1}{d}$ (iv) Product of three variational constant is 1 8. 5 day
 9. 3 metre 10. $y = 2x - \frac{3}{x}$ 12. Rs. 16250 13. 5:9 14. 15
 16. (A) (i) d (ii) d (iii) b (iv) b (v) a (B) (i) false (ii) true (C) (i) z (ii) y^n (iii) x
 17. (i) $y^2 = 4ax$ (ii) 1 (iii) direct variation (v) 8:27

Chapter - 14

Let me do it myself :

Application :

2. Sulekha gets ` 3500, Joynal gets ` 3150, and Shibu gets ` 4500 5. ` 14100, ` 15900, ` 13200 8. Monisha gives ` 2300 and Rajat gives ` 4600.

10. Nivedita gets ` 3250 and Uma gets ` 2925.

Let us work out -14 [Page : 202]

1. My profit part of ` 6,300 profit of mala ` 10,500
 2. Priyam, Supriya and Bulu will pay loss amount ` 900, ` 600 and ` 1500 respectively.
 3. Profit part of Masud ` 5000 and profit part of Shova ` 7500.
 4. Three friends will pay ` 500 ` 600 ` 700.
 5. Profit part of Dipu ` 4700, profit part of Rabeya ` 4400 and profit part of Megha ` 5300
 6. Three friends will get profit ` 2240, ` 2800 ` 3360 respectively.
 7. Cash in hand of three friends are ` 3600 & 4800 ` 3000; 6:8:5
 8. Three friends will get ` 10560, ` 10274, ` 8426
 9. Pradipbabu gets ` 12956 and Aminabibi gets ` 14760.
 10. Niyamatchacha will get profit ` 10000, Karabididi will get profit ` 9000.
 11. Srikanta gets ` 15660, Saifuddin gets ` 19575, Peter gets ` 3915.
 12. After 5 months
 13. ` 36500, ` 35000 and ` 39500.
 14. ` 6800 15. Puja gets ` 2550, Uttam ` 2610, Meher gets ` 300.
 16. (A)(i)b (ii)a (iii)c (iv)a (v)a (B)(i)False (ii)False (C)(i)Two(ii)simple(iii)compound
 17. (i) ` 1500 (ii) 8:12:15 (iii) ` 4000 (iv) ` 480 (v) 4000

Chapter - 15

Let us work out -15.1 [Page : 209]

1. 48° Let me do it myself : 21.1 1. 12 cm. 2. 5 cm. 3. $\angle APB = 60^\circ$, $\angle APO = 30^\circ$

Let us work out -15.2 [Page : 218]

1. 15 cm. 2. 60° 11. (A) (i) a (ii) d (iii) a (iv) d (v) c (B) (i) True (ii) False
 (C) (i) bysector (ii) 4 (iii) transverse 12. (i) 30° (ii) 14 (iii) 2 cm. (iv) 4 cm. (v) 12 cm.

Chapter - 16

Let me do it myself : Application : 2. $9\frac{3}{7}$ Sq m. 4. $1178\frac{4}{7}$ Sq cm. 6. 1100 Sq cm.
 8. 25 cm. 10. 577.5 cubic dcm. 14. 6:25

Let us work out -16 [Page : 227]

1. 1131.43 Sq cm.(approx.), 1838.57 Sq cm.(approx.) 2. (i) 1.232 m^3 (ii) 1617 m^3
 3. $1571\frac{3}{7}$ Sq cm., $2828\frac{4}{7}$ Sq cm., $6285\frac{5}{7}$ cubic cm. 4. $452\frac{4}{7}$ Sq cm.; $402\frac{2}{7}$ cubic cm.
 5. 13 cm.. 6. 38.5 m. 7. ` 866.25 8. 12 cm.; $314\frac{2}{7}$ cubic cm. 9. 4 m., $37\frac{5}{7}$ cubic metre,
 ` 211.20 10. 15 m. 11. 14 cm.; 17.5 cm. 12. 74.18 cubic metre (approx); 80.54 Sq
 (approx) m.(approx) 13. (A) (i) c (ii) d (iii) a (iv) d (v) b (B) (i) False (ii) True
 (C) (i) BC (ii) $\frac{3V}{A}$ (iii) 3:1 14. (i) 5 cm. (ii) 2:1 (iii) 3; (iv) $\frac{1}{9}$ (v) 9:8

Chapter - 18

Let us work out -18.1 [Page : 235]

1. (i) similar (ii) similar (iii) equilateral (iv) equal
 2. (i) True (ii) False (iii) True (iv) True (v) False

Let me do it myself : Application : 2. 6 cm.

Let us work out -18.2 [Page : 244]

1. (i) 6 unit (ii) 9 unit (iii) 4 unit 2. (i) no (ii) yes 11. (A) (i) b (ii) c (iii) d (iv) a
 (v) d (B) (i) false (ii) true (C) (i) proportional (ii) equal (iii) proportional
 12. (i) Isocelstriangle (ii) 3:5 (iii) $x = 9$ (iv) $x = \frac{15}{4}$, $y = \frac{16}{3}$ (v) $\frac{3}{1}$

Let us work out -18.3 [Page : 253]

1. First pair is similar 2. $\angle A = 40^\circ$ 3. 42 m.

Let me do it myself : Application : 21. 10.8 cm.

Let us work out -18.4 [Page : 259]

1. 12.8 cm. 2. $BD = 8 \text{ cm.}$, $AB = 4\sqrt{5} \text{ cm.}$ 7. (A) (i) c (ii) b (iii) c (iv) a (v) d
 (B) (i) false (ii) true (iii) false (C) (i) Similar (ii) 5.4 8. (i) 12 cm. ii) 40 cm.
 (iii) 16 cm. (iv) 8 cm. (v) 50°

Chapter - 19

Let me do it myself :

Application : 3. 7 cm.

Let us work out -19 [Page : 265]

1. 693 kg. 2. 20 cm. 3. 5 cm. 4. 3:4 5. 4096 6. 60 7. 19404 cubic cm.
 8. 3:4 9. 8; 4.851 cubic dm. 10. 50.4 cm. 11. 12 dm. 12. 14 cm. 13. 2.1 dm.
 14. 1:2:3 15. $30\frac{1}{3}$ cm. 16. 3.08 cubic metre 0.84 cubic metre 17. (A) (i) a (ii) b (iii) b (iv) d (v) a (B) (i) false (ii) false (C) (i) cylinder (ii) cylinder (iii) equal
 18. (i) 5 cm. (ii) 1 : 2 (iii) 3 : 1 : 2 (iv) $1 : \sqrt{3}$ (v) 2 : 1

Chapter - 20

Let me do it myself :

Application : 2. $\frac{\pi}{6}$ 7. $\frac{\pi}{8}$ 12. $\frac{5}{2}$ radian 16. $62^\circ 32' 33''$ or $\frac{35\pi}{44}$ 18. $94^\circ 27' 24''$

Let us work out -20 [Page : 276]

1. (i) $13^\circ 52'$ (ii) $1^\circ 45' 12''$ (iii) $6' 15''$ (iv) $27^\circ 5'$ (v) $72^\circ 2' 24''$
 2. (i) $\frac{\pi}{3}$ (ii) $\frac{3\pi}{4}$ (iii) $-\frac{5\pi}{6}$ (iv) $\frac{2\pi}{5}$ (v) $\frac{\pi}{8}$ (vi) $-\frac{25\pi}{72}$ (vii) $\frac{47}{160}\pi$ (viii) $\frac{6041\pi}{27000}$
 3. $\frac{2\pi}{5}, \frac{2\pi}{5}, \frac{\pi}{5}$ 4. $81^\circ, 9^\circ$ 5. $100^\circ, \frac{5\pi}{9}$ 6. $75^\circ, 60^\circ; \frac{5\pi}{12}, \frac{\pi}{3}$ 7. $\frac{4\pi}{9}$ 8. $\frac{\pi}{16}$ 9. $75^\circ, \frac{5\pi}{12}$
 10. Two complete revolution clock wise and makes an angle 195°
 11. $\angle ABD = \frac{\pi}{8}, \angle BAD = \frac{3\pi}{8}, \angle CBD = \frac{\pi}{8}, \angle BCD = \frac{3\pi}{8}$ 12. $\frac{2\pi}{3}, \frac{\pi}{6}, \frac{\pi}{6}$ 13. $60^\circ, \frac{\pi}{3}$
 14. (A) (i) d (ii) d (iii) b (iv) b (v) a (B) (i) True (i) False (C) (i) constant
 (ii) $57^\circ 16' 22''$ (approx) (iii) $\frac{5\pi}{8}$ 15. (i) $\frac{\pi}{180}$ (ii) $26^\circ 24' 45''$ (iii) $\frac{5\pi}{18}$
 (iv) 200 cm. (v) $\frac{\pi}{6}$

Chapter - 22

Let us work out -22 [Page : 289]

1. (i) Rightangled triangle | 2. 21 m. 3. 16 cm. 15. (A) (i) c (ii) c (iii) b
 (iv) b (v) c (B) (i) True (ii) False (C) (i) sum (ii) 8 (iii) 5 16. (i) 90° (ii) 26 cm.
 (iii) $5\sqrt{3}$ cm. (iv) 90° (v) 2.4 cm.

Chapter - 23

Let me do it myself :

Application : 3. 15°

Let us work out -23.1 [Page : 295]

1. $\frac{3}{5}, \frac{3}{4}$ 2. $\frac{7}{25}, \frac{24}{25}, \frac{7}{24}, \frac{25}{7}$ 3. $\frac{21}{29}, \frac{20}{29}, \frac{20}{29}, \frac{21}{29}$
 4. $\sin \theta = \frac{24}{25}, \tan \theta = \frac{24}{7}, \operatorname{cosec} \theta = \frac{25}{24}, \sec \theta = \frac{25}{7}, \cot \theta = \frac{7}{24}$
 5. $\tan \theta = \frac{1}{2}, \sec \theta = \frac{\sqrt{5}}{2}$ 7. $\cos A = \frac{8}{17}, \operatorname{cosec} A = \frac{17}{15}$ 8. $\frac{\sqrt{5}}{2}$
 9. (i) False (ii) False (iii) False (iv) True (v) False (vi) True

Let me do it myself :

Application : 9. 60 m. 12. $45^\circ, 45^\circ$

Let us work out -23.2 [Page : 302]

1. 3 2. $\angle ACB = 60^\circ, \angle BAC = 30^\circ$ 3. $BC = 10 \text{ cm.}, AB = 10\sqrt{3} \text{ cm.}$
 4. $PQ=QR=3 \text{ m.}$ 5. (i) $\frac{1}{2}$ (ii) 0 (iii) 1 (iv) $3\frac{1}{3}$ (v) $\frac{6+\sqrt{6}}{3}$ (vi) 0 (vii) 0
 (viii) $\frac{5}{2\sqrt{3}}$ (ix) 1 7. (i) $\frac{1}{\sqrt{3}}$ (ii) $2\frac{2}{3}$ (iii) $\pm \frac{1}{2}$ 8. $\frac{1}{3}, -\frac{1}{3}$ 11. $0^\circ, 60^\circ$

Let me do it myself :

Application : 23. No 26. $\cot \theta = \frac{\cos \theta}{\sqrt{1-\cos^2 \theta}}, \operatorname{cosec} \theta = \frac{1}{\sqrt{1-\cos^2 \theta}}$ 28. $\frac{7}{5}$ 34. $\frac{1}{2}$
 36. $4x^2 + 25y^2 = 9$ 38. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ 41. 2

Let us work out -23.3 [Page : 311]

1. (i) $\frac{5}{7}$ (iii) 2 2. (i) $\operatorname{cosec} \theta = \frac{1}{\sin \theta}, \tan \theta = \frac{\sin \theta}{\sqrt{1-\sin^2 \theta}}$
 (ii) $\operatorname{cosec} \theta = \frac{1}{\sqrt{1-\cos^2 \theta}}, \tan \theta = \frac{\sqrt{1-\cos^2 \theta}}{\cos \theta}$
 3. (i) $\frac{1}{2}$ (ii) $\sqrt{2} + 1$ (iii) 0 (iv) 0 (v) $\frac{17}{13}$ (vi) $\sqrt{2}$ (vii) $\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (viii) $\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}$
 (ix) $\frac{4}{3}$ (x) $\frac{1}{2}$ (xi) $\frac{3}{2}$ (xii) 1 (xiii) $\frac{4}{\sqrt{3}}, \pm \frac{2}{\sqrt{3}}$ or, $\sqrt{3}, \frac{1}{\sqrt{3}}$ (xiv) $\frac{13}{12}$
 4. (i) $\frac{1}{\sqrt{5}}$ (ii) $\frac{1}{\sqrt{3}}$ 5. (i) $\frac{x^2}{4} + \frac{y^2}{9} = 1$ (ii) $25x^2 - y^2 = 9$
 6. (i) $\frac{n}{\sqrt{m^2+n^2}}, \frac{\sqrt{m^2+n^2}}{m}$ 9. (A) (i) c (ii) a (iii) c (iv) d (v) c
 (B) (i) True (ii) False (C) (i) 4 (ii) $\frac{1}{2}$ (iii) $\frac{1}{2}$
 10. (i) 4, 30° (ii) 0 (iii) 12 (iv) 1 (v) 45°

Chapter - 24

Let me do it myself : Application : 2. 1 13. $\frac{p}{\sqrt{p^2+q^2}}$

Let us work out -24 [Page : 316]

1. (i) 1 (ii) 1 (iii) 1 12. (A) (i) b (ii) c (iii) c (iv) b (v) a (B) (i) True (ii) False
(C) (i) $\sqrt{3}$ (ii) 1 (iii) $\cos B$ 13. (i) 1 (ii) 9° (iii) 3 (iv) 1 (v) 9°

Chapter - 25

Let me do it myself : Application : 2. $20\sqrt{3}$ m. 5. 30° 10. 40 m.

Let us work out -25 [Page : 325]

1. $20\sqrt{3}$ m. 2. $3\sqrt{3}$ m. 3. $75\sqrt{3}$ m. 4. 21 m. 5. 8 m., 24 m.
6. (i) $4\sqrt{3}$ m. (ii) $2\sqrt{3}$ m. (iii) $2(3+\sqrt{3})$ m. (iv) 6 m. 7. 20.784 m.
8. 7.098 m. 9. $19\sqrt{3}$ m. 10. 450 m., $150\sqrt{3}$ m. 11. 64 m., $16\sqrt{3}$ m.
12. $125\sqrt{3}$ m., $125\sqrt{2}$ m., in first case 13. 2180 m. 14. 17.19 m. (approx.)
15. 30° 16. 81.96 m. (approx) 17. $25\sqrt{3}$ m. 18. 42 dm., $42\sqrt{3}$ dm.
19. 6 km./h. 20. 11.83 m./sec. (approx.) 21. $200(\sqrt{3}+1)$ m. 22. 60 m., $15\sqrt{3}$ m.
23. (i) $250\sqrt{3}$ m. (ii) $500\sqrt{3}$ m. 24. (A) (i) b (ii) a (iii) c (iv) c (v) a (B) (i) False
(ii) True (C) (i) decrease (ii) equal (iii) greater 25. (i) 30 m. (ii) 60° (iii) 45° (iv) 15° (v) 30°

Chapter - 26

Let me do it myself : Application : 9. 18 12. 36.4

Let us work out -26.1 [Page : 340]

1. 17.43 years (approx) 2. 4.24 3. 20 4. 8 5. 54.72 6. 43.4 years 7. (i) 25 (ii) 40.3
8. (i) 100.89 (approx) (ii) 51.5 9. (i) 75.75 (ii) 36.24 (approx) 10. 20 11. 41.1 years
12. 35.67 (approx) 13. 31.67 (approx) 14. 11.69 (approx)

Let me do it myself : Application : 16. (i) 10 (ii) 12.5 19. 4 21. 29.17 (approx) 24. $x=8$, $y=7$

Let us work out -26.2 [Page : 349]

1. $\sqrt{107}$ 2. 9.5 years 3. 54.5 4. 8 5. 47 kg. 6. 22 mm. 7. 2 8. $\sqrt{53.33}$ (approx)
9. 153.41 cm. (approx) 10. 35.67 (approx) 11. 19.67 (approx) 12. 18.36 (approx)
13. 83 14. 40 15. $x=9$, $y=16$

Let me do it myself : Application : 28. 70.59 (approx)

Let us work out -26.3 [Page : 358]

2. 46.69 (approx) 4. 138.57 (approx)

Let me do it myself : Application : 30. (ii) 9 (iii) 102, 104 (iv) 5 33. 14.4

Let us work out -26.4 [Page : 364]

1. $\sqrt{15}$, $\sqrt{17}$ 2. 131 cm. 3. (i) 4 (ii) 10, 15 4. 4, 6 5. 18.90 years (approx)
6. 21.76 (approx) 7. 17.38 (approx) 8. 78.44 (approx) 9. (A) (i) d (ii) b (iii) d (iv) a (v) b
(B) (i) false (ii) false (C) (i) centraly (ii) a. $\bar{x}$ (iii) equal 10. (i) 20 (ii) 82 (iii) 6 (iv) 27 (v) 30–40

গণিতের পরিভাষান্তর (Terminology of Mathematics)

অর্ধবৃত্ত	- Semicircle	গড়	- Mean
অধমর্গ বা দেনাদার	- Debtor	গোলক	- Sphere
অধিবৃত্তাংশ	- Major Segment	ঘনক	- Cube
অধিচাপ	- Major Arc	চক্রবৃদ্ধি সুদ	- Compound Interest
অনুপাত	- Ratio	চাপ	- Arc
অন্তর্বৃত্ত	- Incircle	ছোটোবৃত্তকলা	- Minor Sector
অর্ধগোলক	- Hemisphere	তল	- Surface
অংশীদারি কারবার	- Partnership Business	তির্যক উচ্চতা	- Slant Height
অবিন্যস্ত তথ্য	- Ungrouped Data	ত্রিকোণমিতি	- Trigonometry
অবনতি কোণ	- Angle of Depression	দ্বিঘাত সমীকরণ	- Quadratic Equation
আয়তঘন	- Rectangular Parallelopiped or cuboid	দ্বিঘাত করণী	- Quadratic Surd
আয়তন	- Volume	দৈর্ঘ্য	- Length
আকার	- Size	দৃষ্টিরেখা	- Line of Sight
আকৃতি	- Shape	নমুনা	- Sample
আয়তলেখ	- Histogram	নিরূপক	- Discriminant
আসল	- Capital/Principal	পরিধি	- Circumference
উত্তমর্গ বা পাওনাদার	- Creditor	প্রস্থ	- Breadth
উচ্চতা	- Height	প্রান্তিকী	- Edge
উত্তরপদ	- Consequent	পূর্বপদ	- Antecedent
উপচাপ	- Minor Arc	পরিসংখ্যা	- Frequency
উন্নতি কোণ	- Angle of Elevation	পরিসংখ্যা বহুভুজ	- Frequency Polygon
উপবৃত্তাংশ	- Minor Segment	পরিসংখ্যা বিভাজন	- Frequency Distribution
উপপাদ্য	- Theorem	তালিকা	- Table
এককেন্দ্রীয় বৃত্ত	- Concentric Circles	পূরক কোণ	- Complementary Angle
একান্তর প্রক্রিয়া	- Alternendo	পার্শ্বতলের ক্ষেত্রফল	- Area of the Lateral Surface
কর্ণ	- Diagonal	বহুপদী সংখ্যামালা	- Polynomial
কল্পিত গড়	- Assumed Mean	বক্রতলের ক্ষেত্রফল	- Area of the Curved Surface
ক্রমযৌগিক পরিসংখ্যা	- Cumulative Frequency	বীজ	- Root
ক্রমযৌগিক পরিসংখ্যা রেখা	- Cumulative frequency curve or ogive	বৃত্ত	- Circle
ক্রম-বিচ্যুতি পদ্ধতি	- Step-deviation method	বৃত্তাকার ক্ষেত্র	- Circular Region
কেন্দ্রীয় প্রবণতার পরিমাপ	- Measures of Central tendency	বৃত্তচাপ	- Arc
গুরুঅনুপাত	- Ratio of Greater Inequality	বৃত্তাংশ	- Segment

বৃত্তকলা	- Sector
বড়ো বৃত্তকলা	- Major sector
বৃত্তস্থ চতুর্ভুজ	- Cyclic Quadrilateral
ব্যাস	- Diameter
ব্যাসার্ধ	- Radius
বিপরীত অনুপাত	- Inverse Ratio
বৈষম্যানুপাত	- Ratio of inequality
বিপরীত বা ব্যস্তপ্রক্রিয়া	- Invertendo
ব্যস্তভেদ	- Inverse variation
বৃত্তের ছেদক	- Secant
বৃত্তীয় পদ্ধতি	- Circular system
বিন্যস্ত তথ্য	- Grouped data
বহুভূয়িষ্ঠক	- Multimodal
ভূমি	- Base
ভূমির ক্ষেত্রফল	- Area of the Base
ভাগ প্রক্রিয়া	- Dividendo
ভেদ	- Variation
ভেদ ধ্রুবক	- Variation constant
ভারযুক্ত গড়	- Weighted mean
মাত্রা	- Dimension
মূলধন	- Capital/Principal
মধ্যসমানুপাতী	- Mean Proportional
মধ্যমা	- Median
যৌগিক অনুপাত বা	- Compound ratio or Mixed ratio
মিশ্র অনুপাত	
যোগ প্রক্রিয়া	- Componendo
যোগ-ভাগ প্রক্রিয়া	- Componendo and Dividendo
যৌগিক ভেদ	- Joint variation
যৌগিক গড়	- Arithmetic mean
রাশিবিজ্ঞান	- Statistics
লঘু অনুপাত	- Ratio of Less Inequality
লম্ববৃত্তাকার চোঙ	- Right circular cylinder
লম্ববৃত্তাকার শঙ্কু	- Right circular cone
লভ্যাংশ বণ্টন	- Distribution of profit
শীর্ষবিন্দু	- Vertex
শঙ্কুর শীর্ষবিন্দু	- Apex
শ্রেণিসীমা	- Classlimit

শ্রেণিসীমানা	- Classboundary
শ্রেণিমধ্যক	- Mid value of the class
যান্ত্রিক পদ্ধতি	- Sexagesimal system
সমাধান	- Solution
সরল সুদ	- Simple interest
সর্বসম বৃত্ত	- Equal circles
সমবৃত্তস্থ	- Conccyclic
সমদ্বিবাহু ট্রাপিজিয়াম	- Isosceles Trapezium
সমগ্রতলের ক্ষেত্রফল	- Whole surface area
সমানুপাত	- Proportion
সমহার বৃদ্ধি	- Uniform rate of growth
সমহার হ্রাস বা অপচয়	- Uniform rate of decrease or depreciation
সংযোজন প্রক্রিয়া	- Addendo
সংখ্যাগুরুমান বা ভূয়িষ্ঠক	- Mode
সংখ্যাগুরুমানের শ্রেণি	- Modal class
সম্পাদ্য	- Construction
স্পর্শক	- Tangent
স্পর্শবিন্দু	- Point of contact
সদৃশ	- Similar
সদৃশতা	- Similarity
সদৃশকোণী	- Equiangular
সাম্যানুপাত	- Ratio of Equality
সাংখ্যমান	- Numerical value
সর্বস্বিমূল	- Amount
সুদ	- Interest
সুদের হার	- Rate of Interest

Teachers' guidelines

- National Curriculum Framework (NCF) 2005 advises that a student should relate her/his life in school to life outside school. This vision recommends that the education of a student goes beyond books since the knowledge limited to books creates a gap between school and society. On the basis of the vision of NCF has designed the present curriculum, syllabus and text book. Moreover, it is recommended that a student's education is not only subject-oriented but that he or she is able to find connections among various topics.
- It is our hope that teachers will use this text book according to the guide lines and recommendation stated below.
- Presently, education is student-oriented; the role of a teacher is just to help the student. Teachers should keep in mind that he or she learns a great deal from her family, surroundings and society. Prior to teaching, care must be taken by the teachers to encourage students to share their previous experience on any given topic; please ensure that the student's thinking is not hindered; that he or she has freedom of thought.
- A textbook is one of the aids in a student's education but not the only one. Many teaching aids should be used to make the learning process interesting.
- In the learning of Mathematics, it is essential that a student uses real and concrete objects to develop abstract ideas. Otherwise mathematics as a subject becomes an object of fear to children.
- Teachers should start a chapter by framing practical problems related to the student's familiar environment. Next, a logical conception regarding the chapter through some practical activities should be formed, if possible. Students should work on the abstract problems after they achieve clarity of reason and logic.
- Teacher should observe how far the students can understand the different units by themselves from their books. If they are unable to understand on their own, only then should they be helped, gradually, in a manner that they are able to solve problems on their own.
- The teachers should introduce each unit to the students in the manner of story-telling, so that they do feel they are being taught.
- Collaborative learning helps the students in their learning. Teachers should encourage this in the classroom.
- The present system curriculum does not attempt to impose text or burden the student with information, but aims at knowledge construction under the good supervision of the teachers. The moment the pupils are able to construct knowledge he/she will gradually try to discover mathematics in things around him/her and thus the subject will become enjoyable.
- Teacher should pay due attention to ensure that the students are efficient at solving problems mentally. If the pupils practise mental-mathematics for every chapter then their thinking, logical reasoning and ability of calculation will improve rapidly.
- At the time of teaching, teacher should prepare a list of all learning possibilities that appear in the chapter. For example, in the chapter of quadratic equations in one variable
 - 1) Concept of quadratic expressions.
 - 2) Concept of quadratic equations.
 - 3) Concept of quadratic equation of the form $ax^2 + bx + c = 0$, ($a \neq 0$)
 - 4) Concept of the choice of $a \neq 0$ in the equation $ax^2 + bx + c = 0$
 - 5) Concept of the type of equation when $a = 0$ in $ax^2 + bx + c = 0$

- 6) Concept of the solution of the quadratic equation by factorisation.
 - 7) Concept of the number of solutions of a quadratic equation.
 - 8) Concept of roots of a quadratic equation.
 - 9) Concept of the formula of Sridhar Acharyya.
 - (10) Concept of the nature of the roots when the discriminant is zero or greater than zero.
 - (11) Concept of the formation of the quadratic equation when the roots are known.
- All units must have open ended questions:
 - e.g., a) Write down a quadratic equation whose two roots are natural numbers.
 - b) Write down one value of the total profit if the ratio of the principals is 3 : 2 : 5.
 - c) Consider the length, breadth and height of a cuboid which are irrational numbers but the volume is a rational number.
 - d) Write down the lengths of three line segments which are the sides of a triangle such that the circumcentre of the triangle lies on a side of the triangle.
 - If the teachers can form more such open ended questions for the students then it will help them to grasp the concepts easily.
 - Student should not learn the process of a sum by rote learning. Teacher should attentive to a student for a problem solving by logical reasoning.
 - Teachers should take care of that students who solve mathematical problems earlier than general do not sit idle. Students who grasp concepts earlier than others, should be helped to further exercise her/his reasoning by assignment of more difficult sums while students who take more time to grasp should be encouraged to improve and expand their reasoning slowly.
1. Syllabus and content of class X also have been changed to harmonize with All India Board and Council's syllabus and contents.
 2. To adjust with the syllabus and contents of class XI some new chapters have been incorporated in class X syllabus and contents.
 3. Arithmetic, Algebra, Geometry, Mensuration, Trigonometry and Statistics are not in separate areas in this class X 'Ganit Prakash' book. Because chapters of arithmetic are related with algebra and chapters of geometry are related with mensuration. For example, in compound interest we use quadratic equations and in trigonometry we need properties of similarity of triangles.

Learners shall apply the rules and formulae after knowing the causes not on the base of rote learning. So different chapters are arranged in this way.
 4. Multiple Choice Type Questions and Short Answer Type Questions are at the end of each chapter. Because learners should know the chapter well first and solve any complex problem easily after that.
 5. Teachers should advise to use the text-book more fruitfully, taking into account the practical problems of the classroom and surroundings, so that the book becomes a source of logical and joyful learning in future.

Lesson Planning

Month	Subject
January	1 QUADRATIC EQUATIONS IN ONE VARIABLE 2 SIMPLE INTEREST 3 THEOREMS RELATED TO CIRCLE
February	4 RECTANGULAR PARALLELOPIPED OR CUBOID 5 RATIO AND PROPORTION 6 COMPOUND INTEREST AND UNIFORM RATE OF INCREASE OR DECREASE
March	7 THEOREMS RELATED TO ANGLES IN A CIRCLE 8 RIGHT CIRCULAR CYLINDER 9 QUADRATIC SURD 10 THEOREMS RELATED TO CYCLIC QUADRILATERAL
April	11 CONSTRUCTION : CONSTRUCTION OF CIRCUMCIRCLE AND INCIRCLE OF A TRIANGLE 12 SPHERE 13 VARIATION 14 PARTNERSHIP BUSINESS 15 THEOREMS RELATED TO TANGENT TO A CIRCLE
May & June	16 RIGHT CIRCULAR CONE 17 CONSTRUCTION : CONSTRUCTION OF TANGENT TO A CIRCLE 18 SIMILARITY
July	19 REAL LIFE PROBLEMS RELATED TO DIFFERENT SOLID OBJECTS 20 TRIGONOMETRY : CONCEPT OF MEASUREMENT OF ANGLE 21 CONSTRUCTION : DETERMINATION OF MEAN PROPORTIONAL 22 PYTHAGORAS THEOREM
August	23 TRIGONOMETRIC RATIOS AND TRIGONOMETRIC IDENTITIES 24 TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLE
September & October	25 APPLICATION OF TRIGONOMETRIC RATIOS : HEIGHTS & DISTANCES 26 STATISTICS : MEAN , MEDIAN , OGIVE , MODE

First Summative Evaluation

Sample Question

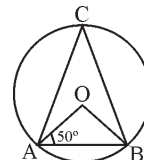
Time : 1 hr 30 m

Full Marks : 40

1. Choose the correct answer :

1×6=6

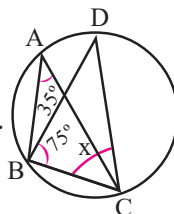
- (i) The compound interest and simple interest are equal at the same rate of interest per annum in
 - (a) 1 year (b) 2 years (c) 3 years (d) 4 years
- (ii) If the ratio of principal and simple interest is 10 : 3, for 5 years, rate of interest per annum will be
 - (a) 3 (b) 30 (c) 6 (d) 12
- (iii) Which is not a quadratic equation given below
 - (a) $2x - 3x^2 = 3x^2 + 5$ (b) $(2x+3)^2 = 2(x^2-5)$
 - (c) $(\sqrt{3}x+2)^2 = 3x^2-7$ (d) $(x-2)^2 = 5x^2+2x-3$
- (iv) If the roots are equal of the quadratic equation $4x^2 + 6kx + 9 = 0$, the value of k is
 - (a) 2 or 0 (b) -2 or 0 (c) 2 or -2 (d) only 0
- (v) In the adjoining figure, $\angle OAB = 50^\circ$ of a circle with centre O and if C is any point on the circle, the value of $\angle ACB$ is (a) 50° (b) 40° (c) 80° (d) 100°
- (vi) AB and CD are two equal chords of a circle with centre O. If $\angle AOB = 70^\circ$, the value of $\angle COD$ is (a) 110° (b) 70° (c) 35° (d) 80°



2. Answer the following questions

2×5=10

- (i) In how many years the interest will be $\frac{3}{5}$ th of principal at the rate of interest 10% simple interest per annum?
- (ii) If $p:q = 5:7$ and $p-q = -4$, what is the value of $(3p-2q)$?
- (iii) Two parallel chords of a circle with centre O lie opposite to the centre. If $AB = 10$ cm, $CD = 24$ cm, and if distance between two chords AB and CD is 17 cm, find the radius of that circle.



- (iv) In the adjoining figure find the value of x.
- (v) If the volume of a cube is 512 cubic cm., find the total surface area of that cube.

3. In how many years will ₹ 1,00,000 amount to ₹ 1,33,100 at 10% compound interest per annum?

or

The length of tree increases by 20% in every year ; if the present length of the tree is 28.8 metre, let us find what was the length of tree 2 years before ?

5

4. **Solve. [any one]**

3×1=3

(i) $(2x+1) + \frac{3}{2x+1} = 4, (x \neq -\frac{1}{2})$

(ii) $\frac{1}{(x-2)(x-4)} + \frac{1}{(x-4)(x-6)} + \frac{1}{(x-6)(x-8)} + \frac{1}{3} = 0, (x \neq 2, 4, 6, 8)$

5. Sulata drew a right-angled triangle having the hypotenuse of 13 cm. length, and difference between other two sides is 7 cm. Find the length of other two sides of right-angled triangle drawn by Sulata.

or

4

If a two digit positive number is multiplied by its unit digit, the value of product is 189 and the tens digit is twice of its unit digit, find the unit digit of the number.

6. If a, b, c, d are in continued proportion, prove that $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$

or

If $a = \frac{\sqrt{5}+1}{\sqrt{5}-1}$ and $ab = 1$, find the value of $\left(\frac{a}{b} + \frac{b}{a}\right)$

3

7. Prove that the angle which an arc of a circle subtends at the centre, is double of any angle subtended by it on the circle.

or

Prove that line drawn through the centre of circle bisects a chord, which is not a diameter, is perpendicular to the chord.

5

8. A reservoir of 21 dcm. length, 11 dcm. breadth and 6 dcm. depth is filled half with water. How much dcm will the water level be raised, if 100 iron balls each of which having 21 cm diameter be completely immersed into it ?

or

The ratio of height of a right circular solid cylinder and length of radius of base is 3 : 1. If the volume of the cylinder is 1029π cubic cm, what will be the total surface area of the cylinder?

4

Let 's Match

1. (i) a (ii) c (iii) c (iv) c (v) b (vi) b 2. (i) 6 year (ii) 2 (iii) 13 cm (iv) 70° (v) 384 sq cm
3. 3 year or, 20 m 4. (i) $x=0$ and $x=1$ (ii) $x=5$ and $x=5$ 5. 12 cm and 5 cm. or, 3 6. or 7 8.
3 dm. or, 1232 sq cm

Second Summative Evaluation

Sample Question

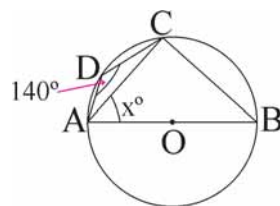
Time : 1 hr 30 m

Full Marks : 40

1. Choose the correct answer :

1×7=7

- (i) Sum of two roots of the equation $5x^2 - 2x + 1 = 0$ is
 (a) 2 (b) 1 (c) $\frac{1}{5}$ (d) $\frac{2}{5}$
- (ii) Samir invests ` 4000 for 3 months and Amita invests ` 3000 for 5 months in a business. Profit will be distributed in the ratio
 (a) 4 : 3 (b) 3 : 4 (c) 4 : 5 (d) 5 : 4
- (iii) $x \propto \frac{1}{y}$ and $y = \frac{2}{5}$ when $x = 5$; $x = \frac{1}{6}$, the value of y is
 (a) $\frac{1}{3}$ (b) 6 (c) 12 (d) 18
- (iv) In the adjoining figure, AB is the diameter of a circle with centre O. If $\angle ADC = 140^\circ$ and $\angle CAB = x^\circ$, the value of x is (a) 80° (b) 40° (c) 50° (d) 30°
- (v) If length of tower and length of the shadow of a stick having length 20 meters are 50 meters and 10 meters respectively, the length of tower is
 (a) 120 m. (b) 250 m. (c) 25 m. (d) 100 m.
- (vi) If height of a solid right circular cone is 15 cm and the length of diameter of the base is 16 cm. then area of lateral surface is
 (a) 120π sq cm (b) 240π sq cm (c) 136π sq cm (d) 130π sq cm
- (vii) If the total surface area of solid hemisphere is 147π , the length of radius of the base of hemisphere is (a) 14 cm. (b) 7 cm. (c) 21 cm. (d) 7.5 cm.



2. Answer the following questions

2×4=8

- (i) If $A \propto \frac{1}{C}$ and $C \propto \frac{1}{B}$, find the variation relation between A and B

(ii) Fill in the blanks.

- (a) If a straight line intersects the circle at the two points, this straight line is called _____ of the circle.
- (b) The length of radii of two circles are 4cm and 5cm. If two circles touch each other externally, the distance between two centres is _____ cm.
- (iii) If the surface area of a sphere is S and volume is V, write the value of $\frac{S^3}{V^2}$

(iv) The lateral surface area of a cone is $\sqrt{5}$ times of its base area. Find the ratio between height of the cone and length of radius of its base.

3. Shova, Masud and Rabeye started a business with capitals ₹ 3000, ₹ 3500, and ₹ 2500 respectively. They have decided to divide $\frac{1}{3}$ part of total profit equally and the remaining profit in the ratio of their capitals invested. What will be the share of profit of each of them if the amount of profit at the end of year is ₹ 810. 5

or

Shakil and Mohuya jointly started a business with capitals of ₹ 30,000 and ₹ 50,000 respectively. After 6 months Shakil invested ₹ 40,000 more in a business, but Mohuya withdrew ₹ 10,000 for personal need. If the profit amount is ₹ 19000 at the end of year, how much share of profit each of them should get?

4. If $a \propto b$ and $b \propto c$, show that $a^3b^3 + b^3c^3 + c^3a^3 \propto abc (a^3 + b^3 + c^3)$ 3

or

Solve : $\frac{x-2}{x+2} + 6 \left(\frac{x-2}{x-6} \right) = 1, (x \neq -2, 6)$

5. Prove that if two circles touch each other externally, then the point of contact will lie on the line joining the two centres. 5

or

Prove that if a perpendicular is drawn from the vertex containing the right angle of a right-angled triangle on the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and are similar to each other.

6. $\angle BAC = 1$ right angle of a right-angled triangle ABC. AD is perpendicular on hypotenuse BC. Prove that $\frac{\Delta ABC}{\Delta ACD} = \frac{BC^2}{AC^2}$ 3

7. Draw a triangle of which the length of one side is 6.8 cm and measures of two angled adjacent to this side are 60° and 75° and draw the circumcircle of this triangle. 5

or

Draw a circle with centre O having radius of 2.5 cm. length. Take a point P outside the circle at a distance of 5 cm. from 'O'. Draw two tangents to the circle from the point P.

8. Three spheres made of copper with radii of 3 cm., 4 cm., and 5 cm. are melted and a large sphere is made. What is the length of radius of the large sphere. 4

or

If length of radius of base is 12 metre and height is 5 meter, what will be the cost to make a tent in the shape of right circular cone at the rate of ₹ 3.50 per sq meter.

Let's match

1. (i) d (ii) c (iii) c (iv) c (v) d (vi) c (vii) b 2. (i) $A \propto B$ (ii) a. b. 9 cm (iii) 36π
(iv) 2:1 3. ₹ 270, ₹ 300, ₹ 240 or, Shakil gets ₹ 10,000 and Mohuya gets ₹ 9,000
4. or, $x=0$ and $x=\frac{2}{3}$ 8. 6 cm or 1716 cm

Third Summative Evaluation

Sample Question

Time : 3 hr

Full Marks : 90

1.A. Choose the correct answer : (Answer all Questions)

1×6=6

- (i) Interest on principal amount 'b' at the rate of simple interest a% per annum for C months is
(a) $\frac{abc}{1200}$ (b) $\frac{abc}{100}$ (c) $\frac{abc}{200}$ (d) $\frac{abc}{120}$
- (ii) If the roots of quadratic equation $kx^2 - 5x + k = 0$ are real and equal, the value of k is
(a) ± 5 (b) $\pm \frac{5}{2}$ (c) $\pm \frac{2}{5}$ (d) ± 2
- (iii) If $\alpha = 90^\circ$, $\beta = 30^\circ$, the value of $\sin(\alpha - \beta)$ is
(a) 1 (b) $\frac{1}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{2}}$
- (iv) The tangent PA has drawn to a circle with centre O from the external point P. If PA = 4 cm, OP = 5 cm, the radius of circle is
(a) 5 cm. (b) 4 cm. (c) 6 cm. (d) 8 cm.
- (v) If the ratio of volumes of two cubes is 8 : 125, the ratio of total surface area of two cubes will be
(a) 2 : 5 (b) 4 : 25 (c) 4 : 5 (d) 2 : 25
- (vi) The median of 8, 15, 10, 11, 7, 9, 11, 13, 16 is
(a) 15 (b) 10 (c) 11.5 (d) 11

B. Fill in the blanks (Answer any five)

1×5=5

- (i) If the rate of compound interest of the principal 'p' is 2r% per annum and the interest is compounded half yearly then the amount in n years is $p(1 + \frac{r}{2})^{2n}$.
- (ii) 50% of a = 20% of b = 25% of c then a : b : c = 2 : 5 : _____
- (iii) If $\alpha = 90^\circ$ and $\beta = 30^\circ$ then the value of $\sin(\alpha - \beta)$ is _____
- (iv) If a straight line intersects a circle at two points then the straight line is called _____
- (v) If the radius of a solid hemisphere is r unit then the whole surface area will be _____ sq. unit.
- (vi) When a distribution has two modes then it is called _____.

C. Write true or false (Answer any five)

1×5=5

- (i) In a partnership business the ratio of the capitals of Anware, Amal and David is 3:2:5 then the ratio of profit will be 5:2:3.

- (ii) A is directly proportional to B and B is directly proportional to C then A is directly proportional to C.
- (iii) If $\alpha + \beta = 90^\circ$ then $\sin^2\alpha + \sin^2\beta = 1$
- (iv) If $\angle A = \angle D$, $\angle B = \angle F$ and $\angle C = \angle E$ of two triangles ABC and DEF then $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
- (v) If the ratio of radii of two solid spheres is 2:3 then the ratio of whole surface area of the spheres is 2:3 then the ratio of whole surface area of the spheres will be 4:9 .
- (vi) Median of a data can be determined from the graph of cumulative frequency.

2. Answer the questions (Answer Any ten) :

2×10=20

- (i) If a sum of money at 5% compound interest per annum for 2 years becomes ₹ 615, find the principal amount.
- (ii) The price of an object is decreased by 10% in every year. The present price of that object is ₹ 182. Find the price of that object in 2 years ago.
- (iii) x and y are two variables. Corresponding values related to them are :
 $x = 6, y = 9$; $x = 4, y = 6$; $x = 12, y = 18$; $x = 3.6, y = 5.4$; Determine the relation of variation between x and y with reasons.
- (iv) If the two roots of the quadratic equation $x^2 + 8x + 2 = 0$ are α and β find the value of $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$
- (v) A cylinder and a sphere are of equal radii, their volumes are also equal. Find out the ratio of the diameter and height of the cylinder.
- (vi) If the volume of a solid hemisphere is 144π , what is its diameter ?
- (vii) The distance of a chord from the centre of a circle of radius 17cm is 8cm. Find the length of the chord.
- (viii) A circle with centre O has a diameter AOC. B is a point of the circle and $\angle ACB = 50^\circ$; AT is a tangent to the circle at the point A where the points B and T are on same side of the diameter AC. Find the value of the $\angle BAT$.
- (ix) In the triangle ABC, the straight line which is parallel to the side BC intersects the sides AB and AC at the points D and E respectively. If $AD:DB = 5:4$ then find $AE:AC$.
- (x) If $\tan\theta \cdot \tan 2\theta = 1$, find the value of $\cos 2\theta$.
- (xi) The length of radius of a circle is 14 cm. Find the circular measures of the angle at the centre by an arc of this circle 66 cm in length.

(xii)

x_i	3	5	8	9	11	13
f_i	6	8	5	p	8	4

If the mean of the above table is 8, find the value of p.

3. At what rate of interest will ₹ 60,000 amount to ₹ 69,984 for 2 years. 5

or

At the starting of the year Arun and Ajoy started a business jointly with capitals of ₹ 24000 and ₹ 30,000 respectively. After some month Arup invested ₹ 12000 more in that business. If the profit after the end of year was ₹ 14030 and Arun got profit share of ₹ 7130. Determine after how many months Arun invested the amount.

4. Solve : $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$, $[x \neq 0, -(a+b)]$ 3

or

The velocity of a boat is 8 km/h in the still water. A boat takes 5 hours to cover 15 km downstream and 6 km upstream. Find the velocity of current.

5. Simplify : $\frac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} - \frac{3\sqrt{3}}{\sqrt{2}+\sqrt{5}} + \frac{2\sqrt{2}}{\sqrt{3}+\sqrt{5}}$ 3

or

If 5 men can harvest jute cultivated in 10 bighas of land in 12 days, how many farmers can harvest jute cultivated in 18 bighas in 9 days—Determine this by applying variation theorem.

6. If, $\frac{ay-bx}{c} = \frac{cx-az}{b} = \frac{bz-cy}{a}$, prove that $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ 3

or

if a, b, c are in continued proportion, prove that $a^2b^2c^2 \left(\frac{1}{a^3} + \frac{1}{b^3} + \frac{1}{c^3} \right) = a^3 + b^3 + c^3$

7. State and prove Pythagoras theorem. 1+4

or

Prove that the opposite angles of a cyclic quadrilateral are supplementary. 5

8. ABCD is a trapezium of which $AB \parallel DC$; a straight line parallel AB intersects to AD and BC at the points E and F respectively. Prove that $AE : ED = BE : FC$. 3

or

In a cyclic quadrilateral ABCD if $AB=DC$, prove that $AC=BD$.

9. Find the value of $\sqrt{22}$ geometrically. 5

or

Draw the triangle having the side of 6.5 cm. length and measures of two angles adjacent to this side are 50° and 75° . Draw the in-circle of that triangle.

10. Answer any two questions. 3×2=6

(i) Find the value of x

$$(x+1) \cot^2 \frac{\pi}{6} = 2 \cos^2 \frac{\pi}{3} + \frac{3}{4} \sec^2 \frac{\pi}{4} + 4 \sin^2 \frac{\pi}{6}$$

(ii) Show that, $\operatorname{cosec}^2 22^\circ \cot^2 68^\circ = \sin^2 22^\circ + \sin^2 68^\circ + \cot^2 68^\circ$

(iii) If $\cos \theta = \frac{x}{\sqrt{x^2+y^2}}$, show that $x \sin \theta = y \cos \theta$

11. Answer any two questions.**4×2=8**

- A hemisphere and a cone have an equal base and their heights are also equal, find the ratio of their volumes and ratio of their curved surface areas.
- Find the volume of a right circular cone obtained from a wooden cube of 4.2 dcm edge by wasting minimum quantity of wood.
- Height of a right circular cylinder is twice of its radius. If the height would be 6 times of its radius, then the volume of the cylinder would be greater by 539 cubic dm. Find the height of the cylinder.

- 12.** The angle of elevation and the angle of depression of the top and foot of the monument, when observed from a point on the roof of a five-storied building of Mihir are 60° and 30° respectively. If the height of building is 16 metre, what will be the height of the monument ?

5**or**

When the angle of elevation of the sun changes from 45° to 60° , the length of shadow of a telegraph post changes to 4 metre. When the angle of elevation of the sun is 30° , then find the length of shadow of the telegraph post.

13. Answer any two questions.**4×2=8**

- Shakil babu would bring mangoes kept in 50 packing boxes in the retail market. These boxes contained varying number of mangoes. The following is the distribution of mangoes according to the number of boxes.

Number of mangoes	50 - 52	52 - 54	54 - 56	56 - 58	58 - 60
Number of boxes	5	14	16	10	5

Find the mean number of mangoes kept in 50 packing boxes.

- Find the mean of heights of students from the following distribution table :

Height (cm)	135-140	140-145	145-150	150-155	155-160	160-165
Number of students	7	12	20	24	20	17

- Find the mode of the following frequency distribution table :

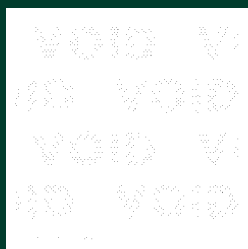
Class interval	0-5	5-10	10-15	15-20	20-25	25-30	30-35
Frequency	5	12	18	28	17	12	8

Let's Match

1.A (i) a (ii) b (iii) c (iv) c (v) b (vi) d **1.B** (i) 100 (ii) 4 (iii) 90° (iv) transversal (v) $3\pi^2$ (vi) Bi **1.C.** (i) F (ii) T (iii) T (iv) F (v) T (vi) T **2.** (i) 6000 (ii) 200 (iii) Direct proportion ratio of x and y is constant (iv) -4 (v) 3:2 (vi) 12 cm. (vii) 30 cm (viii) 50° (ix) 5:9 (x) $\frac{1}{2}$ (xi) $\frac{3\pi}{2}$ (xii) 10 **3.** 8% or, 5 months later **4.** $x=-a$ and $x=-b$ or, $1\frac{3}{5}$ km/h **5.** 0 or, 12 person **10.** (i) 0 **11.** (i) 2:1, 2:1 (ii) 19404 cc. (iii) 7 dm **12.** 64 m or, $6(\sqrt{3}+1)$ m. **13.** (i) 54.84 (ii) 152.29 cm. approx. (iii) 17.38 approx.



सत्यमेव जयते



No tree was cut for printing this book

No.

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